Priceless Consumption *

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ABSTRACT

Priceless consumption (PC) is a type of consumption that is not available in the marketplace and, therefore, is absent from aggregate consumption measures. We propose an estimation methodology to recover PC from its effects on the composition of observable consumption. The estimation shows that PC is economically significant: It is highly volatile and has become increasingly scarce and valuable over the past decades. As an application, we show that using the recovered true aggregate consumption series, which includes PC, significantly improves the fit of the standard consumption-CAPM with power-utility.

*July 2020. We would like to thank Marianne Andries (Adam Smith discussant), Michael Brennan, Max Croce, Winston Dou, Wayne Ferson, Gary Gorton, Christian Julliard, Toomas Laarits (AFA discussant), Sydney Ludvigson (MFS discussant), Igor Makarov, Gregor Matvos, Yao Zeng, and Motohiro Yogo for helpful comments and Erica Jiang for excellent research assistance. Special thanks to Mindy Xiaolan for her contribution in an early stage of this project. We thank Alexi Savov for sharing the municipal solid waste series with us. We would also like to thank seminar participants at the University of Michigan (Ross School of Business), the University of Washington (Foster School of Business), the London School of Economics, Cambridge University (Judge Business School), University of Oxford (Saïd Business School), University of Manchester (Alliance Manchester Business School), Warwick University (Warwick Business School), Bocconi University, the UT Austin finance department brown bag seminar, the 2019 Adam Smith Asset Pricing Conference, the 14th Macro Finance Society Workshop, and the 2020 American Finance Association Meeting for the many helpful comments.

The starting point of this paper is the fact that not everything with consumption value is accounted for in aggregate consumption measures.[1] We denote *priceless consumption* (PC) all non-pecuniary drivers of utility, which, although being left out of aggregate measures, do affect the composition of aggregate marketable consumption.[2] This paper proposes a methodology to recover the PC series, which are latent, from observable consumption data. The recovered series show that: (i) The shadow value of PC is significant, highly volatile, and has been increasing over the decades; (ii) the rising value of PC represents bad news for households since it is driven by the growing scarcity of this good, which complements other types of consumption; and (iii) PC contains information about aggregate wellbeing that is priced in financial assets yet absent from official measures of national consumption in the U.S.

This paper shows that we can recover information about latent PC through its effect on the dynamics of the composition of marketable consumption. To illustrate the idea, consider the case of sunshine.[3] One could learn something about the amount of sunshine consumed (i.e., enjoyed) by a group of individuals over a given month by observing changes in their relative expenditures on sunglasses, which complement sunshine, and umbrellas, which substitute it. Extending this idea beyond a specific case is challenging because we do not know ex-ante what PC is made of, and, therefore, we do not know ex-ante how it affects the composition of marketable consumption. To address this challenge, we propose a flexible structural approach that allows us to recover PC and quantify its effects on the composition of marketable consumption simultaneously.

We propose a structural model based on an endowment economy in which agents derive

[1]See Kennedy (1968) and Stiglitz, Sen, and Fitoussi (2010) for discussions of the limitations of using national account measures as indicators of societal wellbeing.

[2]Illustrative examples of non-pecuniary drivers of utility considered in the literature are: the quality of non-working time (Becker (1965)), education (Lazear (1977) and Eckstein and Wolpin (1999)), work (Akerlof (1982) and Hagedorn and Manovskii (2008)), good health (Jack and Suri (2014)), social connections (Ambrus, Mobius, and Szeidl (2014)), religious activities (Azzi and Ehrenberg (1975)), and charitable donations (Benabou and Tirole (2011)).

[3]Although sunshine is most likely not a significant component of latent consumption, it satisfies the defining properties of PC: Sunshine is nonmarketable, provides consumption value (at least during cold winter days), and, as we argue here, affects the composition of marketable consumption.

utility from the consumption of differentiated goods, each produced by a Lucas tree, and have access to a possibly complete set of financial assets. One of the trees provides the PC good, which differs from the other goods in that it is *nonmarketable* (i.e., it cannot be traded or otherwise transferred between agents) and is thus latent. The solution of the representative agent's problem determines a demand system spanning all available economic goods.

We find two conditions in which the parameters of the model can be identified and the dynamics of PC recovered without the use of any information about PC. The first necessary condition for the recoverability of PC from observable data is related to the smallest partition of the set of marketable goods in the economy. We find that three is the minimum number of different marketable goods required.[4]

The second necessary condition for the recoverability of PC from observable data is the use of a sufficiently flexible functional form for the consumption aggregator. We assume that the consumption aggregator is parameterized by a transcendental logarithmic (translog) function (Christensen, Jorgenson, and Lau, 1975). The key feature of the chosen aggregator, which is absent from the more commonly used constant elasticity of substitution (CES) class of aggregators, is that it allows for variation in EOS across the different goods. In particular, a strict variation across the EOS between PC and the marketable goods implies that the relative prices and relative expenditure shares of the marketable goods are differently affected by changes in the quantity of PC, which we show to be necessary for the recoverability of PC.[5]

The broad procedure to estimate the demand system is as follows. To map the consumption goods from the model to the consumption data from the U.S. Bureau of Economic Analysis (BEA), we use the traditional three-way partition of the set of marketable consumption goods: durables, nondurables, and services. We use the BEA series spanning the

[4]We elaborate on this requirement in Section 1.3.

[5]As we discuss in Section 1.5.2, a CES aggregator is overly restrictive for our methodology since it implies that every pair of goods in the economy has the same EOS value. The Cobb-Douglas is the most restrictive member of the CES class and implies that any pair of goods has unit EOS. Under a CES aggregator, the relative prices and thus the expenditure shares of marketable goods are unaffected by PC, which, therefore, cannot be recovered.

period from 1929 to 2018 on the quantity, price, and expenditure share of each of these goods. The existence of the durable good implies that the parameters of the translog consumption aggregator and the power utility function must be estimated simultaneously. For this reason, we also include financial moments in the estimation. We use a two-stage GMM methodology that targets the shares of the marketable consumption goods, the real risk-free rate, and the market, size, and value factors from Fama and French (1993). We then use the estimated parameter values and the data on the marketable consumption goods to obtain the model-implied series for the quantity and shadow value of PC for the period 1930 to 2018.

The recovered latent series from the estimation indicates that the shadow expenditure share of PC is economically sizeable and has increased over time. At the start of the sample, PC has a negative shadow expenditure share of marketable consumption of around -11% in 1930.[6] After a steady increase throughout the sample, the shadow expenditure share of PC is around 63% of the marketable consumption expenditure.[7] Translating these shares into real 2018 U.S. Dollars, the per-capita shadow value of PC has increased from around −$800 in 1930 to around $27,000 in 2018. As a reference, the marketable per-capita real expenditures increased from around $7,000 in 1930 to around $43,000 in 2018.

The increase in the shadow expenditure share of PC over the sample period is in stark contrast to the negative trend in the quantity of PC. The increase in the shadow expenditure share of PC is driven by a drastic rise in the shadow price of PC. The price increase dominates the quantity decrease and thus leads to an increase in the shadow value of PC because the PC good has no strong substitute among marketable goods.[8] The complementarity between

[6] In contrast with the CES case, the solution of a demand system based on a translog aggregator does not rule out negative prices. For a marketable good, a negative price would represent an arbitrage opportunity and thus an anomaly in a perfect market. For a nonmarketable good, however, a negative shadow price is not only possible but also economically meaningful. To illustrate this last claim, consider sunshine again: A rational and fully informed individual might be willing to pay to receive an additional hour of sunshine in a cold winter day and also willing to pay to avoid an extra hour of sunshine during a hot summer day.

[7] In terms of shares of the shadow total consumption expenditure, which includes both marketable and latent consumption, the figures are -13% in 1930 and 38% in 2018.

[8] We document in Section E of the accompanying Online Appendix the result from an estimation of the elasticities of substitution between PC and the marketable goods.

PC and marketable goods implies that the drop in the quantity of PC has a positive effect on the shadow expenditure share of PC.

The decline in the quantity of PC implies that the aggregate consumption reported in the U.S. national accounts (henceforth denoted *Market-C* since it only includes marketable consumption) overestimates the growth of the true aggregate consumption (henceforth denoted *Total-C* since it includes all types of consumption, marketable or otherwise) between 1930 and 2018.[9] The reported Market-C increased at an annualized rate of 4.68% (3.58% for per-capita Market-C). In contrast, the Total-C declined, with an annualized growth rate of -0.15% (-1.26% for per capita consumption).

The definition of PC as the universe of non-pecuniary drivers of utility that affect observable consumption allows it to be measured but does not answer the question of what PC is made of specifically. Although a definitive answer to this question is impossible, we investigate some potential candidate sources of PC. Specifically, we analyze the relation between PC and survey data on how households spend their time, and with a number of wellbeing and distress indicators. The findings of the investigation indicate that social forms of leisure, job security, and financial stability are likely components of PC, while nonsocial forms of leisure, home production, and work are not.

The growth rate of Total-C, which includes PC, is significantly more volatile than the growth rate of Market-C. A natural question is whether the higher volatility of the growth rate of the broader measure of consumption represents, as we should expect under our working hypothesis, a component of systematic risk that is missing from aggregate measures of consumption growth. To investigate this question, we use the recovered series of Total-C to check, in the spirit of the equivalent exercise in Savov (2011), whether the standard consumption-based CAPM (i.e., the C-CAPM from Breeden, Gibbons, and Litzenberger (1989)) with power utility can rationalize the average market excess return (i.e., risk premia)

[9] While Total-C represents a consumption quantity, Market-C represents both a quantity and a value. While Market-C is constructed from the value of marketable consumption (i.e., the aggregate real expenditure in durables, nondurables, and services), Market-C is also widely used as proxy for the *quantity* of aggregate marketable consumption. Samuelson and Swamy (1974) discuss the assumptions under which the real consumption expenditure is a valid quantity index.

with a plausible coefficient of risk aversion.

We estimate different specifications of the C-CAPM and find that using the Total-C series, which includes PC, significantly improves the performance of the standard C-CAPM with power utility. In particular, we find that the C-CAPM based on Total-C can explain the equity risk premium with a coefficient of relative risk aversion of around 12, which is significantly lower than that of the C-CAPM based on the standard aggregate measures of (marketable) consumption (between 26 and 47). The coefficient that we estimate is also lower than that of the C-CAPM based on the quantity of municipal solid waste series as a proxy for the aggregate consumption quantity proposed by Savov (2011) (around 17). These findings are consistent with the hypothesis that PC contains information about systematic, priced risk that is absent from the Market-C.

Related Literature. This paper is related to the vast literature in economics and finance in which consumption decisions, and hence consumption data, take a central role. More specifically, our empirical application is closely related to the consumption-based approach to asset pricing. Many papers in this literature use Market-C or its components (e.g., durables, nondurables, and services).[10] We argue in this paper that useful information about the true aggregate consumption (i.e., Total-C), which includes PC, is left out from the aggregate consumption reported in national accounts (i.e., Market-C). We present support for this statement by showing that our estimated PC series contains information relevant for asset pricing.

A closely related paper is Savov (2011), in that our work also attempts to address measurement problems of consumption data. Savov (2011) proposes a proxy for aggregate consumption quantity (i.e., municipal solid waste) that is less subject to smoothing in its construction than Market-C and that possibly also captures nonmarket consumption that produces waste, such as that from home production.[11] We do not focus on issues related to the

[10] See, for example, Cochrane (2011) for an overview of the consumption-based asset pricing literature.

[11] In the spirit of Savov (2011), Da, Yang, and Yun (2015) propose the use of residential electricity as a proxy for household consumption in an asset pricing model.

marketable consumption measures reported in the national accounts. We focus instead on an entire class of consumption that is latent and altogether missing from aggregate measures of economic activity.

This paper also contributes to the strand of the asset pricing literature that studies heterogeneous goods. Examples of this literature include Ait-Sahalia, Parker, and Yogo (2004) (luxury goods and basic goods), Yogo (2006) (durables and nondurables), Piazzesi, Schneider, and Tuzel (2007) (housing and non-housing consumption), Binsbergen (2016) (several consumption goods).[12] Our paper also considers the asset pricing implications of preferences that imply imperfect substitutability across heterogeneous goods. The main departure point from this literature is to consider that a significant fraction of the Total-C is nonmarketable and thus unaccounted for in Market-C. In addition, we use the dynamics of heterogeneous marketable goods to extract information about the latent portion of Total-C and not directly in our asset pricing tests.

An additional departure point from the heterogeneous-goods asset pricing literature is in the modeling choice of the consumption aggregator. We show in the paper that the CES consumption aggregator, which is the most commonly used in the literature (e.g., Yogo (2006)), is overly restrictive and cannot be used to retrieve information about latent consumption from the data. Moreover, a CES aggregator is unable to explain the joint dynamics of durable goods, nondurable goods, and services. Our results show that the translog consumption aggregator, in particular when combined with PC, addresses the limitations of the CES aggregator at explaining the dynamics of the marketable consumption goods. In particular, we show that the scaled mean absolute error between empirical and model-implied expenditure shares decreases from an average of around 20% with the CES aggregator (without PC) to a significantly better average fit of around 3% with the translog aggregator with PC.

[12] A less directly related yet important strand of the literature studies production-based asset pricing models that consider the production of heterogeneous goods. A couple of illustrative examples from this literature are Gomes, Kogan, and Yogo (2009) and Gorodnichenko and Weber (2016).

1 Model

1.1 Setup

The model is based on an endowment economy populated by a unit mass of consumers that are rational and fully informed. Since consumers are identical in terms of preferences and outcomes, we do not index individuals to denote per-capita variables. The economy produces an output flow $\mathbf{F}_t \equiv \{F_t^{\mathrm{D}}, F_t^{\mathrm{N}}, F_t^{\mathrm{S}}, F_t^{\mathrm{L}}\} \in \mathbb{R}^4_{\geq 0}$ of goods from four productive technologies: a durable good (D), a nondurable good (N), a "service good" (S), and a latent PC good (L). The PC good is evenly distributed across individuals and is nonmarketable. With the exception of the durable good, all other goods are perishable and cannot be stored for later use. The stock of the durable good follows the law of motion given by

$$Q_t^{\mathrm{D}} = (1-\delta)Q_{\mathrm{t}-1}^{\mathrm{D}} + F_t^{\mathrm{D}}, \tag{1}$$

where $0 < \delta \leq 1$ is the annual depreciation rate of the durable good.

Consumers' preferences are represented by the function $\mathcal{U}[\mathbf{Q}_t]$ over the consumption of the four goods, $\mathbf{Q}_t \equiv \{Q_t^{\mathrm{D}}, Q_t^{\mathrm{N}}, Q_t^{\mathrm{S}}, Q_t^{\mathrm{L}}\} \in \mathbb{R}^4_{\geq 0}$.[13] We assume that the function $\mathcal{U}[.]$ has the nested structure $\mathcal{U}[\mathbf{Q}_t] \equiv \mathrm{U}[\mathrm{C}[\mathbf{Q}_t]]$. The outer function $U[.]$ represents preferences over the current period's total consumption, $C_t \equiv \mathrm{C}[\mathbf{Q}_t]$. The inner function, $\mathrm{C}[\mathbf{Q}_t]$, which we denote *consumption aggregator*, represents preferences over the four individual consumption goods.

Except for the non-marketability of the PC good, the economy has perfectly competitive product and financial markets. The product market prices of the nondurable and service goods, P_t^{N} and P_t^{S}, the shadow price of the PC good, P_t^{L}, and all other monetary values in the model are expressed in terms of units of the durable good, which is the numeraire good in the economy. The price of the durable good is omitted in what follows since $P_t^{\mathrm{D}} = 1$.

[13] Although we refer to a Q_t^{D} as the time-t quantity *consumed* of the durable good, this terminology should be qualified: Consistent with the law of motion in Equation (1), the quantity Q_t^{D} is not used up every period and instead provides a flow of durable-good services to consumers.

Financial markets are, as in Breeden (1979), effectively complete, frictionless, and free of arbitrage opportunities. We denote financial markets as *effectively complete* in the sense that individuals can trade any financial asset that they would optimally want to trade, although they might not be able to trade other assets (i.e., assets that would not be traded even if markets were complete). We choose the assumption of effective completeness to allow for the possibility that the PC tree itself cannot be owned and traded. However, making the stronger assumption of market completeness would not affect the model solution, estimation, or any of the results presented.

1.2 Individual Consumer's Problem

Here we characterize the optimization problem of the individual consumer. The consumer's maximization problem is given by

$$\max_{\{\mathbf{Q}_{t+s},\mathbf{A}_{t+s}\}_{s=0}^{\infty}} \sum_{s=0}^{\infty} \beta^s \, \mathbb{E}_t \left[\mathrm{U}[C_{t+s}]\right], \tag{2a}$$

subject to the law of motion in Equation (1) and to the constraints

$$\sum_{j=0}^{n} A_{j,t} + X_t^{\text{DNS}} + \Delta_t^{\text{L}} P_t^{\text{L}} \leq \sum_{j=0}^{n} A_{j,t-1} R_{j,t}, \tag{2b}$$

$$Q_t^{\text{N}} \leq F_t^{\text{N}}, \tag{2c}$$

where $A_{j,t}$ is the amount invested in financial asset j at time t, $R_{j,t}$ is the gross return of the asset between periods $t-1$ and t, $0 < \beta \leq 1$ is a constant that represents the consumer's patience rate,

$$X_t^{\text{DNS}} \equiv F_t^{\text{D}} + Q_t^{\text{N}} P_t^{\text{N}} + Q_t^{\text{S}} P_t^{\text{S}}, \tag{2d}$$

is the time-t expenditure in the marketable consumption goods, Δ_t^{L} and P_t^{L} are the shadow quantity traded and the shadow price of the PC good. Even though the PC good is nonmarketable, the constraints in Equations (2b) and (2c) include the shadow quantity traded Δ_t^{L}, which represents an infinitesimal value used to define the shadow price P^{L}. Specifically, the shadow price P^{L} is defined as the price level for the PC good that would make the individual optimally refrain from trading it (i.e., $\Delta^{\text{L}} \to 0$) if doing so were possible.

1.3 General Solution

Following Lucas (1978), we map the solution of an individual consumer's problem in Equation (2) directly to the general demand system that defines market prices in terms of aggregate quantities. The motivation for the mapping is as follows. Since agents are identical both in terms of preferences and outcomes, the solution to the individual agent's problem in Equation (2) also represents the general demand system that underlies the estimation procedure used to recover PC. To see this, note that the solution $\mathbf{Q}_{i,t} = \mathbf{Q}_t$ of the individual consumer's problem in Equation (2) given market prices $\mathbf{P}_t^{\text{mkt}}$ is analogous to the outcome of a competitive equilibrium in which market prices $\mathbf{P}_t^{\text{mkt}}$ clear product markets such that aggregate endowment of goods equals the aggregate consumption of goods, since $\int_0^1 \mathbf{Q}_{i,t} di = \mathbf{Q}_t$ and $\int_0^1 \mathbf{F}_{i,t} di = \mathbf{F}_t$. The same argument follows for the aggregation in financial markets.

We define the expenditure share S_t^{G} on a marketable good $G \in \{D, N, S\}$ as the ratio of the expenditure on the good (i.e., $X_t^{\text{G}} \equiv F_t^{\text{G}} P_t^{\text{G}}$) and the Market-C (i.e., $X_t^{\text{DNS}} = X_t^{\text{D}} + X_t^{\text{N}} + X_t^{\text{S}}$). For the PC good, we define the shadow expenditure share S_t^{L} as the ratio of the shadow value of PC (i.e., $X_t^{\text{L}} \equiv F_t^{\text{L}} P_t^{\text{L}}$) over the Market-C, X_t^{DNS}. From these definitions and from Equation (2d), the optimal (actual or shadow) expenditure share in each of the goods is given by

$$S_t^{\mathrm{D}} = \frac{F_t^{\mathrm{D}}}{F_t^{\mathrm{D}} + Q_t^{\mathrm{N}} P_t^{\mathrm{N}} + Q_t^{\mathrm{S}} P_t^{\mathrm{S}}}, \tag{3a}$$

$$S_t^{\mathrm{N}} = \frac{Q_t^{\mathrm{N}} P_t^{\mathrm{N}}}{F_t^{\mathrm{D}} + Q_t^{\mathrm{N}} P_t^{\mathrm{N}} + Q_t^{\mathrm{S}} P_t^{\mathrm{S}}}, \tag{3b}$$

$$S_t^{\mathrm{S}} = \frac{Q_t^{\mathrm{S}} P_t^{\mathrm{S}}}{F_t^{\mathrm{D}} + Q_t^{\mathrm{N}} P_t^{\mathrm{N}} + Q_t^{\mathrm{S}} P_t^{\mathrm{S}}}, \tag{3c}$$

$$S_t^{\mathrm{L}} = \frac{Q_t^{\mathrm{L}} P_t^{\mathrm{L}}}{F_t^{\mathrm{D}} + Q_t^{\mathrm{N}} P_t^{\mathrm{N}} + Q_t^{\mathrm{S}} P_t^{\mathrm{S}}}, \tag{3d}$$

where the model-implied prices are given by

$$P_t^{\mathrm{N}} = \frac{Q_t^{\mathrm{D}}}{Q_t^{\mathrm{N}}} \left(\frac{c_{\mathrm{N}}[\mathbf{q}_t]}{c_{\mathrm{D}}[\mathbf{q}_t]} \right) \Upsilon_t, \tag{3e}$$

$$P_t^{\mathrm{S}} = \frac{Q_t^{\mathrm{D}}}{Q_t^{\mathrm{S}}} \left(\frac{c_{\mathrm{S}}[\mathbf{q}_t]}{c_{\mathrm{D}}[\mathbf{q}_t]} \right) \Upsilon_t, \tag{3f}$$

$$P_t^{\mathrm{L}} = \frac{Q_t^{\mathrm{D}}}{Q_t^{\mathrm{L}}} \left(\frac{c_{\mathrm{L}}[\mathbf{q}_t]}{c_{\mathrm{D}}[\mathbf{q}_t]} \right) \Upsilon_t, \tag{3g}$$

$$\text{where} \qquad \Upsilon_t \equiv 1 + (1-\delta)\, \mathbb{E}_t \left[M_{t+1} \right], \tag{3h}$$

and M_{t+1} is the stochastic discount factor (SDF) given by

$$M_{t+1} \equiv \beta \left(\frac{\mathrm{U_C}[C_{t+1}]}{\mathrm{U_C}[C_t]} \right) \left(\frac{\mathrm{C_D}[\mathbf{Q}_{t+1}]}{\mathrm{C_D}[\mathbf{Q}_t]} \right), \tag{3i}$$

$\mathbf{q}_t \equiv \log[\mathbf{Q}_t]$, $\mathrm{c}[\mathbf{q}_t] \equiv \log[\mathrm{C}[\exp[\mathbf{q}_t]]]$, $\mathrm{c_G}[\mathbf{q}_t] \equiv {}^{\partial\, \mathrm{c}[\mathbf{q}_t]}/_{\partial q_t^{\mathrm{G}}}$, $\mathrm{C_G}[\mathbf{Q}_t] \equiv {}^{\partial\, \mathrm{C}[\mathbf{Q}_t]}/_{\partial Q_t^{\mathrm{G}}}$ for $G \in \{D, N, S, L\}$, and $\mathrm{U_C}[C_t] \equiv {}^{d\, \mathrm{U}[C_t]}/_{dC_t}$.

The model-implied expenditure shares presented in Equations (3a)–(3d), which in turn are based on the model-implied prices defined in Equations (3e)–(3g), represent the general demand system that we use to recover PC from the data. The details of the derivation of the general demand system are presented in Appendix A.

1.4 Parametric Solution

Before we present our estimation methodology, we must first specify the functional forms of the consumption aggregator, $\mathrm{C}[\mathbf{Q}_t]$, and the current period's utility over Total-C, $\mathrm{U}[C_t]$.

1.4.1 Functional Form for the Consumption Aggregator

We specify a transcendental logarithmic (translog) functional form for the consumption aggregator C[.], which we denote $\mathrm{C}^{\mathrm{TL}}[.]$.[14] The translog functional form used is formalized in the assumption below:

Assumption 1. The consumption aggregator $\mathrm{C}[\mathbf{Q}_t]$ *is defined in logs by the translog function given by*

$$\log[C^{\mathrm{TL}}[\exp[\mathbf{q}_t]]] = c^{\mathrm{TL}}[\mathbf{q}_t] \equiv \mathbf{q}_t \times \mathbf{a} + \frac{1}{2}\left(\mathbf{q}_t \times \mathbf{b} \times \mathbf{q}_t{}'\right), \tag{4a}$$

$$\textit{where} \qquad \mathbf{a} \equiv \begin{bmatrix} a_{\mathrm{D}} \\ a_{\mathrm{N}} \\ a_{\mathrm{S}} \\ a_{\mathrm{L}} \end{bmatrix} \in [0,1]^{4\times 1}, \quad \textit{and} \quad \mathbf{b} \equiv \begin{bmatrix} b_{\mathrm{DD}} & b_{\mathrm{DN}} & b_{\mathrm{DS}} & b_{\mathrm{DL}} \\ b_{\mathrm{ND}} & b_{\mathrm{NN}} & b_{\mathrm{NS}} & b_{\mathrm{NL}} \\ b_{\mathrm{SD}} & b_{\mathrm{SN}} & b_{\mathrm{SS}} & b_{\mathrm{SL}} \\ b_{\mathrm{LD}} & b_{\mathrm{LN}} & b_{\mathrm{LS}} & b_{\mathrm{LL}} \end{bmatrix} \in \mathbb{R}^{4\times 4}. \tag{4b}$$

Without additional restrictions, the translog aggregator presented in Equations (4a)–(4b) contains 20 free parameters. To reduce the number of free parameters in the estimation, we follow the literature and impose two standard restrictions to the translog function. The first parameter restriction, which is formalized in Assumption 2, is to impose symmetry in the matrix $\mathbf{b}$.

Assumption 2. The parameter matrix $\mathbf{b}$ *in Equations* (4a)–(4b) *is symmetric around its main diagonal.*

[14]See Christensen, Jorgenson, and Lau (1973) and Christensen et al. (1975) for seminal discussions of the transcendental logarithmic function.

The second parameter restriction is the normalization of the vector **a** and the matrix **b**, which is formalized in Assumption 3.

Assumption 3. The vector **a** *is normalized as follows:*

$$\sum_{\mathrm{i}\in\{\mathrm{D,N,S,L}\}} a_{\mathrm{i}} = 1. \tag{5a}$$

The parameter matrix **b** *is normalized as follows:*

$$\sum_{\mathrm{i}\in\{\mathrm{D,N,S,L}\}} b_{\mathrm{G,i}} = 0, \quad \forall G \in \{D, N, S, L\}. \tag{5b}$$

The restrictions in Assumption 2 are introduced solely to remove redundancy in the parameter space and have no relevant economic implications. The parameter restrictions in Assumption 3 are critical for the estimation because they make the consumption aggregator C[.] homothetic over the four consumption goods. Homotheticity implies that the expenditure shares of the marketable goods are unaffected by changes in the true aggregate income, which is latent because of the shadow income from the PC endowment. Note that homotheticity over the four consumption goods implies that preferences should appear, as in the data (e.g., Eichenbaum and Hansen (1990)), non-homothetic over the three marketable goods (i.e., if we ignore the existence of PC).

We implement Assumptions 2 and 3 by restricting the coefficient vector **a** and the coefficient matrix **b** as follows:

$$a_{\mathrm{L}} = 1 - a_{\mathrm{D}} - a_{\mathrm{N}} - a_{\mathrm{S}}, \tag{6a}$$

$$\mathbf{b} = \begin{bmatrix} b_{\mathrm{DD}} & b_{\mathrm{DN}} = b_{\mathrm{ND}} & b_{\mathrm{DS}} = b_{\mathrm{SD}} & b_{\mathrm{DL}} = -b_{\mathrm{D}} \\ b_{\mathrm{ND}} & b_{\mathrm{NN}} & b_{\mathrm{NS}} = b_{\mathrm{SN}} & b_{\mathrm{NL}} = -b_{\mathrm{N}} \\ b_{\mathrm{SD}} & b_{\mathrm{SN}} & b_{\mathrm{SS}} & b_{\mathrm{SL}} = -b_{\mathrm{S}} \\ b_{\mathrm{LD}} = -b_{\mathrm{D}} & b_{\mathrm{LN}} = -b_{\mathrm{N}} & b_{\mathrm{LS}} = -b_{\mathrm{S}} & b_{\mathrm{LL}} = b_{\mathrm{D}} + b_{\mathrm{N}} + b_{\mathrm{S}} \end{bmatrix}, \tag{6b}$$

where

$$b_{\mathrm{D}} = b_{\mathrm{DD}} + b_{\mathrm{ND}} + b_{\mathrm{SD}}, \tag{6c}$$
$$b_{\mathrm{N}} = b_{\mathrm{ND}} + b_{\mathrm{NN}} + b_{\mathrm{SN}}, \tag{6d}$$
$$b_{\mathrm{S}} = b_{\mathrm{SD}} + b_{\mathrm{SN}} + b_{\mathrm{SS}}. \tag{6e}$$

Assumptions 2 and 3 effectively reduce the number of free parameters from 20 to 9.

1.4.2 Functional Form for the Utility over Total Aggregate Consumption

The final assumption needed to estimate the model is over the functional form of the utility function U[.], which represents preferences over the current period's Total-C and defines the expected utility over current and expected future Total-C, $\sum_{s=0}^{\infty} \beta^s \, \mathbb{E}_t \left[\mathrm{U}[C_{t+s}]\right]$. In Assumption 4 we specify that the function U[.] has a power functional form:

Assumption 4.

$$\mathbb{E}_t \left[\mathrm{U}[C_{t+s}]\right] = \beta^s \, \mathbb{E}_t \left[\frac{C_{t+s}^{1-\gamma}}{1-\gamma} \right], \tag{7}$$

where $\gamma \geq 1$ is the coefficient of relative risk aversion (RRA), which is constant in the power class of utility functions.

Assumptions 1–4 imply the Euler equation given by

$$\mathbb{E}_t \left[M_{t+1}^{\mathrm{TL}} R_{j,t+1} \right] = 1, \tag{8a}$$

where R_{j} is the gross return on financial asset j and M^{TL} is the parametric SDF given by

$$M_{t+1}^{\mathrm{TL}} \equiv \beta \left(\frac{\mathrm{C}^{\mathrm{TL}}[\mathbf{Q}_{t+1}]}{\mathrm{C}^{\mathrm{TL}}[\mathbf{Q}_t]} \right)^{-\gamma} \left(\frac{\mathrm{C}_{\mathrm{D}}^{\mathrm{TL}}[\mathbf{Q}_{t+1}]}{\mathrm{C}_{\mathrm{D}}^{\mathrm{TL}}[\mathbf{Q}_t]} \right). \tag{8b}$$

From Equation (8a), we have that the model-implied risk-free rate between periods t and $t+1$, R_{t+1}^{f}, is given by

$$R_{t+1}^{\mathrm{f}} = \mathbb{E}_t \left[M_{t+1}^{\mathrm{TL}} \right]^{-1}. \tag{9}$$

1.4.3 Demand System

The translog functional form for the consumption aggregator presented in Section 1.4.1 and the power functional form for the utility over current Total-C presented in Section 1.4.2 jointly imply the following parametric form for the general model solution presented in Section 1.3:

$$S_t^{\text{D,TL}} = \frac{F_t^{\text{D}}}{F_t^{\text{D}} + Q_t^{\text{N}} P_t^{\text{N,TL}} + Q_t^{\text{S}} P_t^{\text{S,TL}}}, \tag{10a}$$

$$S_t^{\text{N,TL}} = \frac{Q_t^{\text{N}} P_t^{\text{N,TL}}}{F_t^{\text{D}} + Q_t^{\text{N}} P_t^{\text{N,TL}} + Q_t^{\text{S}} P_t^{\text{S,TL}}}, \tag{10b}$$

$$S_t^{\text{S,TL}} = \frac{Q_t^{\text{S}} P_t^{\text{S,TL}}}{F_t^{\text{D}} + Q_t^{\text{N}} P_t^{\text{N,TL}} + Q_t^{\text{S}} P_t^{\text{S,TL}}}, \tag{10c}$$

$$S_t^{\text{L,TL}} = \frac{Q_t^{\text{L}} P_t^{\text{L,TL}}}{F_t^{\text{D}} + Q_t^{\text{N}} P_t^{\text{N,TL}} + Q_t^{\text{S}} P_t^{\text{S,TL}}}, \tag{10d}$$

where $P^{\text{N,TL}}$, $P^{\text{S,TL}}$, and $P^{\text{L,TL}}$ are the model-implied (market or shadow) prices with the translog aggregator of the nondurable, service, and PC good, given by

$$P_t^{\text{N,TL}} = \left(\frac{Q_t^{\text{D}}}{Q_t^{\text{N}}}\right) \left(\frac{a_{\text{N}} - b_{\text{N}}(q_t^{\text{L}} - q_t^{\text{N}}) + b_{\text{ND}}(q_t^{\text{D}} - q_t^{\text{N}}) + b_{\text{SN}}(q_t^{\text{S}} - q_t^{\text{N}})}{a_{\text{D}} - b_{\text{D}}(q_t^{\text{L}} - q_t^{\text{D}}) + b_{\text{ND}}(q_t^{\text{N}} - q_t^{\text{D}}) + b_{\text{SD}}(q_t^{\text{S}} - q_t^{\text{D}})}\right) \Upsilon_t^{\text{TL}}, \tag{10e}$$

$$P_t^{\text{S,TL}} = \left(\frac{Q_t^{\text{D}}}{Q_t^{\text{S}}}\right) \left(\frac{a_{\text{S}} - b_{\text{S}}(q_t^{\text{L}} - q_t^{\text{S}}) + b_{\text{SD}}(q_t^{\text{D}} - q_t^{\text{S}}) + b_{\text{SN}}(q_t^{\text{N}} - q_t^{\text{S}})}{a_{\text{D}} - b_{\text{D}}(q_t^{\text{L}} - q_t^{\text{D}}) + b_{\text{ND}}(q_t^{\text{N}} - q_t^{\text{D}}) + b_{\text{SD}}(q_t^{\text{S}} - q_t^{\text{D}})}\right) \Upsilon_t^{\text{TL}}, \tag{10f}$$

$$P_t^{\text{L,TL}} = \left(\frac{Q_t^{\text{D}}}{Q_t^{\text{L}}}\right) \left(\frac{a_{\text{L}} - b_{\text{D}}(q_t^{\text{D}} - q_t^{\text{L}}) - b_{\text{N}}(q_t^{\text{N}} - q_t^{\text{L}}) - b_{\text{S}}(q_t^{\text{S}} - q_t^{\text{L}})}{a_{\text{D}} - b_{\text{D}}(q_t^{\text{L}} - q_t^{\text{D}}) + b_{\text{ND}}(q_t^{\text{N}} - q_t^{\text{D}}) + b_{\text{SD}}(q_t^{\text{S}} - q_t^{\text{D}})}\right) \Upsilon_t^{\text{TL}}, \tag{10g}$$

where $a_{\text{L}} = 1 - a_{\text{D}} - a_{\text{N}} - a_{\text{S}}$, and where the model-implied consumption good price adjustment Υ_t^{TL} is given by

$$\Upsilon_t^{\text{TL}} = 1 + (1 - \delta)\, \mathbb{E}_t \left[M_{t+1}^{\text{TL}}\right]. \tag{10h}$$

The parametric solution above, which is based on the translog aggregator, shows that the prices and expenditure shares of the marketable goods respond to changes in q^{L}. An implication of this fact is that the PC quantity and shadow price series can, in principle, be recovered from the system in Equations (10a)–(10g).

1.5 Discussion of Modelling Choices and Model Solution

The goal of the model estimation presented in Section 2 is to recover the quantity consumed, Q_t^{L}, the shadow price P_t^{L}, and the shadow expenditure share S_t^{L} of PC from observable series in the data. This section provides the intuition for why the estimation requires: (i) at least three marketable consumption goods in the underlying model; (ii) a functional form for the consumption aggregator more flexible than that of the CES consumption aggregator; and (iii) the use of both financial and consumption moments.

1.5.1 Why Three Marketable Goods?

The demand system in Equations (10a)–(10g) provides the intuition for why three is the minimum number of goods required to recover the latent PC series: First, note that the last equality in Equations (3a)–(3d) cannot be used in the estimation because the share S_t^{L}, the quantity Q_t^{L}, and the shadow price P_t^{L} are unobservable. Moreover, one out of the first three equalities in Equations (3a)–(3d) is redundant, since, by construction, $S_t^{\text{D}} + S_t^{\text{N}} + S_t^{\text{S}} = 1$. An additional challenge is that the model implied prices in Equations (3a)–(3d) are functions of the unobservable Q_t^{L}, as shown in Equations (3e)–(3g). To produce an expression that can be estimated directly from the data, we must combine the two remaining equalities into a single one that does not involve any unobservable variable. The parameter values from the estimation based on this single equality can be used in any of the equalities in Equations (3a)–(3c) to recover the quantity Q_t^{L}, which can then be plugged back to Equation (3g) to recover the P_t^{L}. Extending this argument to the general case, a model with $N+1$ goods (N marketable goods and the PC good) leads to a demand system with $N-2$ equalities that do not involve any latent variable. Thus, the model must have at least three marketable goods so that at least one equality can be directly estimated. While more than three marketable goods could be used in the estimation, we use three for parsimony and to avoid parameter proliferation.

1.5.2 Why Not a CES Consumption Aggregator?

To motivate the need for a flexible functional form for the consumption aggregator C[.], we show in this section why the commonly used constant elasticity of substitution (CES) functional form, which nests the Cobb-Douglas form, cannot be used to recover the quantity consumed of the PC good Q^{L} from observable data.[15]

Let $C_t = \mathrm{C}^{\mathrm{CES}}[\mathbf{Q}_t]$, where the function $\mathrm{C}^{\mathrm{CES}}[.]$ has the functional form given by

$$C^{\mathrm{CES}}[\mathbf{Q}_t] \equiv (\alpha_{\mathrm{D}}(Q_t^{\mathrm{D}})^{\rho} + \alpha_{\mathrm{N}}(Q_t^{\mathrm{N}})^{\rho} + \alpha_{\mathrm{S}}(Q_t^{\mathrm{S}})^{\rho} + (1-\alpha_{\mathrm{D}}-\alpha_{\mathrm{N}}-\alpha_{\mathrm{S}})(Q_t^{\mathrm{L}})^{\rho})^{\frac{1}{\rho}}, \tag{11}$$

and $\frac{1}{1-\rho} > 0$ is the elasticity of substitution between the goods. When we set $C = C^{\mathrm{CES}}$, the CES-model-implied expenditure shares of the marketable goods from Equations (3a)–(3d) become

$$S_t^{\mathrm{D,CES}} = \frac{F_t^{\mathrm{D}}}{F_t^{\mathrm{D}} + Q_t^{\mathrm{N}} P_t^{\mathrm{N,CES}} + Q_t^{\mathrm{S}} P_t^{\mathrm{S,CES}}}, \tag{12a}$$
$$S_t^{\mathrm{N,CES}} = \frac{Q_t^{\mathrm{N}} P_t^{\mathrm{N,CES}}}{F_t^{\mathrm{D}} + Q_t^{\mathrm{N}} P_t^{\mathrm{N,CES}} + Q_t^{\mathrm{S}} P_t^{\mathrm{S,CES}}}, \tag{12b}$$
$$S_t^{\mathrm{S,CES}} = \frac{Q_t^{\mathrm{S}} P_t^{\mathrm{S,CES}}}{F_t^{\mathrm{D}} + Q_t^{\mathrm{N}} P_t^{\mathrm{N,CES}} + Q_t^{\mathrm{S}} P_t^{\mathrm{S,CES}}}, \tag{12c}$$
$$S_t^{\mathrm{L,CES}} = \frac{Q_t^{\mathrm{L}} P_t^{\mathrm{L,CES}}}{F_t^{\mathrm{D}} + Q_t^{\mathrm{N}} P_t^{\mathrm{N,CES}} + Q_t^{\mathrm{S}} P_t^{\mathrm{S,CES}}}, \tag{12d}$$

where $P^{\mathrm{N,CES}}$, $P^{\mathrm{S,CES}}$, and $P^{\mathrm{L,CES}}$ are the CES-model-implied market or shadow prices of nondurables, services, and PC, respectively, and are given by

[15] Two examples of asset pricing papers that use CES aggregators are Yogo (2006)) and Piazzesi et al. (2007).

$$P_t^{\mathrm{N,CES}} = \frac{\alpha_{\mathrm{N}}}{\alpha_{\mathrm{D}}} \left(\frac{Q_t^{\mathrm{N}}}{Q_t^{\mathrm{D}}} \right)^{\frac{1-\rho}{\rho}} \Upsilon_t, \tag{12e}$$

$$P_t^{\mathrm{S,CES}} = \frac{\alpha_{\mathrm{S}}}{\alpha_{\mathrm{D}}} \left(\frac{Q_t^{\mathrm{S}}}{Q_t^{\mathrm{D}}} \right)^{\frac{1-\rho}{\rho}} \Upsilon_t, \tag{12f}$$

$$P_t^{\mathrm{L,CES}} = \frac{1-\alpha_{\mathrm{D}}-\alpha_{\mathrm{N}}-\alpha_{\mathrm{L}}}{\alpha_{\mathrm{D}}} \left(\frac{Q_t^{\mathrm{L}}}{Q_t^{\mathrm{D}}} \right)^{\frac{1-\rho}{\rho}} \Upsilon_t, \tag{12g}$$

where Υ is defined in Equation (3h). The demand system in Equation (12) shows that, in the CES case, the latent PC series cannot be recovered from the data because the model-implied expenditure shares of the marketable consumption goods are not functions of the latent quantity Q_t^{L}.[16] The intuition for this result is that, in the CES case, the elasticity of substitution is constant not only across time but also across goods. A constant elasticity of substitution across goods implies that the marginal utilities of the marketable goods change in the same proportion in response to a given change in Q_t^{L}. Model-implied prices are based on ratios of marginal utilities (i.e., marginal rates of substitution), so that, in the CES, model-implied prices and expenditure shares of the marketable goods are unaffected by changes in Q_t^{L}.

1.5.3 Why Are Financial Moments Needed in the Model Estimation?

As presented in the next section, the estimation of the model requires the use of observable consumption and financial series from the data. This section shows how this requirement is due to the existence of a durable good in the model.

The durable good is unique in that it has a hybrid consumption good and storage technology nature. To see how this dual role of durable goods matters for the estimation, consider a hypothetical case with depreciation $\delta = 1$ (i.e., all goods are perishable). Under this scenario, $\Upsilon = 1$ in Equation (10h), so that the demand system is independent of the functional

[16]Note that, as discussed in Section 1.5.1, Equations (12d) and (12g) cannot be used directly in the estimation because the shadow price and expenditure share of the PC good are not observable.

form U[.]. Since, in this case, the demand system only depends on the parameters from the consumption aggregator, it can be estimated entirely from observable consumption series. The intuition for this result is that when all consumption goods are perishable, the agent optimization problem can be split into two independent decisions: the intertemporal decision over how much to consume today and how much to save for the future and the intratemporal decision over how to allocate today's consumption budget across different goods.

The empirically relevant case $0 < \delta < 1$ implies that the intertemporal and intratemporal decisions are no longer separable as in the hypothetical case above.[17] This implication can be seen in Equations (8b) and (10a)–(10g), which jointly imply that goods prices will be affected by the agent's impatience rate (i.e., β) and degree of risk aversion (i.e., γ).

2 Estimation Methodology

This section presents the data and the methodology used in the estimation of the parametric model solution shown in Section 1.4.3.

2.1 Data Overview

Here we present an overview of the consumption and financial data used in the model estimation. Appendix B provides the details of the data sources and sample construction.

Table 1 reports the mean, standard deviation, autocorrelation, and cross-correlations of the consumption and financial series used in the empirical analysis. Panel A presents the statistics for 10 marketable consumption series: the log growth of the aggregate marketable consumption expenditures Δx_{DNS} and Δx_{NS}, which includes and excludes durables, respectively; the proxy for aggregate consumption quantity based on municipal solid waste, Δc_G, from Savov (2011); the log growth in the quantity purchased, Δf_{D}, and consumed, Δq_{D}, of the durable good; the quantities consumed of nondurables, Δq_{N}, and services, Δq_{S}; and

[17]This implication, which is unrelated to the existence of PC, is not unique to our model. For instance, an analogous mechanism can be seen in Equation (13) from Yogo (2006).

the change in the expenditure shares of the three marketable goods, ΔS_{D}, ΔS_{N}, and ΔS_{S}. Overall, the statistics show that the growth of the log expenditure, expenditure share, and log quantity of durables is more volatile than that of other goods.

Panel B of Table 1 presents the statistics for the financial series. $R^{\text{f}}_{\text{nominal}}$ is the nominal rate of the 30-day U.S. Treasury Bill. $R^{\text{f}}_{\text{PCE}}$ and $R^{\text{f}}_{\text{durables}}$ are the real risk-free rates defined as $R^{\text{f}}_{\text{nominal}}$ deflated by the PCE index and the price of the durable good, respectively. We report both $R^{\text{f}}_{\text{PCE}}$ and $R^{\text{f}}_{\text{durables}}$ because our empirical analysis requires the use of the risk-free rate deflated by the price of the durable good and not the more standard PCE index. Relative to $R^{\text{f}}_{\text{PCE}}$, $R^{\text{f}}_{\text{durables}}$ is higher on average (2.19% vs. 0.59%) and more volatile (standard deviations of 4.83% vs. 3.96%).[18] $R^{\text{e}}_{\text{MKT}}$, $R^{\text{e}}_{\text{SMB}}$, and $R^{\text{e}}_{\text{HML}}$ are the excess returns of the market, size, and value factor mimicking portfolios from Fama and French (1993).

Panel C of Table 1 presents the correlations between the marketable consumption series and the contemporaneous returns of the financial series. As expected, the growth series of the Market-C and the quantities consumed of the three marketable goods are positively related to the contemporaneous market excess returns.

[Table 1 here]

Figure 1 plots the time series of the quantity and expenditure shares of the three marketable consumption goods between 1930 and 2018. Panel A of Figure 1 presents the time-series of the log quantities. The panel shows a steady increase in the quantity consumed across all three goods, as well as the quantity purchased of the durable good. Panel B of Figure 1 presents the expenditure shares of the three marketable goods. As discussed in Section 2.4.1, these shares are used as targets in the model estimation. The panel shows a low and relatively flat expenditure share on durables, and a gradual shift from nondurable to service consumption, in particular in the post-war period. The significant growth of services

[18] The average nominal price growth rates are 1.21% (durables), 2.66% (nondurables), 3.25% (services), and 2.84% (PCE index). The standard deviations of the nominal price growth rates are 4.13% (durables), 4.67% (nondurables), 3.01% (services), and 3.55% (PCE index).

(from around 40% in 1930 to approximately 74% in 2018) is consistent with the documented rise of the service sector in the U.S. economy (e.g., Buera and Kaboski (2012)).

[Figure 1 here]

2.2 Notation and Definitions

In what follows, unless otherwise noted, all model-implied variables are from the translog-based model, so we drop the superscript TL from these variables. We use the notation $\widehat{\cdot}$ to denote series from the data and also estimated parameter values.

The tuple of all the empirical inputs used in the estimation is $\hat{Z} = \{\hat{Q}^{\text{N}}, \hat{Q}^{\text{D}}, \hat{Q}^{\text{S}}, \hat{S}^{\text{D}}, \hat{S}^{\text{N}}, \hat{S}^{\text{S}}, \hat{F}^{\text{D}}, \widehat{\mathbf{R}}\}$, in which the matrix $\widehat{\mathbf{R}}$ contains the financial series discussed in Section 2.4.2.

The vector $\theta = \{a_{\text{D}}, a_{\text{N}}, a_{\text{S}}, b_{\text{ND}}, b_{\text{SD}}, b_{\text{SN}}, b_{\text{D}}, b_{\text{N}}, b_{\text{S}}, \beta, \gamma\}$ represents the 11 parameters in the demand system presented in Section 1.4.3. The first nine elements of θ are the parameters of translog function and the last two are the parameters that define the power utility over Total-C.

For some model-implied variable H, the notation $\text{H}[\theta|\hat{Z}_t]$ (e.g., $\text{S}^{\text{D}}[\theta|\hat{Z}_t]$) denotes that H is a function of θ given observable data $\hat{Z}_t$. Alternatively, the notation $\text{H}[\hat{Z}_t, \hat{\theta}]$ denotes the estimated value of H at date t.

2.3 Identification Strategy

The high-level objective of the estimation is to use the identities in Equations (10a)–(10c), which define the model-implied expenditure shares of the marketable goods, to identify the parameter vector θ and recover the latent PC series. These identities cannot be estimated directly because they involve the unobservable conditional expectation from Equation (10h) and also because they depend on the variable q_t^{L}, which is latent and has undefined units. We discuss below the procedure used to address these two challenges.

The first challenge in the estimation is related to the conditional expectation in the identity in Equation (10h). This conditional expectation defines the model-implied adjustment

$\Upsilon[\theta|\hat{Z}_t]$ and therefore determines the connection between the parameters of the power utility function and the parameters of the translog consumption aggregator. To eliminate the unobservable conditional expectation, we replace the identity in Equation (10h) with the restriction

$$\Upsilon[\theta|\hat{Z}_t] = \hat{\Upsilon}_t, \tag{13a}$$

$$\text{where} \qquad \hat{\Upsilon}_t \equiv 1 - (1-\hat{\delta})(\hat{R}^{\mathrm{f}}_{\text{durables},t})^{-1} \tag{13b}$$

is the empirical proxy for the adjustment Υ_t based on Equations (9) and (10h), and $\hat{R}^{\mathrm{f}}_{\text{durables},t}$ denotes the real risk-free rate defined as the nominal rate of the 30-day U.S. Treasury Bill deflated by the price of the durable good. In order to pin down the relation between the parameters β and γ and the parameters from the consumption aggregator, we add unconditional financial moments to the estimation, as we discuss in the next section.

The second challenge is related to existence of the latent variable q_t^{L} in the identities in Equations (10a)–(10c). To eliminate the variable q_t^{L} from these identities, we first derive a model-implied quantity of PC, $\mathrm{q}^{\mathrm{L}}[\theta|\hat{Z}_t]$, and then replace q_t^{L} with $\mathrm{q}^{\mathrm{L}}[\theta|\hat{Z}_t]$ in the demand system. Specifically, the model-implied $\mathrm{q}^{\mathrm{L}}[\theta|\hat{Z}_t]$ is defined as the solution for q_t^{L} to the restriction that the difference between the model-implied and empirical ratio of the expenditure shares of the nondurable and durable goods is zero at every point in time, and is given by

$$\begin{aligned} \mathrm{q}^{\mathrm{L}}[\theta|\hat{Z}_t] = {} & \frac{\hat{S}_t^{\mathrm{N}}\left(a_{\mathrm{D}} + b_{\mathrm{D}}\hat{q}_t^{\mathrm{D}} + b_{\mathrm{ND}}(\hat{q}_t^{\mathrm{N}} - \hat{q}_t^{\mathrm{D}}) + b_{\mathrm{SD}}(\hat{q}_t^{\mathrm{S}} - \hat{q}_t^{\mathrm{D}})\right)(\hat{F}_t^{\mathrm{D}}/\hat{Q}_t^{\mathrm{D}})}{b_{\mathrm{D}}\hat{S}_t^{\mathrm{N}}(\hat{F}_t^{\mathrm{D}}/\hat{Q}_t^{\mathrm{D}}) - b_{\mathrm{N}}\hat{S}_t^{\mathrm{D}}\hat{\Upsilon}_t} \\ & - \frac{\hat{S}_t^{\mathrm{D}}\hat{\Upsilon}_t\left(a_{\mathrm{N}} + b_{\mathrm{N}}\hat{q}_t^{\mathrm{N}} + b_{\mathrm{ND}}(\hat{q}_t^{\mathrm{D}} - \hat{q}_t^{\mathrm{N}}) + b_{\mathrm{SN}}(\hat{q}_t^{\mathrm{S}} - \hat{q}_t^{\mathrm{N}})\right)}{b_{\mathrm{D}}\hat{S}_t^{\mathrm{N}}(\hat{F}_t^{\mathrm{D}}/\hat{Q}_t^{\mathrm{D}}) - b_{\mathrm{N}}\hat{S}_t^{\mathrm{D}}\hat{\Upsilon}_t}. \end{aligned} \tag{14}$$

After replacing q_t^{L} by $\mathrm{q}^{\mathrm{L}}[\theta|\hat{Z}_t]$ in the system in Equations (10a)–(10g) we can express the model implied expenditure shares of the marketable goods solely in terms of the input series, $\hat{Z}_t$, and the parameters, θ.

Equations (10a)–(10g) and (14) jointly imply that the functions $\mathrm{q}^{\mathrm{L}}[\theta|\hat{Z}_t]$, $\mathrm{S}^{\mathrm{D}}[\theta|\hat{Z}_t]$, $\mathrm{S}^{\mathrm{N}}[\theta|\hat{Z}_t]$, and $\mathrm{S}^{\mathrm{S}}[\theta|\hat{Z}_t]$ are homogeneous of degree zero in θ. The last step to identify the system is to define units of the quantity of PC (q_t^{L}). Without loss of generality, we set the log quantity of PC at the initial date to zero, $q_1^{\mathrm{L}} = 0$ (i.e., $Q_1^{\mathrm{L}} = 1$). The normalization implies that the quantity of PC that we recover from the data is expressed in units of PC consumed in the first year of the sample (i.e., 1930).[19]

2.4 Estimation Procedure

The estimation is based on a two-stage GMM procedure implemented through the search over θ of the minimum of the loss function given by

$$\hat{\theta} = \underset{\theta}{\operatorname{argmin}}\, \mathrm{g}\left[\theta|\hat{Z}\right] W \,\mathrm{g}\left[\theta|\hat{Z}\right]', \tag{15a}$$

$$\text{where} \qquad \mathrm{g}\left[\theta|\hat{Z}\right] \equiv \left\{\mathrm{g}^{\mathrm{C}}\left[\theta|\hat{Z}\right], \mathrm{g}^{\mathrm{F}}\left[\theta|\hat{Z}\right]\right\} \tag{15b}$$

is a 1×13 vector that combines the consumption ($\mathrm{g}^{\mathrm{C}}[\theta|\hat{Z}]$) and financial ($\mathrm{g}^{\mathrm{F}}[\theta|\hat{Z}]$) vectors of moments. W is a weighting matrix defined as the identity matrix in the first stage and defined in the second stage as the inverse of the diagonal variance matrix of moment errors from the first stage.

In untabulated results, we use simulations to validate the identification strategy and the estimation procedure. Specifically, we use the system in Equations (10a)–(10g) with random parameter values to generate simulated data and verify that the estimation procedure and moments presented here are successful at identifying the values of the parameters used and at recovering the original PC series from the simulation.

The remainder of this section presents the details of the consumption and financial moments used in the estimation.

[19]Note that $Q_t^{\mathrm{L}} > 0$ since the consumption aggregator is defined in logs. A normalization based on the price of PC (e.g., $P_1^{\mathrm{L}} = 1$) would not be without loss of generality since it would impose a restriction on the sign of the shadow value of PC, $X_t^{\mathrm{L}} = Q_t^{\mathrm{L}} P_t^{\mathrm{L}}$, on at least one point in time (e.g., $X_1^{\mathrm{L}} = Q_t^{\mathrm{L}} > 0$).

2.4.1 Consumption Moments

The consumption moments are standard and based on nonlinear least squares (NLLS) analysis.[20] We define the consumption moments from the first-order conditions of the minimization of a loss function, which is defined as the mean of the sum of the squared deviations between the empirical and model-implied expenditure shares of each of the three marketable goods, as given by

$$\min_{\theta} \left\{ \frac{1}{T} \sum_{t=1}^{T} \left(\left(\hat{S}_t^{\mathrm{D}} - S^{\mathrm{D}}[\theta|\hat{Z}_t] \right)^2 + \left(\hat{S}_t^{\mathrm{N}} - S^{\mathrm{N}}[\theta|\hat{Z}_t] \right)^2 + \left(\hat{S}_t^{\mathrm{S}} - S^{\mathrm{S}}[\theta|\hat{Z}_t] \right)^2 \right) \right\}, \tag{16}$$

where T denotes the number of periods in the sample.

The first-order condition (FOC) of the minimization problem in Equation (16) with respect to each of the nine parameters of the translog aggregator define the consumption moments, $\mathrm{g}^{\mathrm{C}}\left[\theta|\hat{Z}_t\right]$, as given by

$$\mathrm{g}^{\mathrm{C}}\left[\theta|\hat{Z}\right] \equiv g^{\mathrm{C,D}} + g^{\mathrm{C,N}} + g^{\mathrm{C,S}}, \tag{17a}$$

$$\text{where} \qquad g^{\mathrm{C,D}} \equiv \frac{2}{T} \sum_{t=1}^{T} \left(\hat{S}_t^{\mathrm{D}} - S^{\mathrm{D}}[\theta|\hat{Z}_t] \right) \frac{\partial S^{\mathrm{D}}[\theta|\hat{Z}_t]}{\partial \theta^{\mathrm{TL}}}, \tag{17b}$$

$$g^{\mathrm{C,N}} \equiv \frac{2}{T} \sum_{t=1}^{T} \left(\hat{S}_t^{\mathrm{N}} - S^{\mathrm{N}}[\theta|\hat{Z}_t] \right) \frac{\partial S^{\mathrm{N}}[\theta|\hat{Z}_t]}{\partial \theta^{\mathrm{TL}}}, \tag{17c}$$

$$g^{\mathrm{C,S}} \equiv \frac{2}{T} \sum_{t=1}^{T} \left(\hat{S}_t^{\mathrm{S}} - S^{\mathrm{S}}[\theta|\hat{Z}_t] \right) \frac{\partial S^{\mathrm{S}}[\theta|\hat{Z}_t]}{\partial \theta^{\mathrm{TL}}}, \tag{17d}$$

where $\theta^{\mathrm{TL}} \equiv \{a_{\mathrm{D}}, a_{\mathrm{N}}, a_{\mathrm{S}}, b_{\mathrm{ND}}, b_{\mathrm{SD}}, b_{\mathrm{SN}}, b_{\mathrm{D}}, b_{\mathrm{N}}, b_{\mathrm{S}}\}$ is a 1×9 row vector of the parameters of the translog aggregator. The 1×9 vector of consumption moments $\mathrm{g}^{\mathrm{C}}\left[\theta|\hat{Z}_t\right]$ presented in Equations (17a)–(17d) represents the nine FOCs of the minimization in Equation (16) (i.e., one moment per parameter of the consumption aggregator).

[20] Specifically, we use as moment conditions in the GMM estimation the first-order conditions of the NLLS (i.e., the partial derivatives of the loss function with respect to each of the parameters should be equal zero at the loss function minimum). See chapter nine of Greene (2003) for an overview of NLLS estimation.

2.4.2 Financial Moments

The financial moments used in the estimation are based on the standard unconditional Euler equation given by

$$\mathbb{E}[M_{t+1}R_{j,t+1}] = 1. \tag{18}$$

Equation (18) is constructed by applying the law of iterated expectations to the conditional Euler Equation (8a). The 1×4 vector of financial moments is defined as

$$\mathrm{g}^{\mathrm{F}}\left[\theta|\hat{Z}\right] \equiv \{g^{\mathrm{F,RFR}}, g^{\mathrm{F,MKT}}, g^{\mathrm{F,SMB}}, g^{\mathrm{F,HML}}\}, \tag{19a}$$

$$\text{where} \qquad g^{\mathrm{F,RFR}} \equiv \frac{1}{T}\sum_{t=1}^{T} \mathrm{M}[\theta|\hat{Z}_{t+1}]\hat{R}^{\mathrm{f}}_{\mathrm{durables},t+1} - 1, \tag{19b}$$

$$g^{\mathrm{F,j}} \equiv \frac{1}{T}\sum_{t=1}^{T} \mathrm{M}[\theta|\hat{Z}_{t+1}]\hat{R}^{\mathrm{e}}_{j,t+1}, \quad \text{for } j \in \{\textsc{mkt}, \textsc{smb}, \textsc{hml}\}, \tag{19c}$$

where M[.] is the parametric SDF from Equation (8b), $\hat{R}^{\mathrm{f}}_{\mathrm{durables}}$ represents the gross nominal risk-free rate deflated with the price of the durable good, and $\hat{R}^{e}_{j}$ represents the excess returns of the market (MKT), size (SMB), and value (HML) factor mimicking portfolios from Fama and French (1993).[21]

3 Estimation Results

3.1 Estimated Parameters and Model Fit

Table 2 reports the point estimates and GMM standard errors of the parameters of the translog consumption aggregator from Equations (4a)–(4b) and the expected utility over Total-C from Equation (7).[22] The parameter estimates reveal that the effect of the PC

[21]See Appendix B.2 for the details of the financial series used.

[22]The derivation of the model-implied standard errors is presented in Appendix C.1.

good on Total-C is different in nature from those of the marketable goods. In particular, in contrast with the corresponding parameters for the marketable goods, the estimated value for the parameter a_{L}, is not statistically different from zero (i.e., the lower bound for the parameter) and the value for parameter b_{LL} is negative. These two values indicate that, if we ignore the interaction between the goods in preferences, the utility is decreasing in the PC good. However, the positive values for the interaction parameters b_{xL} and b_{Lx} suggest that the positive effect on agents' utility of an increase in durables, nondurables, or services is increasing in PC. This result is consistent with the finding that the PC complements the marketable consumption goods.

The estimated relative risk aversion coefficient γ from the power utility function is 10.8 (with 1% confidence interval [8.2, 13.4]), which is high in terms of how individuals make decisions that involve risk (e.g., Campbell (2017)), but at the very low end of values implied by consumption-based asset pricing models with power utility.[23] The constant β, which is related to agents' patience over waiting to consume, is also relatively low at 0.64 (with 1% confidence interval [0.46, 0.82]). We elaborate on the implications of the existence of PC for consumption-based asset pricing models in Section 4.

[Table 2 here]

Figure 2 shows that the estimated model does a good job matching the dynamics of expenditure shares of the three marketable consumption goods. The model-implied expenditure shares match not only the levels and trends but also the higher frequency movements of their empirical counterparts.

To quantify the performance of the model in matching the expenditure shares, we use the goodness-of-fit measure from Belo, Gala, Salomao, and Vitorino (2019), which is defined as the average mean absolute error scaled by the average expenditure share from the data. The values of the goodness-of-fit measure are 3.3% for durable goods, 3.2% for nondurable goods, and 2.8% for services. These errors are quite low, especially in comparison with the fit of the

[23]See Savov (2011) for a discussion of the ranges of values documented in this literature.

model with the more commonly used constant-elasticity-of-substitution (CES) aggregator, as we discuss in Section 3.3.2 below.

[Figure 2 here]

3.2 Properties of the Recovered Latent Consumption Series

To recover the model-implied quantity of the latent PC, we use the tuple of observable series $\hat{Z}_t$ and the vector of estimated parameters $\hat{\theta}$ in Equation (14), as given by

$$\hat{Q}_t^{\mathrm{L}} \equiv \exp\Big[\mathrm{q}^{\mathrm{L}}[\hat{Z}_t, \hat{\theta}]\Big]. \tag{20a}$$

We recover the Total-C series from Equations (1) and (20a), as given by

$$\hat{C}_t \equiv \exp\Big[\mathrm{c}[\hat{Q}_t^{\mathrm{L}}, \hat{Z}_t, \hat{\theta}]\Big]. \tag{20b}$$

Finally, the shadow value of Total-C is obtained from

$$\hat{X}_t^{\mathrm{DNSL}} \equiv \hat{X}_t^{\mathrm{DNS}}\Big(1 + \hat{S}_t^{\mathrm{L}}\Big), \tag{20c}$$

where $\hat{X}_t^{\mathrm{DNS}}$ is the Market-C and $\hat{S}_t^{\mathrm{L}}$ is the recovered shadow expenditure share of PC from Equations (20a) and (10d) as given by

$$\hat{S}_t^{\mathrm{L}} \equiv \mathrm{S}^{\mathrm{L}}[\hat{Q}_t^{\mathrm{L}}, \hat{\theta}, \hat{Z}_t]. \tag{20d}$$

Table 3 presents the summary statistics of the recovered latent consumption series from Equations (20a)–(20d). Panel A of Table 3 shows that the average annual growth rate of Total-C (Δc) is -0.15%, which is lower than that of the shadow value of Total-C (Δx_{DNSL}), which is 2.78%. In contrast, the volatility of the growth rate of Total-C is 8.58%, which is markedly higher than that of the shadow value of Total-C, 3.67%, and that of the Market-C (Δx_{DNS}), which is 2.74% as shown in Table 1. The cause for the gap between the growth rates of Total-C and the shadow value of Total-C is the growing scarcity of PC. The quantity of PC sharply declined over the period. The growth rate of PC (Δq_{L}) has a mean of -17.81% and a volatility of 64.07% over the sample period. The significant decline and high volatility of Δq_{L} do not imply similar properties for the change in the shadow value of PC (ΔS_{L}), which has a mean of 0.86% and a volatility of 3.11%. The correlations between the variables suggest that the latent quantity growth rates (i.e., Δc and Δq_{L}) are negatively correlated with the growth rates of the value-based latent series (i.e., Δx_{DNSL} and ΔS_{L}).

Panel B of Table 3 shows the correlations between the recovered latent consumption series and the marketable consumption series. The table shows that the growth rates of Total-C (Δc) and PC (Δq_{L}) are positively related with all but one (i.e., Δq_{D}) of the growth rates of the marketable quantity series.

Panel C of the table shows the correlations between the recovered marketable consumption series and the financial series. The panel shows that the growth rates of the Total-C and PC are both positively correlated to contemporaneous excess market returns. In contrast, the growth in the shadow expenditure share of PC is negatively correlated to excess market returns. These correlations, which have opposite signs, offset each other so that the growth rate of the shadow value of Total-C is effectively uncorrelated with market returns.

[Table 3 here]

Panels A and B of Figure 3 show the quantity (q_{L}) and the shadow expenditure share (S_{L}) of PC, respectively, along with their corresponding 1% confidence bands. The log quantity of PC declined from its normalized value of zero in 1930 to around -12.5 in 2018. The shadow expenditure share of PC is significant and ranges from around -11% in 1930 to around 63% of the observable consumption expenditure in 2018.

Panels C and D of Figure 3 show the time series of the recovered Total-C (C) and the shadow value of Total-C, ($X_{\mathrm{DNSL}} = X_{\mathrm{DNS}}(1 + S_{\mathrm{L}})$), respectively, with their corresponding 1% confidence bands. The panels show that Total-C and its shadow value both increased between 1930 and around 1996, at which point the series markedly diverge. While the shadow value of Total-C has continued its positive trend, the Total-C itself has been rapidly declining from 1996 until 2018.

[Figure 3 here]

Panel A of Figure 4 presents the series expressed in log trillions of U.S. Dollars, and Panel B presents the series expressed in U.S. Dollars per capita.[24] Panel A of Figure 4 shows that in the first half of the sample (i.e., between 1930 and 1974), the gap between the shadow value of Total-C and the Market-C ranges between a minimum of -\$110 billion in 1932 (-15% of the Market-C) and a maximum of \$160 billion in 1974 (5% of the Market-C), with an average of -\$1 billion. In the second half of the sample (i.e., between 1975 and 2018), the gap ranges between a minimum of \$360 billion in 1981 (8% of the Market-C) and a maximum of \$8.9 trillion in 2018 (63% of the Market-C), with an average of \$2.8 trillion.

Panel B of Figure 4, which presents the series in U.S. Dollars per capita, shows similar general trends. In the first half of the sample, the gap between the per-capita Total-C and per-capita Market-C ranges between a minimum of -\$870 in 1933 and a maximum of \$360 in 1942, with an average of approximately \$0. In the second half of the sample, the gap ranges between a minimum of \$1,500 in 1981 and a maximum of \$25,800 in 2017, with an average of \$9,100.

Panels C and D of Figure 4 present the time-series of the growth rates of Total-C (Δc) and the shadow value of Total-C (Δx_{DNSL}), respectively, juxtaposed with the time-series of the growth rate of Market-C (Δx_{DNS}). Overall, the figure illustrates the finding presented in Table 3, that the growth rate of Total-C is significantly more volatile than that of the shadow value of Total-C or Market-C.

[Figure 4 here]

Turning to the analysis of the cyclical properties of the recovered consumption, Table 4 reports the mean and the standard deviation of the growth rates of the consumption series

[24] All U.S. Dollar values are expressed in real 2018 U.S. Dollars.

conditional on the state of the business cycle. Specifically, we report disaggregated statistics for the samples of the years that experienced recessions according to the National Bureau of Economic Research (NBER) and all other years, which we denote *expansions*.[25]

Panel A of Table 4 shows that the mean and volatility of the recovered consumption growth series are significantly affected by the state of the economy. The growth rate of Total-C (Δc) and the growth of the shadow value of Total-C (Δx_{DNSL}) are strongly procyclical. The averages of these series over expansions (0.95% and 3.72%) are significantly higher than those over recessions (-2.51% and 0.76%). The volatilities of the growth series over expansions (6.49% and 3.02%) are significantly lower than those over recessions (11.7% and 4.16%). The growth rate of the quantity (Δq_{L}) and the growth of shadow expenditure share (ΔS_{L}) of PC have countercyclical means and volatilities. The averages of Δq_{L} are -19.89% over expansions and -13.40% during recessions, with corresponding volatilities of 53.55% and 83.14%. In contrast to the strong cyclicality of Δq_{L}, ΔS_{L} shows relatively small variation over the business cycle, with averages of 0.82% and 0.94% over expansions and recessions, respectively, and corresponding volatilities of 2.78% and 3.77%.

[Table 4 here]

Taken together, the results presented in this section show that PC is economically significant, volatile, and has become increasingly scarce and valuable over the past decades.

3.2.1 What Is PC Made Of?

We defined PC as the set of all non-pecuniary drivers of utility, which are unaccounted for in aggregate measures but that affect the composition of Market-C. This working definition allows PC to be measured but does not answer the question of what PC is made of specifically. It is impossible to provide a definite answer to this question given that the set of utility drivers, documented in the literature or otherwise, that satisfy the definition of PC is possibly

[25] The unconditional means and standard deviations of the consumption series are presented in Tables 1 and 3.

unbounded. However, we present in this section two exploratory analyses that use the recovered latent consumption series to study some potential sources of PC.

We first investigate how the recovered PC series are related to survey data on how households allocate their time. The motivation for this analysis is based on two ideas. The first idea, which can be traced back to Becker's (1965) theory of the allocation of time, is that the utility derived from consumption requires the input of households' time. The second idea is that work and leisure themselves are drivers of utility (e.g., Akerlof (1982) and Hagedorn and Manovskii (2008)) so that these are potential components of PC.

We use data from the American Time Use Survey (ATUS) covering the period 2003 to 2018 to construct an annual series of the average number of hours per individual per day spent in nine time-use activities. The categories that we use are: *Sleeping*; *Eating*; *Household and Care*; *Shopping*; *Working*; *Learning*; *Civic and Volunteering*; *Leisure, Sports, and Social (ex-TV)*; and *TV*. We follow Bertrand and Schanzenbach (2009) and consider the time spent watching TV separately from other forms of leisure because of its importance and because of its nonsocial nature. Appendix B.3 presents the construction details and Panel A of Table 5 presents the summary statistics of the time-use series.

Panel B of Table 5 presents the results of multivariate regressions of each of the nine time-use series on the expenditure shares of durables and nondurables, and shadow expenditure shares of PC. The expenditure share of services is implicitly considered in the regressions since the sum of the expenditure shares of the three marketable goods is one. Panel C of Table 5 presents the results of multivariate regressions similar to the ones in panel B, but in which we use time-series of the log growth of all three marketable consumption goods and PC. The regressions in both panels include a time–trend.

The results in Panels B and C of Table 5 provide complementary pieces of information about PC. Both the quantity growth and the shadow expenditure share of PC are positively related to the more social types of leisure represented by the category *Leisure, Sports, and Social (ex-TV)*. This result is consistent with the hypothesis that leisure and social interactions are forms of PC. Based on the findings in Panels B and C, other potential forms of PC are sleeping and eating, which can be considered close cousins of leisure. In contrast, time spent watching TV is negatively related to the quantity growth but positively related to the shadow expenditure share of PC. The interpretation of these results is that watching TV, which is a nonsocial form of leisure, is a substitute for PC but possibly not a form of PC. Other types of activities that seem to substitute PC are *Working*; and *Household and Care*.

[Table 5 here]

The second analysis investigates potential candidates for the non-pecuniary determinants of utility that make up PC. Specifically, we study the relationship between PC and different wellbeing or distress indicators. The wellbeing indicators are the University of Michigan Consumer Sentiment Index, which is a survey-based measure that tracks consumers' optimism about the near future, and the average earnings from the BEA, which is related to financial stability. The distress indicators are the suicide rate, which proxies for poor mental health, from the CDC/NCHS, the homicide rate, which proxies for the general sense of insecurity, also from the CDC/NCHS, and the unemployment rate from the BLS. We also include the number of hours worked from the BEA, which can be interpreted as an indicator of distress (unemployment or low leisure time) or as an indicator of wellbeing (job security and satisfaction). Panel A of Table 6 presents the summary statistics of the six indicators.

Panel B of Table 6 presents the results of multivariate regressions of the log change in each wellbeing or distress indicator on changes of the expenditure shares of durables and nondurables, and the shadow expenditure share of PC. The panel shows that, in general, the shadow expenditure share of PC is positively related to wellbeing and negatively related to distress. In particular, the slopes of the shadow expenditure share of PC, ΔS_{L}, are positive for suicide rates and unemployment rates and negative for average earnings. ΔS_{L} does not appear to be related to changes in consumer sentiment, homicide rates, and hours worked.

Panel C of Table 6 presents the results of regressions of the indicators of wellbeing or distress on measures of the quantity growth of the different consumption series. In general, we find that, in contrast with the growth in the shadow expenditure share of PC, the quantity growth of PC is generally positively related to wellbeing and negatively related to distress. Specifically, the panel shows that the slope of Δq_{L} is positive for average earnings and negative for unemployment rates. Although negative, the slope of Δq_{L} on suicide rates is not statistically significant. Note that the flipped signs between Panels B and C are unique to PC and do not occur for durables and nondurables.

Overall, the results in Tables 5 and 6 suggest that social forms of leisure, job security, and financial stability are possible sources of PC. In contrast, the analysis indicates that non-social forms of leisure, home production, and work are not components of PC. A hypothesis left for future research is whether the decline in Total-C over the last few decades, shown in Figure 3, is due the decline in job security and shift from more social forms of leisure (e.g., talking in person with family and friends) to less social forms of leisure (e.g., watching TV).[26]

[Table 6 here]

3.3 Model Fit with Alternative Specifications

Do we need PC and a flexible consumption aggregator to capture the consumption expenditure share dynamics in the data? To answer this question, in this section, we present the model fit of two alternative model specifications: a translog-based model and a CES-based model, both without PC.

3.3.1 Translog-Based Model with No PC

We first investigate the importance of the existence of PC for the good fit of our baseline specification, which can be seen in Figure 2. We estimate an alternative translog-based model that does not embed PC but that is otherwise identical to that in our main specification. The consumption aggregator $\mathrm{C}^{\mathrm{TL,NOPC}}[.]$ used in the alternative specification is also defined

[26]This hypothesis relates to that in Di Tella and MacCulloch (2008), p.38: "*Maybe adaptation to income, increased anxiety and job insecurity caused by globalization, stress at work, or the rise of television that has become addictive, have led to flat happiness in the face of sharply rising average incomes.*"

in Equations (4a)–(4b) but in which the coefficients are given by

$$\mathbf{a}^{\text{NOPC}} \equiv \begin{bmatrix} a_{\text{D}} \\ a_{\text{N}} \\ a_{\text{S}} \end{bmatrix} \in [0,1]^{3\times 1}, \quad \text{and} \quad \mathbf{b}^{\text{NOPC}} \equiv \begin{bmatrix} b_{\text{DD}} & b_{\text{DN}} & b_{\text{DS}} \\ b_{\text{ND}} & b_{\text{NN}} & b_{\text{NS}} \\ b_{\text{SD}} & b_{\text{SN}} & b_{\text{SS}} \end{bmatrix} \in \mathbb{R}^{3\times 3}, \tag{21}$$

and subject to the parameter restriction $a_{\text{S}} = 1 - a_{\text{D}} - a_{\text{N}}$. We do not impose additional restrictions to the parameters in Equation (21) given the evidence discussed in Section 1.4.1 against the homotheticity of preferences defined over marketable goods.

Panel A of Figure 5 shows that the fit of the translog-based model without PC is significantly worse than that of the baseline model with PC, reported in Figure 2. The most significant difference in performance from removing PC seems to be the ability to explain the dynamics of the expenditure shares of the durable good. For instance, the values of the scaled MAE from Belo et al. (2019) are 3.3% with PC vs. 20.1% without PC. The differences in the goodness-of-fit measures for the expenditure shares of nondurables (3.2% vs. 5.4%) and services (2.8% vs. 3.6%) are more modest, but also in favor of the model with PC.

3.3.2 CES-Based Model

Next, we investigate the importance to the model-fit of using a translog consumption aggregator relative to a less flexible but simpler CES aggregator. Since a CES-based model with PC cannot be identified, we estimate a model that is identical to that discussed in Section 1.5.2 but in which the consumption aggregator is defined over marketable goods only, as given by $C^{\text{CES}}[\mathbf{Q}_t] \equiv \left(a_{\text{D}}(Q_t^{\text{D}})^{\rho} + a_{\text{N}}(Q_t^{\text{N}})^{\rho} + (1 - a_{\text{D}} - a_{\text{N}})(Q_t^{\text{S}})^{\rho}\right)^{1/\rho}$, where ${}^{1}/_{1-\rho} > 0$ is the elasticity of substitution between the goods.

Panel B of Figure 5 shows the fit of the CES-based model. The general goodness-of-fit of the CES-based model at explaining the marketable consumption expenditure shares is worse than that of the translog-based model without PC, which can be seen in Panel A of Figure 5. The scaled MAE of the fit of the expenditure shares of durable goods is 40.7% in the CES case vs. 5.4% in the translog model without PC. The scaled MAE values for nondurables are 8.2% (CES), and 5.4% (translog with no PC), and the scaled MAE values for services are 11.1% (CES) and 3.6% (translog with no PC). These results show that the greater flexibility of the translog aggregator is not only needed for the recovery of PC but also to explain the dynamics of marketable consumption.

[Figure 5 here]

4 Application: Consumption-CAPM

Table 2 shows that the estimated model is successful at matching consumption moments and implies an RRA coefficient of around 11. A natural question is whether the relatively low RRA estimated value is, as predicted by our model, due to the more comprehensive nature of Total-C relative to the Market-C series typically used in the asset pricing literature, or due to a low relative estimation weight of the financial moments relative to that of the consumption moments. To answer the question, in this section, we abstract away from the consumption-goods markets and focus on the fit of the model on financial moments only. This focus also allows for a more direct comparison between our findings and those from the asset pricing literature.

The approach that we use here is based on the standard empirical implementation of the consumption-based asset pricing model (C-CAPM). Specifically, we compare five different specifications of the C-CAPM, one with PC and four without PC, following the procedure in Savov (2011). We fix the patience constant β of the power utility function from Equation (7) and estimate the implied coefficient of RRA by minimizing the pricing errors of Equation (18) in explaining the excess returns of the market portfolio, and in an additional analysis, the returns of the 25 size-B/M portfolios from Fama and French (1993).

Table 7 presents the results of the exercise. The first specification considers an aggregate consumption measure that includes PC. Specification (1), which is the baseline, uses the recovered series of Total-C presented in Section 3. For comparison, we also test the standard C-CAPM model using four additional specifications without PC: Specification (2), based on the translog aggregator without PC discussed in Section 3.3.1; specification (3) based on the CES aggregator without PC discussed in Section 3.3.2; specification (4) based on the standard proxy for consumption based on the Market-C in nondurables and services, and specification (5), which is based on the garbage-based proxy for the quantity of aggregate consumption from Savov (2011).

Panel A of Table 7 presents the estimation results using the market portfolio as the test asset. All models shown in the panel can generate a zero pricing error so that the implied RRA estimates are informative about the relative performance of the different specifications. The high implied RRA using Market-C (specification 4), of around 47, confirms the documented unplausibly high levels of risk aversion required to explain the equity premium with power utility estimated with real marketable consumption expenditures. The estimate of RRA from specification 5, which is about 17, precisely matches that presented in Savov (2011).

The new insight from the Panel A of Table 7, and one of the main contributions of our paper, is that using Total-C (specification 1) leads to a relatively low implied coefficient of RRA, around 12.6, which is consistent with the estimated value of 11 presented in Table 2. This result is likely to follow from the high volatility of Total-C relative to the Market-C. Despite the lower estimate of RRA, the standard errors presented suggest that we cannot reject the hypothesis that our RRA estimates are statistically different from that in the garbage-based model. Specification (2) shows that a model based on the translog aggregator but without PC implies an extremely high RRA of 61. Specification (3) shows that a model based on the CES aggregator, in which, as discussed, the existence of PC is irrelevant in the estimation, leads to results very similar to those of the standard C-CAPM (i.e., specification 4), in which consumption is proxied by real marketable consumption expenditures (i.e., Market-C).

Panel A of Table 7 also presents the estimated real risk-free rates implied by the five specifications and confirms the challenge faced by the canonical C-CAPM with power utility of simultaneously matching the equity risk premium and the risk-free rate. Despite this difficulty, the implied risk-free rates of our model (specification 1) is around 5%, which is not excessively high, especially in comparison with the implied risk-free rate of around 17% of the successful garbage-based model of Savov (2011) (specification 5).

Panel B of Table 7 presents the results of an exercise similar to that in Panel A but based on the returns of the 25 portfolios of stocks double sorted on market value and book-to-market ratios from Kenneth French's data library. We use a single-stage GMM procedure with an equally-weighted weighting matrix in the estimation. The estimates of RRA are generally consistent with those presented in Panel A. Interestingly, the point estimate of around 11 using the Total-C measure that includes PC (specification 1) is lower than that of the garbage-based model, which is about 22 (specification 5).

Taken together, the results in this section show that the dynamics of the recovered latent consumption series are relevant for economic models that rely on the dynamics of marginal utility over consumption, which are exemplified in the paper by the consumption-based CAPM. Adding PC to standard measures of aggregate consumption helps the ability of the C-CAPM with power utility to rationalize the asset pricing series from the data.

[Table 7 here]

5 Conclusion

Marketable aggregate consumption (Market-C) can only account for goods and services for which prices and quantities can be observed. We denote by priceless consumption (PC) all non-pecuniary drivers of utility that are unaccounted for in Market-C but that affect its composition, and propose a structural estimation methodology to recover the dynamics of PC in the data. Our estimation results suggest that: i) PC is a significant and volatile component of the true aggregate consumption (Total-C); ii) the quantity consumed of PC has been declining; but iii) the shadow expenditure share in PC has been growing over time, which indicates that PC has become an increasingly scarce good. As an application, we use the estimated PC series to provide new empirical evidence in support of the standard C-CAPM with power utility. When use Total-C, which accounts for PC, the empirical fit of the model improves significantly. In particular, the model can match the observed equity risk premium with a relatively low relative risk aversion coefficient of around 12.

A more complete empirical measure of aggregate consumption allows for a better evaluation of the many economic theories in which consumption plays a central role. In addition to the application considered here (i.e., testing the C-CAPM with Total-C), other promising applications include the analysis of the welfare costs of business cycle fluctuations (e.g., Lucas (1987)), the degree of international risk sharing (e.g., Brandt, Cochrane, and Santa-Clara (2006)), the potential link between PC and the Easterlin Paradox (e.g., Easterlin (1974)), and the construction of more comprehensive measures of wellbeing (e.g., Diener (2009)).

Finally, the approach presented in this paper is based on a representative consumer and focuses on the U.S. economy. Future research can extend this approach in several promising directions. Two examples are the study of whether heterogeneous consumers value the latent PC differently and relate the variation to household characteristics, and, in the spirit of Clark, Frijters, and Shields (2008), the study of whether PC varies across countries, and relate the variation to country characteristics. Examining these and other links will help us further understand PC and its economic implications.

References

[1] Ait-Sahalia, Yacine, Jonathan A. Parker, and Motohiro Yogo, 2004, Luxury goods and the equity premium, *Journal of Finance* 59, 2959–3004.

[2] Akerlof, George A., 1982, Labor contracts as partial gift exchange, *Quarterly Journal of Economics* 97, 543–569.

[3] Ambrus, Attila, Markus Mobius, and Adam Szeidl, 2014, Consumption risk-sharing in social networks, *American Economic Review* 104, 149–82.

[4] Azzi, Corry, and Ronald Ehrenberg, 1975, Household allocation of time and church attendance, *Journal of Political Economy* 83, 27–56.

[5] Becker, Gary S., 1965, A theory of the allocation of time, *The Economic Journal* 75, 493–517.

[6] Belo, Frederico, Vito Gala, Juliana Salomao, and Maria Ana Vitorino, 2019, Decomposing Firm Value, working paper.

[7] Benabou, Roland, and Jean Tirole, 2011, Identity, morals, and taboos: Beliefs as assets, *Quarterly Journal of Economics* 126, 805–855.

[8] Bertrand, Marianne, and Diane Whitmore Schanzenbach, 2009, Time use and food consumption, *American Economic Review Papers and Proceedings* 99, 170–76.

[9] Binsbergen, Jules H. Van, 2016, Good-Specific Habit Formation and the Cross-Section of Expected Returns, *Journal of Finance* 71, 1699–1732.

[10] Brandt, Michael W, John H Cochrane, and Pedro Santa-Clara, 2006, International risk sharing is better than you think, or exchange rates are too smooth, *Journal of Monetary Economics* 53, 671–698.

[11] Breeden, Douglas T., 1979, An intertemporal asset pricing model with stochastic consumption and investment opportunities, *Journal of Financial Economics* 7, 265–296.

[12] Breeden, Douglas T., Michael R. Gibbons, and Robert H. Litzenberger, 1989, Empirical tests of the consumption-oriented CAPM, *Journal of Finance* 44, 231–262.

[13] Buera, Francisco J., and Joseph P. Kaboski, 2012, The Rise of the Service Economy, *American Economic Review* 102, 2540–2569.

[14] Campbell, John Y, 2017, *Financial decisions and markets: a course in asset pricing* (Princeton University Press).

[15] Christensen, Laurits R., Dale W. Jorgenson, and Lawrence J. Lau, 1973, Transcendental Logarithmic Production Frontiers, *Review of Economics and Statistics* 55, 28–45.

[16] Christensen, Laurits R., Dale W. Jorgenson, and Lawrence J. Lau, 1975, Transcendental Logarithmic Utility Functions, *American Economic Review* 65, 367–383.

[17] Clark, Andrew E, Paul Frijters, and Michael A Shields, 2008, Relative income, happiness, and utility: An explanation for the easterlin paradox and other puzzles, *Journal of Economic literature* 46, 95–144.

[18] Cochrane, John H, 2011, Discount rates, *Journal of Finance* 66, 1047–1108.

[19] Da, Zhi, Wei Yang, and Hayong Yun, 2015, Household Production and Asset Prices, *Management Science* 62, 387–409.

[20] Di Tella, Rafael, and Robert MacCulloch, 2008, Gross national happiness as an answer to the easterlin paradox?, *Journal of Development Economics* 86, 22–42.

[21] Diener, Edward, 2009, *The science of well-being: The collected works of Ed Diener*, volume 37 (Springer).

[22] Easterlin, Richard A, 1974, Does economic growth improve the human lot? some empirical evidence, in *Nations and households in economic growth*, 89–125 (Elsevier).

[23] Eckstein, Zvi, and Kenneth I. Wolpin, 1999, Why youths drop out of high school: The impact of preferences, opportunities, and abilities, *Econometrica* 67, 1295–1339.

[24] Eichenbaum, Martin, and Lars Peter Hansen, 1990, Estimating models with intertemporal substitution using aggregate time series data, *Journal of Business & Economic Statistics* 8, 53–69.

[25] Fama, Eugene F., and Kenneth R. French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3–56.

[26] Gomes, Joao F., Leonid Kogan, and Motohiro Yogo, 2009, Durability of Output and Expected Stock Returns, *Journal of Political Economy* 117.

[27] Gorodnichenko, Yuriy, and Michael Weber, 2016, Are Sticky Prices Costly? Evidence from the Stock Market, *American Economic Review* 106, 165–199.

[28] Greene, William H., 2003, *Econometric Analysis*, fifth edition (Pearson Education, New Jersey, USA).

[29] Hagedorn, Marcus, and Iourii Manovskii, 2008, The cyclical behavior of equilibrium unemployment and vacancies revisited, *American Economic Review* 98, 1692–1706.

[30] Jack, William, and Tavneet Suri, 2014, Risk sharing and transactions costs: Evidence from Kenya's mobile money revolution, *American Economic Review* 104, 183–223.

[31] Kennedy, Robert F., 1968, *Remarks of Robert F. Kennedy at the University of Kansas, March 18, 1968* (John F. Kennedy Presidential Library and Museum).

[32] Lazear, Edward, 1977, Education: consumption or production?, *Journal of Political Economy* 85, 569–597.

[33] Lucas, Robert E., 1978, Asset prices in an exchange economy, *Econometrica* 46, 1429–1445.

[34] Lucas, Robert E, 1987, *Models of business cycles*, volume 26 (Basil Blackwell Oxford).

[35] Piazzesi, Monika, Martin Schneider, and Selale Tuzel, 2007, Housing, consumption and asset pricing, *Journal of Financial Economics* 83, 531–569.

[36] Samuelson, P. A., and S. Swamy, 1974, Invariant economic index numbers and canonical duality: Survey and synthesis, *American Economic Review* 64, 566–593.

[37] Savov, Alexi, 2011, Asset pricing with garbage, *Journal of Finance* 66, 177–201.

[38] Stiglitz, Joseph E, Amartya Sen, and Jean-Paul Fitoussi, 2010, *Mismeasuring our lives: Why GDP doesn't add up* (The New Press).

[39] Yogo, Motohiro, 2006, A Consumption-Based Explanation of Expected Stock Returns, *Journal of Finance* 61.

Appendix

A Details of the Demand System Derivation

Applying the Bellman principle, we can express the maximization problem in Equation (2a), which involves decisions over an infinite series of consumption quantities and portfolio allocations into a recursive maximization problem that only involves decisions over the current consumption and portfolio allocation, as given by

$$V_t = \max_{\{\mathbf{Q}_t, \mathbf{A}_t\}} \mathrm{U}[C_t] + \beta \mathbb{E}_t[V_{t+1}], \tag{A1}$$

subject to the law of motion in Equation (1) and to the constraints in Equations (2b)–(2c).

The Lagrangian function associated with the maximization problem in Equation (A1) is given by

$$\mathcal{L}_t = \mathrm{U}[C_t] + \beta \mathbb{E}_t[V_{t+1}] - \lambda_t \left(\sum_{j=0}^{n} A_{j,t} + X_t^{\text{DNS}} + \Delta_t^{\text{L}} P_t^{\text{L}} - \sum_{j=0}^{n} A_{j,t-1} R_{j,t} \right). \tag{A2}$$

The first order conditions of the Lagrangian function in Equation (A2) w.r.t. $q_t^{\text{D}} \equiv \log[Q_t^{\text{D}}]$, $q_t^{\text{N}} \equiv \log[Q_t^{\text{N}}]$, $q_t^{\text{S}} \equiv \log[Q_t^{\text{S}}]$, Δ_t^{L}, and $A_{j,t}$ are given by

$$\frac{\partial \mathcal{L}_t}{\partial f_t^{\text{D}}} = U_{\text{C}}[C_t] C_t c_{\text{D}}[\mathbf{q}_t] + \beta\, \mathbb{E}_t \left[\left(\frac{\partial V_{t+1}}{\partial q_t^{\text{D}}} \right) \right] - \lambda_t Q_t^{\text{D}} = 0, \tag{A3a}$$

$$\frac{\partial \mathcal{L}_t}{\partial q_t^{\text{N}}} = U_{\text{C}}[C_t] C_t c_{\text{N}}[\mathbf{q}_t] - \lambda_t P_t^{\text{N}} Q_t^{\text{N}} = 0, \tag{A3b}$$

$$\frac{\partial \mathcal{L}_t}{\partial q_t^{\text{S}}} = U_{\text{C}}[C_t] C_t c_{\text{S}}[\mathbf{q}_t] - \lambda_t P_t^{\text{S}} Q_t^{\text{S}} = 0, \tag{A3c}$$

$$\frac{\partial \mathcal{L}_t}{\partial \Delta_t^{\text{L}}} = U_{\text{C}}[C_t] C_t c_{\text{L}}[\mathbf{q}_t] - \lambda_t P_t^{\text{L}} Q_t^{\text{L}} = 0, \tag{A3d}$$

$$\frac{\partial \mathcal{L}_t}{\partial A_{j,t}} = \beta\, \mathbb{E}_t \left[\left(\frac{\partial V_{t+1}}{\partial A_{j,t}} \right) \right] - \lambda_t = 0, \tag{A3e}$$

where the first condition follows from the fact that $({Q_t^{\text{D}}}/{F_t^{\text{D}}}){\partial q_t^{\text{D}}}/{\partial f_t^{\text{D}}} = 1$. The envelope conditions of the Lagrangian function in Equation (A2) w.r.t. q_{t-1}^{D} and $A_{j,t-1}$ are given by

$$\frac{\partial V_t}{\partial q^{\text{D}}_{t-1}} = (1-\delta)\lambda_t, \tag{A4a}$$

$$\frac{\partial V_t}{\partial A_{j,t-1}} = -R_{j,t}\lambda_t. \tag{A4b}$$

The first order conditions and the envelope conditions in Equations (A3) and (A4) jointly imply the model-implied prices presented in Equations (3e)–(3g).

B Sample Construction Details

B.1 Consumption Data

We obtain all the marketable consumption data from the Bureau of Economic Analysis (BEA). The empirical series of the prices and expenditures of marketable goods are from the BEA's NIPA tables. Specifically, we use the series for durable goods (*DDUR*), nondurable goods (*DNDG*), and services (*DSER*) from BEA tables 2.3.4 (prices) and 2.3.4 (expenditures). The data is at the annual frequency and spans the period 1929 to 2018. The expenditures series are scaled by the total U.S. population series produced by the U.S. Census Bureau and retrieved from the Federal Reserve Bank of St. Louis (FRED, series code *POP*). We construct series of quantities purchased of the durable ($\hat{F}^{\text{D}}_t$), nondurable ($\hat{F}^{\text{N}}_t$), and service goods ($\hat{F}^{\text{S}}_t$) as the ratios of expenditures and prices of the respective goods. The quantities consumed of the nondurable and service goods in a given year equal the quantities purchased by agents of these goods (i.e., $\hat{Q}^{\text{N}}_t = \hat{F}^{\text{N}}_t$ and $\hat{Q}^{\text{S}}_t = \hat{F}^{\text{S}}_t$). Since the durable good is nonperishable, we have that the quantity purchased ($\hat{F}^{\text{D}}_t$) is different than the stock ($\hat{Q}^{\text{D}}_t$) of this good. We create an empirical series for the quantity stock of the durable good, $\hat{Q}^{\text{D}}_t$, parallel to that in the model, Q^{D}_t. We set the quantity stock of durables in 1929 using the quantity flow of durables in that year and the ratio of the expenditure and net stock of consumer durables at current cost from the BEA's Fixed Assets Accounts Table 1.1. To be consistent with the model, the stock of durables for the years 1930 onward is constructed iteratively using the law of motion in Equation (1) with a durable good depreciation rate

$\hat{\delta} = 17.27\%$, which is the annualized quarterly depreciation rate for durable goods from Gomes et al. (2009).

B.2 Price Indexes and Financial Data

Since the durable good is the numeraire in the model, we construct two sets of series of the observable goods: one using durables as the numeraire good and one in real 2018 U.S. Dollars. To construct the series based on durables as the numeraire good, we scale all prices and expenditures series with the price index series for the durable good and multiply the resulting series with the durable price index value of 2018. The risk-free rate from Equation (9), which represents the rate of a bond that pays a fixed number of units of the numeraire good, is not observable in the data. We approximate the real risk-free rate expressed in units of durable goods with the annualized nominal Treasury Bill rate from Kenneth French's data library deflated by the change in the price index of the durable good over the following year. The error from approximating the expected price change with the realized price change should be small, given that the price growth of the durable good has a low covariance with Market-C growth and is highly predictable.[27] We use the PCE price index to construct the series based on real 2018 U.S. Dollars. We construct a series of aggregate expenditures in marketable goods by summing the expenditures of durable, nondurable, and service goods. Finally, we construct expenditure shares series by dividing the expenditures of a given good by the aggregate expenditure in marketable goods.

The market factor and 25 size-B/M portfolio data used in most of the analyses are from Fama and French (1993) and were obtained from Kenneth French's website. The exceptions are the analyses involving the municipal solid waste series from Savov (2011), in which the financial data used was provided by Alexi Savov.

B.3 Time-Use Series

The American Time Use Survey (ATUS) data is obtained from the U.S. Bureau of Labor Statistics and covers the period 2003 to 2018. The ATUS dataset includes more than 400

[27] In our sample, the price growth for the durable good has low volatility (4.13% nominal and 2.05% real) and low correlation (0.17 and -0.11) with the aggregate real expenditure growth, and high autocorrelation (0.75 and 0.64).

detailed categories of time use. To reduce the complexity of the analysis, we group the detailed categories into the nine time-use activity categories presented in Table 5: sleeping and napping (*Sleeping*), eating and drinking (*Eating*), household activities, grooming, and caring for and helping household and non-household members (*Household and Care*), shopping in person, online, or by phone (*Shopping*), work-related activities (*Working*), education-related activities (*Learning*), religious and volunteering activities (*Civic and Volunteering*), activities related to leisure (except for watching TV), sports, and socializing (*Leisure, Sports, and Social (ex-TV)*), and watching TV (*TV*). We construct a series of yearly average across all households of the number of daily hours spent in activities in each of the nine groups considered. The nine series account for over 23.5 hours per household per day on average.

C Model-Implied Standard Errors

This appendix presents the calculation of the standard errors of the parameter estimates in Table 2 and standard errors of the recovered consumption series used in the confidence intervals in Figure 3.

C.1 Standard Errors of the Parameter Estimates

The vector of GMM standard errors of $\hat{\theta}$ is given by $\mathrm{SE}[\hat{\theta}] = \sqrt{\frac{\mathrm{diag}[\Gamma_\theta]'}{T}}$, where Γ_θ is the covariance matrix of parameter estimates given by

$$\Gamma_\theta \equiv T^{-1} \left(\mathbf{d}_{\mathrm{g}}[\hat{\theta}]' \, \mathrm{I}_{13} \mathbf{d}_{\mathrm{g}}[\hat{\theta}]\right)^{-1} \mathbf{d}_{\mathrm{g}}[\hat{\theta}]' \, \mathrm{I}_{13} W \, \mathrm{I}_{13} \mathbf{d}_{\mathrm{g}}[\hat{\theta}] \left(\mathbf{d}_{\mathrm{g}}[\hat{\theta}]' \, \mathrm{I}_{13} \mathbf{d}_{\mathrm{g}}[\hat{\theta}]\right)^{-1}, \tag{C5a}$$

T is the number of periods in the sample, W is the 13×13 second stage weighting matrix from Equation (15), I_{13} is a 13×13 identity matrix, and $\mathbf{d}_{\mathrm{g}}[\hat{\theta}] \equiv \left.\frac{\partial \, \mathrm{g}[\theta|\hat{Z}_t]}{\partial \theta}\right|_{\theta=\hat{\theta}}$ is a 13×11 Jacobian matrix of the 1×13 moment vector in Equation (15) with respect to the 11×1 parameter vector θ. We calculate the standard errors of the parameters that are not in the vector θ (i.e., a_{L}, b_{DD}, b_{NN}, b_{SS}, b_{LL}) from their definitions in Equations (6a), (6b), and (6c) and from the covariance matrix of the parameter estimates in Equation (C5a).

C.2 Standard Errors of the Recovered Latent Consumption Series

We use the delta method to construct the model-implied standard error of each the recovered latent consumption series. Let $h_t \equiv \mathrm{h}[\hat{Z}_t, \hat{\theta}]$ denote the time-t value of one of the four model-implied series Q_t^{L}, C_t, X_t^{DNSL}, and S_t^{L} in Equations (20a)–(20d). The standard error of $\hat{h}_t \equiv \mathrm{h}[\hat{Z}_t, \hat{\theta}]$ is given by

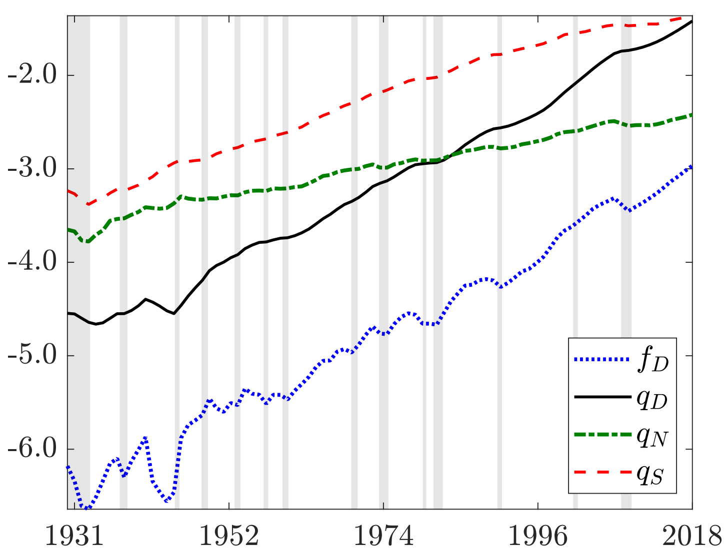
$$\mathrm{SE}[\hat{h}] = \sqrt{\left(\mathbf{d}_{\mathrm{h}}[\hat{Z},\hat{\theta}]'\,\mathrm{I}_{11}\,\mathbf{d}_{\mathrm{h}}[\hat{Z}_t,\hat{\theta}]\right)^{-1}\mathbf{d}_{\mathrm{h}}[\hat{Z},\hat{\theta}]'\,\mathrm{I}_{11}\,\Gamma_\theta\,\mathrm{I}_{11}\,\mathbf{d}_{\mathrm{h}}[\hat{Z},\hat{\theta}]\left(\mathbf{d}_{\mathrm{h}}[\hat{Z},\hat{\theta}]'\,\mathrm{I}_{11}\,\mathbf{d}_{\mathrm{h}}[\hat{Z},\hat{\theta}]\right)^{-1}}, \tag{C6}$$

where Γ_θ is the covariance matrix of parameter estimates from Equation (C5a), I_{11} is a 11 $\times$ 11 identity matrix, and $\mathbf{d}_{\mathrm{h}}[\hat{Z},\hat{\theta}] \equiv \frac{1}{T}\sum_{t=1}^{T} \frac{\partial\,\mathrm{g}[\hat{Z}_t,\hat{\theta}]}{\partial h}\Big|_{h=\hat{h}_t}$ is a 11 $\times$ 1 vector.

Figure 1
Quantity and Expenditure Shares of the Marketable Goods

Panel A shows the time series of the quantities of the marketable consumption goods. f_{D} and q_{D} denote the log quantity flow and log quantity in stock of durables, respectively, q_{N} denotes the log quantity consumed of nondurables, and q_{S} denotes the log quantity consumed of services. Panel B shows the consumption expenditure shares S of the marketable consumption goods. Shaded vertical bars denote recessions as reported by the NBER. The sample is at the annual frequency and covers the period from 1930 to 2018.

Panel A: Log Quantities



Panel B: Expenditure Shares

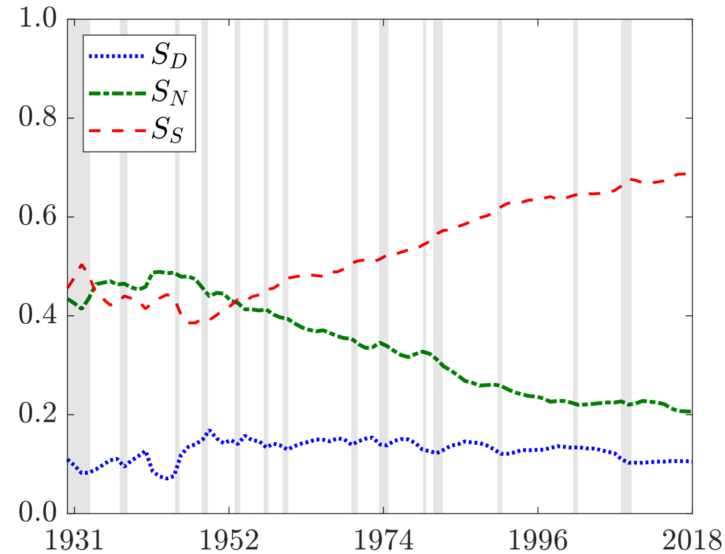


Figure 2
Estimated Model Fit

This figure presents the time series of the expenditure shares of the marketable consumption goods from the data and the estimated model. *MAE* is the mean absolute error between the expenditure shares in the data and from the estimated model. Shaded vertical bars denote recessions as reported by the NBER. The sample is at the annual frequency and covers the period from 1930 to 2018.

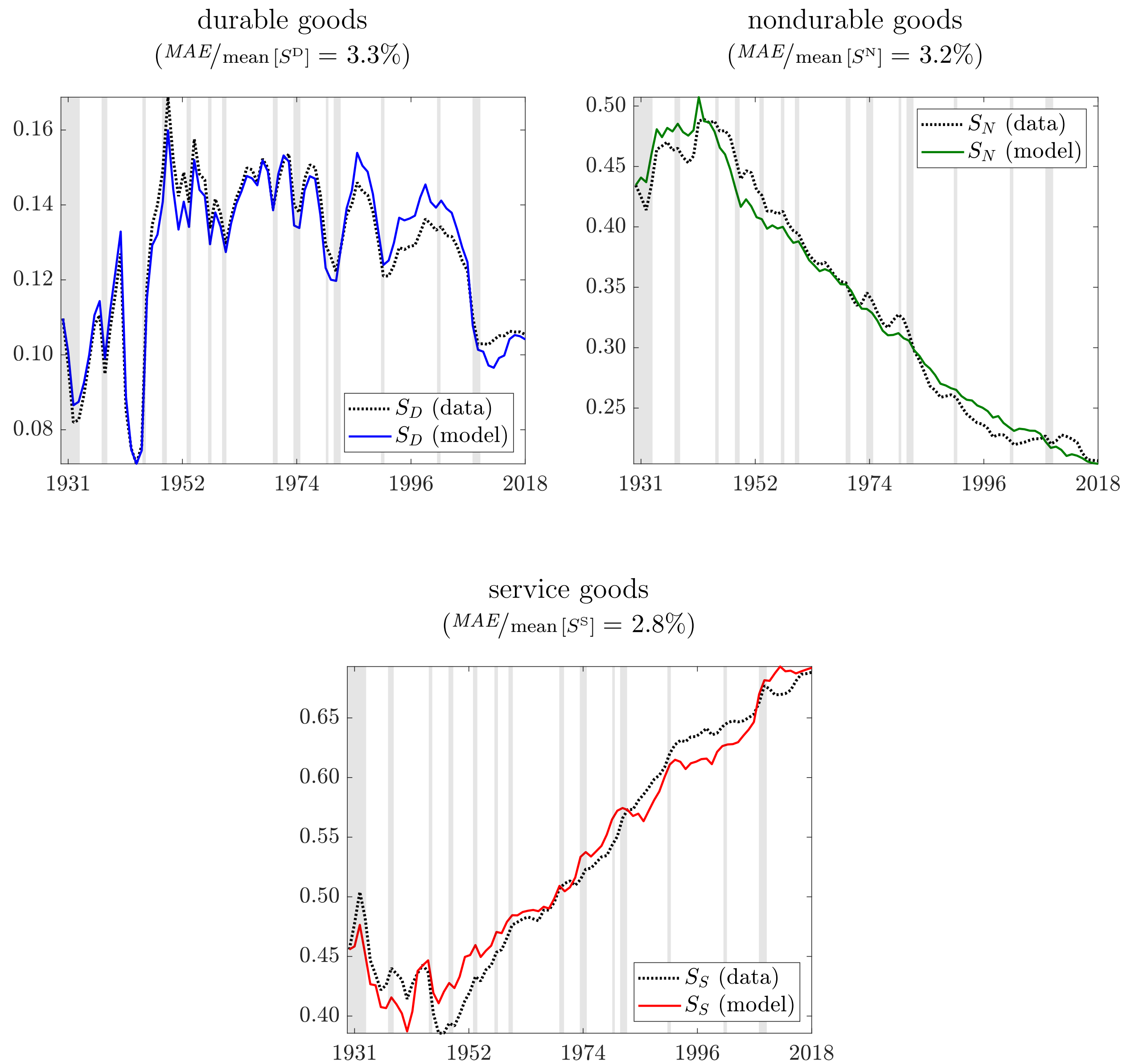


Figure 3
Recovered Latent Consumption Series

This figure shows the latent consumption series recovered from the model estimation. Total-C is the true aggregate consumption. The standard errors used in the confidence intervals are presented in Appendix C.2. Shaded vertical bars denote recessions as reported by the NBER. The sample is at the annual frequency and covers the period from 1930 to 2018.

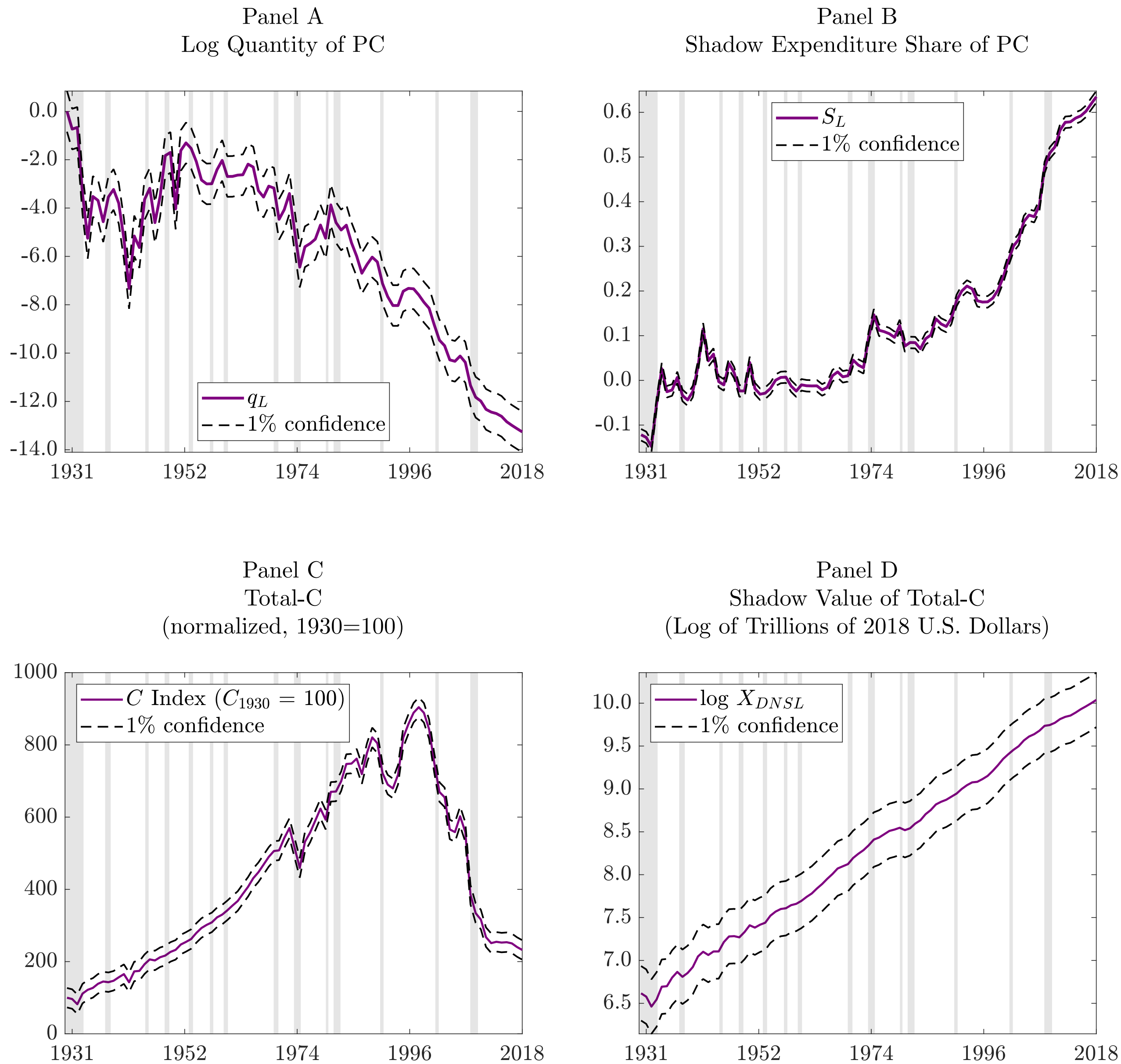


Figure 4
Recovered True Aggregate Consumption vs. Measured Aggregate Consumption

The figures overlays the recovered latent true aggregate consumption (Total-C) and the aggregate marketable consumption (Market-C). Shaded vertical bars denote recessions as reported by the NBER. The sample is at the annual frequency and covers the period from 1930 to 2018.

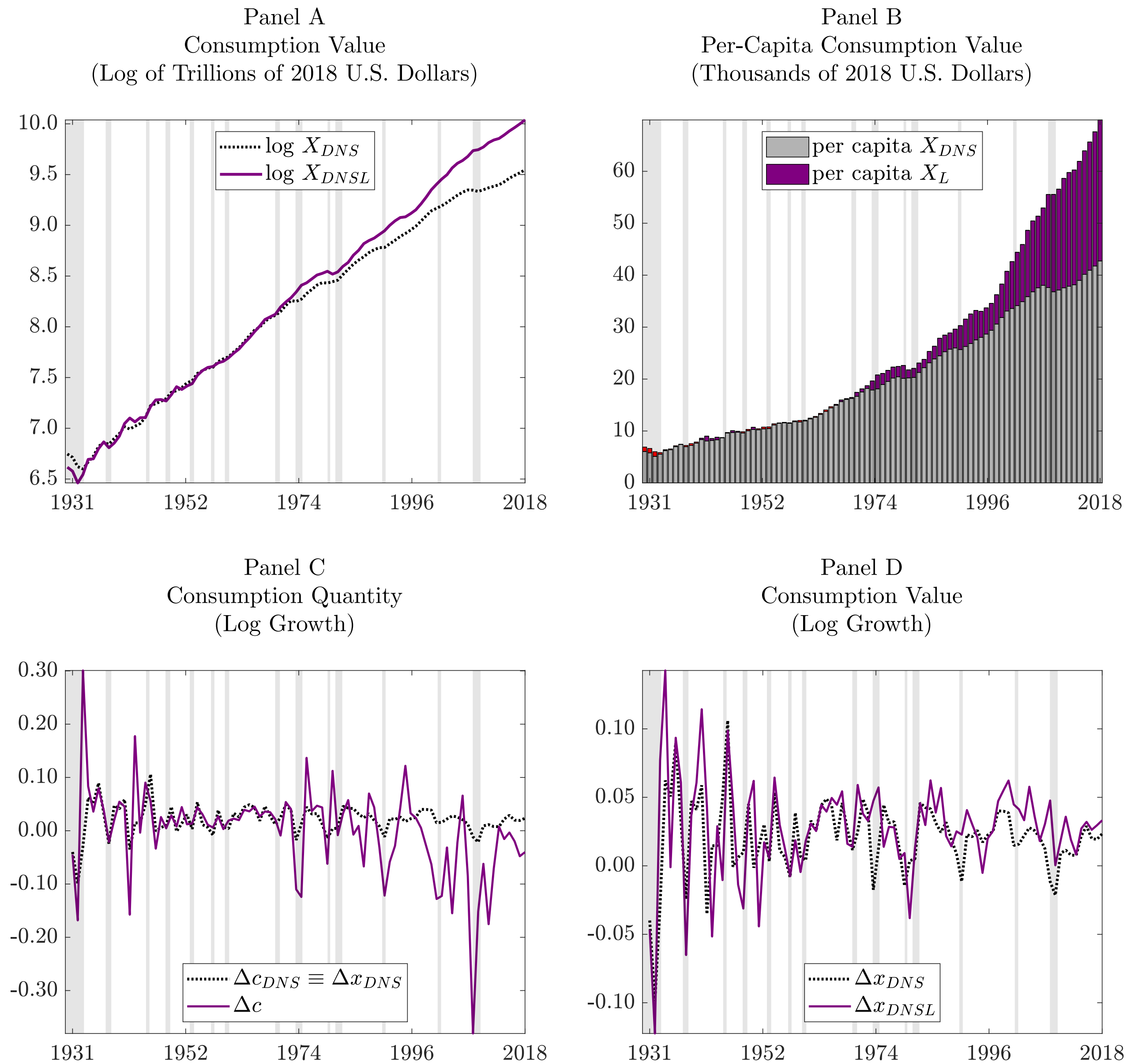
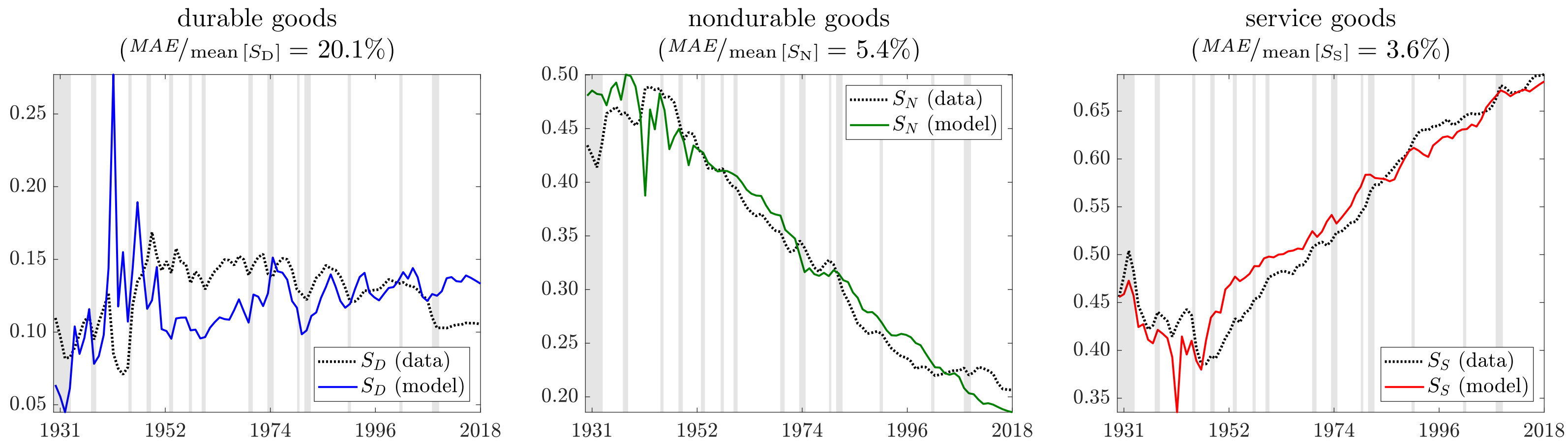


Figure 5
Alternative Specifications without PC

This figure shows the fit of two alternative model specifications: The first based on the translog consumption aggregator without PC (Panel A) and the second based on the CES consumption aggregator without PC $(a_{\mathrm{D}}(Q_t^{\mathrm{D}})^\rho + a_{\mathrm{N}}(Q_t^{\mathrm{N}})^\rho + (1 - a_{\mathrm{D}} - a_{\mathrm{N}})(Q_t^{\mathrm{S}})^\rho)^{1/\rho}$ (Panel B). *MAE* is the mean absolute error between the expenditure shares in the data and from the estimated model. The sample is at the annual frequency and covers the period from 1930 to 2018.

Panel A: Translog Consumption Aggregator (without PC)



Panel B: CES Consumption Aggregator (without PC)

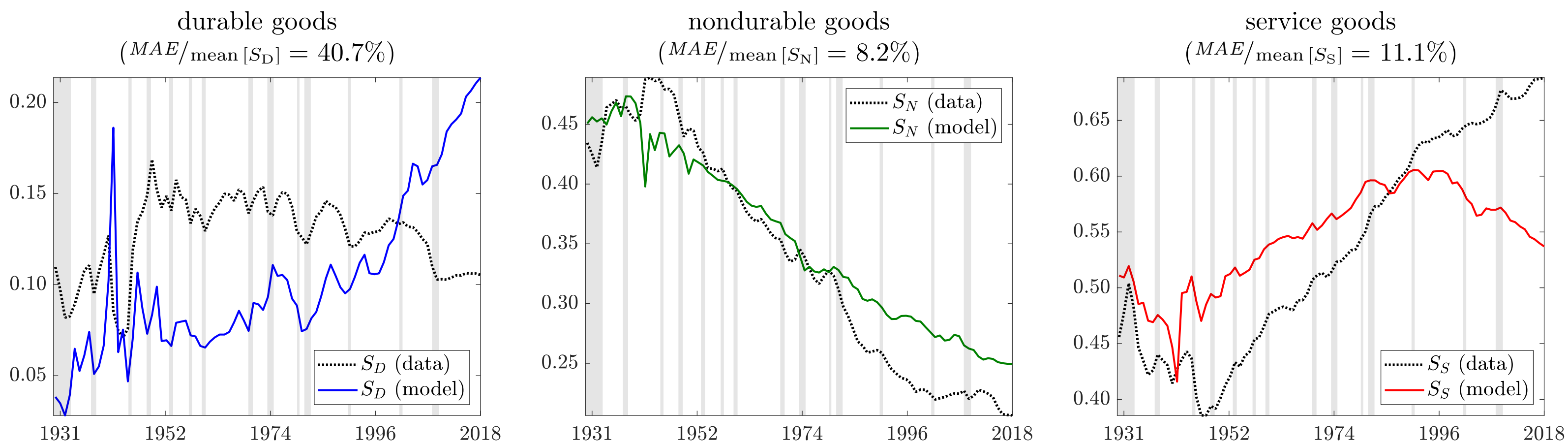


Table 1
Summary Statistics of the Marketable Consumption and Financial Series

This table presents summary statistics of the empirical series of marketable consumption and financial series. Panel A presents statistics for the marketable consumption series. The notation used to label the remaining consumption series is as follows: x denotes log real expenditures in 2018 U.S. Dollars, f denotes log quantity purchased, q denotes log quantity consumed, and S represents expenditure shares. The subscript in a variable denotes the good(s) considered in the series: D = durables, N = nondurables, and S = services. c_{G} is the proxy for aggregate consumption quantity based on the municipal solid waste (i.e., *garbage*) series from Savov (2011) and obtained from the author. Panel B presents the statistics for the financial series. $R^{\mathrm{f}}_{\mathrm{nominal}}$ is the nominal rate of the 30-day U.S. Treasury Bill. $R^{\mathrm{f}}_{\mathrm{PCE}}$ and $R^{\mathrm{f}}_{\mathrm{durables}}$ are the real risk-free rates defined as $R^{\mathrm{f}}_{\mathrm{nominal}}$ deflated by the PCE index and the price of the durable good, respectively. $R^{\mathrm{e}}_{\mathrm{MKT}}$, $R^{\mathrm{e}}_{\mathrm{SMB}}$, and $R^{\mathrm{e}}_{\mathrm{HML}}$ are the market, size, and value factor mimicking portfolio returns from Fama and French (1993) and obtained Kenneth French's website. Panel C presents the correlations between the marketable consumption and the financial series. The sample is at the annual frequency and covers the period 1930 to 2018, except for the series of municipal solid waste growth, Δc_{G}, which covers the period 1960 to 2006.

Panel A: Marketable Consumption Series

				Correlations						
Series	Mean	S.D.	AC[1]	Δx_{DNS}	Δx_{NS}	Δc_{G}	Δf_{D}	Δq_{D}	Δq_{N}	Δq_{S}
Δx_{DNS}	2.07	2.74	0.33	1.00	0.95	0.51	0.91	0.59	0.89	0.85
Δx_{NS}	2.08	2.05	0.32	0.95	1.00	0.54	0.77	0.64	0.88	0.89
Δc_{G}	1.47	2.88	-0.15	0.51	0.54	1.00	0.41	0.23	0.50	0.47
Δf_{D}	3.65	11.52	0.31	0.91	0.77	0.41	1.00	0.58	0.80	0.61
Δq_{D}	3.55	3.05	0.78	0.59	0.64	0.23	0.58	1.00	0.59	0.41
Δq_{N}	1.40	2.46	0.26	0.89	0.88	0.50	0.80	0.59	1.00	0.63
Δq_{S}	2.13	1.99	0.39	0.85	0.89	0.47	0.61	0.41	0.63	1.00
ΔS_{D}	-0.00	0.98	0.26	0.83	0.62	0.33	0.94	0.36	0.68	0.55
ΔS_{N}	-0.26	0.77	0.43	-0.43	-0.23	-0.04	-0.50	0.14	-0.22	-0.36
ΔS_{S}	0.26	0.95	0.20	-0.58	-0.54	-0.39	-0.64	-0.64	-0.64	-0.30

Table 1
Summary Statistics of the Marketable Consumption and Financial Series (Cont'd)

Panel B: Financial Series

				Correlations					
Series	Mean	S.D.	AC[1]	$R^f_{nominal}$	R^f_{PCE}	$R^f_{durables}$	R^e_{MKT}	R^e_{SMB}	R^e_{HML}
$R^f_{nominal}$	3.39	3.18	0.83	1.00	0.49	0.35	-0.14	-0.07	0.02
R^f_{PCE}	0.59	3.96	0.73	0.49	1.00	0.82	0.03	-0.30	0.07
$R^f_{durables}$	2.19	4.83	0.67	0.35	0.82	1.00	-0.10	-0.31	0.08
R^e_{MKT}	8.28	20.21	-0.09	-0.14	0.03	-0.10	1.00	0.28	-0.27
R^e_{SMB}	3.68	13.51	0.27	-0.07	-0.30	-0.31	0.28	1.00	-0.04
R^e_{HML}	4.96	14.35	-0.10	0.02	0.07	0.08	-0.27	-0.04	1.00

Panel C: Correlations Between Marketable Consumption and Financial Series

	Correlations					
Series	$R^f_{nominal}$	R^f_{PCE}	$R^f_{durables}$	R^e_{MKT}	R^e_{SMB}	R^e_{HML}
Δx_{DNS}	-0.32	0.03	-0.14	0.47	0.23	0.05
Δx_{NS}	-0.33	0.04	-0.11	0.43	0.22	-0.06
Δc_G	-0.19	0.00	-0.21	0.59	0.19	-0.15
Δf_D	-0.31	0.07	-0.04	0.46	0.16	0.18
Δq_D	-0.46	-0.02	0.15	0.16	0.08	0.13
Δq_N	-0.38	-0.05	-0.16	0.47	0.30	0.01
Δq_S	-0.21	0.02	-0.23	0.28	0.16	-0.07
ΔS_D	-0.22	0.02	-0.17	0.41	0.18	0.22
ΔS_N	-0.25	-0.22	0.05	-0.07	-0.11	-0.24
ΔS_S	0.59	0.24	0.17	-0.46	-0.11	-0.02

Table 2
Estimated Parameters of the Translog Consumption Aggregator

This table presents the point estimates and standard errors of the estimated parameters from the demand system. Panel A presents the estimated parameter values for the translog consumption aggregator

$$\mathrm{c}[\mathbf{q}_t] \equiv \log[\mathrm{C}[\exp[\mathbf{q}_t]]] = \mathbf{q}_t \times \mathbf{a} + \frac{1}{2}\left(\mathbf{q}_t \times \mathbf{b} \times \mathbf{q}_t{}'\right).$$

Panel B presents the estimated parameters from the expected utility over Total-C

$$\mathbb{E}_t\left[\mathrm{U}[C_{t+s}]\right] \equiv \beta^s\,\mathbb{E}_t\left[\frac{C_{t+s}^{1-\gamma}}{1-\gamma}\right].$$

The derivation of the standard errors is presented in Appendix C.1.

Panel A: Parameters from the Consumption Aggregator				
	vector **a**			
	a_{D}	a_{N}	a_{S}	a_{L}
	0.193	0.472	0.334	0.000
	(0.020)	(0.021)	(0.001)	(0.042)
	matrix **b**			
	b_{xD}	b_{xN}	b_{xS}	b_{xL}
b_{Dx}	0.094	-0.093	-0.020	0.019
	(0.015)	(0.003)	(0.010)	(0.001)
b_{Nx}	-0.093	0.312	-0.221	0.002
	(0.003)	(0.014)	(0.011)	(0.000)
b_{Sx}	-0.020	-0.221	0.230	0.012
	(0.010)	(0.011)	(0.022)	(0.001)
b_{Lx}	0.019	0.002	0.012	-0.033
	(0.001)	(0.000)	(0.001)	(0.002)

Panel B: Parameters from the Expected Utility over Total Consumption	
relative risk aversion γ	patience constant β
10.82	0.64
(1.02)	(0.07)

Table 3
Summary Statistics of the Recovered Latent Consumption Series

This table presents summary statistics of the latent consumption series recovered by the model estimation. The variable c, which is based on the translog aggregator in Equations (4a)–(4b), denotes the log true total consumption (Total-C). x_{DNSL} denotes the log shadow value (in 2018 U.S. Dollars) of Total-C. The variables q_{L} and S_{L} represent the log quantity and the shadow expenditure share of priceless consumption (PC). The remaining variables are defined in Table 1. The sample is at the annual frequency and covers the period 1930 to 2018, except for the series of municipal solid waste growth, Δc_{G}, which covers the period 1960 to 2006.

Panel A: Recovered Latent Consumption Series

				Correlations			
Series	Mean	S.D.	AC[1]	Δc	Δq_{L}	Δx_{DNSL}	ΔS_{L}
Δc	-0.15	8.58	0.24	1.00	0.76	-0.52	-0.91
Δq_{L}	-17.81	64.07	-0.04	0.76	1.00	-0.63	-0.92
Δx_{DNSL}	2.78	3.67	0.38	-0.52	-0.63	1.00	0.66
ΔS_{L}	0.86	3.11	0.11	-0.91	-0.92	0.66	1.00

Panel B: Correlations Between Recovered Latent and Marketable Consumption Series

	Correlations									
Series	Δx_{DNS}	Δx_{NS}	Δc_{G}	Δf_{D}	Δq_{D}	Δq_{N}	Δq_{S}	ΔS_{D}	ΔS_{N}	ΔS_{S}
Δc	0.42	0.40	0.42	0.31	-0.08	0.49	0.38	0.36	-0.25	-0.17
Δq_{L}	0.36	0.38	0.47	0.31	0.08	0.48	0.21	0.24	0.05	-0.38
Δx_{DNSL}	0.41	0.42	0.00	0.39	0.55	0.21	0.46	0.31	-0.13	-0.25
ΔS_{L}	-0.41	-0.36	-0.42	-0.35	0.09	-0.51	-0.25	-0.38	0.24	0.21

Panel C: Correlations Between Recovered Latent Consumption and Financial Series

	Correlations					
Series	$\text{R}^{\text{f}}_{\text{nominal}}$	$\text{R}^{\text{f}}_{\text{PCE}}$	$\text{R}^{\text{f}}_{\text{durables}}$	$\text{R}^{\text{e}}_{\text{MKT}}$	$\text{R}^{\text{e}}_{\text{SMB}}$	$\text{R}^{\text{e}}_{\text{HML}}$
Δc	-0.00	-0.11	-0.49	0.31	0.10	-0.14
Δq_{L}	-0.14	-0.25	-0.48	0.47	0.20	-0.24
Δx_{DNSL}	-0.24	0.20	0.36	-0.04	-0.03	0.19
ΔS_{L}	0.00	0.16	0.49	-0.41	-0.20	0.16

Table 4
Cyclical Properties of Consumption

This table reports the disaggregated means and standard deviations of the growth rates of the recovered and marketable consumption series over expansions and recessions. The marketable and recovered latent consumption series presented are defined in Tables 1 and 3, respectively. We include a year in the *Recessions* sample if any month within the year is classified as a recession by the NBER's Business Cycle Dating Committee. The sample is at the annual frequency and covers the period 1930 to 2018, except for the series of municipal solid waste growth, Δc_{G}, which covers the period 1960 to 2006.

	Expansions		Recessions	
Series	Mean	S.D.	Mean	S.D.
Panel A: Recovered Latent Consumption Series				
Δc	0.95	6.49	-2.51	11.70
Δq_{L}	-19.89	53.55	-13.40	83.14
Δx_{DNSL}	3.72	3.02	0.76	4.16
ΔS_{L}	0.82	2.78	0.94	3.77
Panel B: Marketable Consumption Series				
Δx_{DNS}	3.03	2.10	0.02	2.84
Δx_{NS}	2.77	1.41	0.58	2.40
Δc_{G}	2.08	2.16	-0.14	3.88
Δf_{D}	6.59	11.57	-2.65	8.66
Δq_{D}	4.14	2.77	2.30	3.28
Δq_{N}	2.22	1.99	-0.37	2.46
Δq_{S}	2.73	1.36	0.84	2.49
ΔS_{D}	0.22	1.01	-0.49	0.71
ΔS_{N}	-0.26	0.79	-0.27	0.74
ΔS_{S}	0.03	0.89	0.76	0.88

Table 5
Time Use and Consumption

This table investigates the relation between households' allocation of time and consumption. Panel A reports the summary statistics of the time-use series. The time-use series represent the average daily number of hours allocated in nine groups of activities across U.S. households. The time-use series are from the American Time Use Survey (*ATUS*) and obtained from the Bureau of Labor Statistics (see Appendix B.3 for details). Panel B presents the results of multivariate regressions of the time-use series on consumption shares. Panel C presents the results of multivariate regressions of the time-use series on consumption quantities. All regressions include a time index to control for the trend in the dependent variable. The independent variables are defined in Tables 1 and Table 3. The sample covers the period 2003 to 2018. Heteroskedasticity and autocorrelation consistent standard errors are shown in parentheses. Significance levels are denoted by (* = 10% level), (** = 5% level), and (*** = 1% level).

	Time-Use Activity Category								
	Sleep	Eat	Household and Care	Purchase	Work	Education	Civic and Religious	Leisure, Sports, and Social (ex-TV)	TV
Panel A: Summary Statistics									
Mean	8.69	1.22	2.54	0.76	3.61	0.47	0.32	3.19	2.73
S.D.	0.10	0.03	0.06	0.03	0.10	0.02	0.02	0.09	0.09
AC[1]	0.92	0.73	0.66	0.84	0.67	-0.26	0.27	0.75	0.72
Panel B: Time-Use and Consumption Shares									
S_{D}	2.38	-0.03	2.85	0.44	-3.42	0.58	-2.73***	11.75***	-0.66
	(1.98)	(1.28)	(1.93)	(0.84)	(4.33)	(2.26)	(0.51)	(2.83)	(3.50)
S_{N}	0.18	1.91	-7.24***	-0.97	5.56	-1.18	-0.19	6.76***	-1.91
	(1.84)	(1.28)	(1.79)	(0.88)	(3.66)	(1.96)	(0.75)	(2.27)	(2.60)
S_{L}	0.50	0.16	0.56	-0.23	-3.28***	0.39	-0.27	1.35**	1.72***
	(0.63)	(0.27)	(0.63)	(0.25)	(0.80)	(0.68)	(0.20)	(0.59)	(0.57)
Trend	Y	Y	Y	Y	Y	Y	Y	Y	Y
R^2 (%)	90.7	65.3	80.7	87.8	56.7	2.4	46.4	80.8	83.0
Panel C: Time-Use and Consumption Quantities									
Δf_{D}	-0.18	0.60***	-0.84**	-0.52***	-0.99	0.21	0.28	1.37**	0.46
	(0.31)	(0.17)	(0.37)	(0.15)	(0.72)	(0.33)	(0.26)	(0.64)	(0.56)
Δq_{N}	0.12	-2.41***	6.70***	2.18***	-1.80	-0.33	-0.84	-2.75	-2.28
	(1.33)	(0.85)	(1.06)	(0.66)	(2.74)	(1.10)	(1.07)	(2.73)	(2.22)
Δq_{S}	0.92	0.37	-3.69**	-0.43	8.69***	-0.62	-0.40	0.54	-1.41
	(1.18)	(1.11)	(1.88)	(0.61)	(2.84)	(1.07)	(0.77)	(2.32)	(1.49)
Δq_{L}	0.06***	0.03**	-0.13***	-0.02	0.06	0.00	-0.01	0.10**	-0.08*
	(0.02)	(0.01)	(0.05)	(0.02)	(0.06)	(0.05)	(0.02)	(0.05)	(0.04)
Trend	Y	Y	Y	Y	Y	Y	Y	Y	Y
R^2 (%)	93.7	73.1	82.8	89.9	61.7	8.8	40.8	83.7	85.1

Table 6
Wellbeing and Distress Indicators

This table investigates the relation between shocks to wellbeing and distress indicators and consumption growth. Panel A reports the summary statistics of the indicators. Panel B presents the results of multivariate regressions of the indicators on consumption shares. Panel C presents the results of multivariate regressions of the indicators on consumption quantities. All regressions include a time-trend control. *Consumer Sentiment* is the University of Michigan Consumer Sentiment Index (MCSI), which is obtained from the FRED/St. Louis Federal Reserve Bank. *Suicide Rate* and *Homicide Rate* are the age-adjusted number suicides and homicides per 100,000 residents across all sample groups from the CDC/NCHS National Vital Statistics. *Hours Worked* and *Average Earnings* are the accumulated annual percentage change in the Nonfarm Business Sector Hours and Compensation series from the BEA. *Unemp Rate* is the unemployment rate of people over 16 years of age obtained from the BLS. The independent variables are defined in Tables 1 and 3. Heteroskedasticity and autocorrelation consistent standard errors are shown in parentheses. Significance levels are denoted by (* = 10% level), (** = 5% level), and (*** = 1% level).

	Wellbeing/Distress Indicator (Log Growth)					
	Consumer Sentiment	Suicide Rate	Homicide Rate	Hours Worked	Average Earnings	Unemp Rate
Type	wellbeing	distress	distress	both	wellbeing	distress
Sample Period	1953-2018	1981-2018	1981-2018	1948-2018	1948-2018	1948-2018
Panel A: Summary Statistics						
Mean	0.28	0.39	-1.31	-0.23	6.07	-0.93
S.D.	9.66	2.22	6.60	0.68	3.28	15.26
AC[1]	0.03	0.31	0.12	0.03	0.41	0.50
Panel B: Indicators and Consumption Shares						
ΔS_{D}	7.15***	0.33	-3.32	0.70***	1.87*	-22.31***
	(2.29)	(1.30)	(3.02)	(0.19)	(0.98)	(2.92)
ΔS_{N}	-4.97*	-0.34	0.35	0.10	1.55*	-17.55***
	(2.57)	(1.05)	(2.74)	(0.22)	(0.92)	(3.21)
ΔS_{L}	-0.68	0.38**	-0.03	0.01	-0.51**	2.22***
	(0.50)	(0.15)	(0.71)	(0.03)	(0.23)	(0.60)
R^2 (%)	45.4	8.9	4.8	24.5	22.7	60.3
Panel C: Indicators and Consumption Quantities						
Δf_{D}	1.24***	0.19**	-0.89*	0.08***	-0.01	-1.20***
	(0.33)	(0.09)	(0.46)	(0.03)	(0.14)	(0.43)
Δq_{N}	-0.51	-0.33	2.89*	-0.04	-0.27	-1.98
	(1.53)	(0.64)	(1.67)	(0.11)	(0.48)	(1.46)
Δq_{S}	-1.53	-0.91***	-0.31	-0.07	1.78***	-0.15
	(0.97)	(0.33)	(0.83)	(0.10)	(0.31)	(1.78)
Δq_{L}	0.01	-0.01	-0.03	-0.00	0.01**	-0.08***
	(0.02)	(0.01)	(0.04)	(0.00)	(0.01)	(0.02)
R^2 (%)	30.2	23.3	10.0	22.9	45.7	59.5

Table 7
Estimates of Relative Risk Aversion

This table presents the estimation of the coefficient of relative risk aversion (RRA) and the real risk-free rate (R^f) implied by different consumption-based model specifications. The RRA coefficient is the parameter γ of the power utility function shown in Equation (7). Specification (1) is based on the model estimation presented in Section 2. Specifications (2) and (3) are based on the translog and CES aggregators without PC and are discussed in Section 3.3. Specification (4) is the standard C-CAPM specification, which is based on the real consumption expenditures of nondurables and services as a proxy for the quantity of aggregate consumption. Specification (5) is based on the proxy for aggregate consumption quantity based on municipal solid waste data from Savov (2011) and provided by the author. The tests in Panel A are based on the excess returns of the market portfolio, and the tests in Panel B are based on the 25 portfolios of stocks double sorted by size and B/M. The parameter value β is obtained from the respective model estimations in specifications (1)–(3) and set to 0.95 in specifications (4) and (5). The model-implied real risk-free rate, R^f, and the root mean square error (*RMSE*) are expressed in percentages. The sample covers the period from 1930 to 2018 in specifications (1)–(4) and from 1960 to 2006 in the specification (5). GMM standard errors are shown in parentheses.

	PC	Specifications without PC			
	1	2	3	4	5
aggregator or proxy for consumption	translog	translog	CES	real consump. expend.	garbage
Panel A: Market Factor					
RRA (γ)	12.68	Inf	47.19	46.61	17.32
	(4.45)	(99.00)	(14.80)	(14.49)	(8.68)
Implied r^f	4.98	-Inf	-0.64	1.88	17.19
Panel B: 25 Size-B/M Portfolios					
RRA (γ)	11.12	40.08	26.80	26.19	22.31
	(4.47)	(21.49)	(13.29)	(13.06)	(9.41)
Implied r^f	19.61	9.54	40.33	46.82	13.65
RMSE	3.43	3.61	4.69	4.64	3.85