Black Economic Empowerment Contracts and Risk Incentives

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Abstract

After the fall of apartheid in South Africa, Black Economic Empowerment (BEE) emerged as the central policy aimed at redressing the imbalances of the past by fairly transferring financial and economic resources to the majority of its citizens. We develop a dynamic capital structure model of the firm in an environment where regulatory constraints are imposed on firms to incentivize them to draw in impoverished citizens, who, without subsidies, could not otherwise join the shareholder base. We explore these schemes and focus on how they affect risk-taking within the firm.

Keywords: Government Policy and Regulation, Ownership Structure, Capital Structure, Risk-Taking.

JEL codes: G320, G380, L51

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Over the past two decades, one of the most wide ranging and ambitious affirmative action policies in the world has been enacted in South Africa. Black Economic Empowerment (BEE) emerged as a central policy of the newly elected government after the dismantling of apartheid. Its aim was to redress the imbalances of apartheid by substantially and fairly transferring the ownership, management and control of South Africa’s financial and economic resources to the majority of its citizens. The financial mechanisms and regulations put in place to facilitate this transfer of wealth have been extraordinary. The purpose of this paper is to closely examine the economic incentives created by the regulatory environment that led to firms undertaking major corporate restructuring.

According to Becker (1993), the costs to minorities are substantial when they act against majorities. He claims the downfall of the apartheid regime was driven by these excessive costs. If this were true, then the removal of these costs, brought about by the transition to democracy, should have been rapid. However, removing the misallocation of resources left by apartheid is complex, and, given highly imperfect markets, a change in regime did not by itself lead to rapid change. Most blacks do not have access to capital, have limited access to educational opportunities, and racial stereotypes, that influence hiring decisions, abound.

To remove these obstacles, the BEE commission, headed by Cyril Ramaphosa, now the president of South Africa, decided a “big push” could solve these problems by creating policies that would allow blacks to accumulate sufficient wealth so that the underdevelopment trap, in which they were stuck, would be broken (Acemoglu, Gelb, and Robinson (2007)). To accomplish this, the policies were focused on rewarding firms that moved rapidly towards having large black ownership stakes and strong affirmative action policies.

The regulations have largely been successful in inducing listed companies in South Africa to undertake major corporate restructuring in order to facilitate the transfer of sizable ownership stakes to previously disadvantaged groups that otherwise would be unable to participate. For example, of the 100 largest firms in South Africa, 83 have conducted BEE deals. These deals, according to Intellidex (Theobald, Tambo, Makuwerere, and Anthony (2015)), collectively generated a net 32 billion dollars of value to the beneficiaries, which, the report states, is almost twice the total corporate income tax taken in South Africa in 2014, or sufficient to purchase the entire stock of planted agricultural land and machinery in the country. The size of individual deals can be quite large. Anglo American’s deals alone have been estimated to create value of over 5 billion dollars, making it the largest BEE deal value creator of all companies. The top panel in Figure 1 shows a histogram of the value of a sample of 81 BEE deals.

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1The term “Black”, as defined in the legislation and used throughout this paper, is a generic term referring to African, Coloured (mixed race) and Indian South African citizens, but may also refer to “previously disadvantaged individuals”, including women and the physically challenged of all races.
2Examples included the transfer of at least 30% of productive land to blacks and collective organizations, 25% ownership of Johannesburg Stock Exchange listed shares, and 50% of government procurement directed to black owned companies (see Theobald, Tambo, Makuwerere, and Anthony (2015)).
3For an excellent discussion of the issues associated with BEE formations see Acemoglu, Gelb, and Robinson (2007).
While the policies of BEE have had a huge impact on firms doing business in SA, the overall objectives of developing a more inclusive economy, have not as yet been met, with the Gini coefficient in South Africa now the highest in the world (Sulla and Zikhali (2018)).

Many studies have investigated the pros and cons of the BEE regulations. Most economic studies have attempted to measure the effectiveness of BEE regulations by simply recording the changes in wealth, income and wage levels across racial groups and assessing whether productivity and inclusiveness has improved over time. The underlying assumption is that if these metrics have improved, then there is support for their effectiveness, while failing to find improvement implies that the regulations have not met their goals.

Surprisingly, to our knowledge, few studies have focused on the microeconomic implications of these regulations at the firm level. The only BEE studies we found that focus on the firm are event studies surrounding the announcement of BEE deals. If BEE transactions are perceived by the market to represent an increase in the future earnings potential of the firm or a reduction in the riskiness of future earnings then the announcement of a BEE transaction should result in an increase in the share price of the firm. Strydom, Christison, and Matias (2009) examine 254 BEE transactions and find no support for negative abnormal returns and some support for positive abnormal returns.

Some papers have focused on modeling the value of BEE deals under the assumption of an exogenous vendor stock price (West and West (2009)). To our knowledge, however, there are no studies that provide an analysis of the BEE deal process that explicitly models the trade-offs between the regulatory advantages of doing the deal against the associated costs. Our goal is to model this BEE process. With a model in place, we can assess how the original shareholders and financiers are incentivized to do the deal. We are particularly interested in how BEE policies impact risk-taking behavior at the firm level. This is an important question because it relates to how firms view, and act on, growth opportunities, capital investment and investment financing decisions in the presence of a regulatory regime focused on reducing inequalities among the races of South Africa.

BEE regulations provide preferential treatment for firms that have altered the composition of their shareholders (and supply chains) and are certified as BEE compliant. Since disadvantaged parties, represented by BEE groups, cannot afford to purchase equity, the rational response is for firms to work with investment banks to create financing arrangements that enable these parties to participate in the market. Transactions are facilitated by providing non-recourse debt to BEE groups that purchase equity stakes in the empowering company, using the equity as collateral. These groups then rely on equity appreciation and dividends to repay the debt. Most BEE transactions rely on the use of Special Purpose Vehicles (SPVs) to facilitate the deal. The financiers typically include banks, that usually have senior priority, and the firm, or vendor, that is seeking BEE certification.

Our models provide values and fair credit spreads for both the bank and vendor loans that are provided to the SPV. We also value the complex claims held by the BEE groups who have voting rights on the shares, but, as we will see, have limited ownership rights. Finally, and perhaps
most importantly, we develop explicit relationships that allow the trade-offs of costs to the original
shareholders, who ultimately are subsidizing payments to the BEE group, against benefits, provided
by improved access into markets that open up as a result of being BEE compliant.

We draw on a large literature devoted to dynamic capital structure models that trade off benefits
of the tax shield against bankruptcy costs in an environment where the firm is faced with growth
options. In this literature the timing of exercise of growth options, together with the financing of
these future investments, have a direct influence in the design of the current capital structure.4
Our models follow along these lines, but, in addition to taxes and deadweight bankruptcy costs, we
explicitly model the BEE regulations that alter the standard trade-off model in capital structure
studies.

With a model in hand we find that the BEE regulations alter incentives for the original share-
holders. The modern corporation, with its limited liability, allows owners to take on risky en-
trepreneurial activities that otherwise would lie dormant. Moreover, on top of limited liability,
most governments provide tax advantages for the use of debt that allow owners to leverage their
risk. Government policies that induce rational equityholders to become risk-averse could have
negative consequences for entrepreneurial activity, and this could manifest itself in conservative
policies put into place by equityholders of the firm. Our model shows that the rational response
to the regulatory environment created in South Africa, is to induce shareholders of the firm to
become risk-averse, even in the face of limited liability, and even in the face of a capital structure
that includes significant debt. An understanding of how this happens is certainly of interest and
leads to interesting implications for policy makers who are attempting to encourage entrepreneurial
activities rather than hinder risk-taking.

Our paper should be of interest to financial economists for several reasons. Not only do our
models build on the stream of capital structure analyses to include frictions imposed by BEE
regulations, but also on a stream of literature involving stock based lending. Rather than sell
shares, borrowers provide their shares as collateral to lenders and in return receive a lower interest
rate. In these transactions the dividends from the stock may be used to pay down the loan. A
very interesting literature has emerged regarding how these deals can be valued by translating the
problem to that of evaluating complex American call options with a strike price dependent on time
(Xia and Zhou (2007) and Grasselli and Gómez (2013)). The typical assumption in this literature
is that both parties involved in the transaction are independent of the firm issuing the stock. In
our case, the parties to the stock based loans are not independent of the firm issuing the stock.
Indeed, the SPV uses its full holdings of vendor equity as collateral to secure the loans. With
no bank funding, the full loan provided by the vendor is secured using the vendor’s equity in the
SPV. This creates interesting issues regarding the determination of an appropriate credit spread.
When the SPV utilizes bank loans as well, also secured with vendor equity, the fair credit spread
computations become even more challenging.

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4In these models, managers who act on behalf of equityholders, have incentives to under-invest or over-invest. The
resulting agency costs influence the cost of debt and feedbacks into the firm’s choice of capital structure. The original
model of Leland has been extended in many directions. Examples include Mello and Parsons (1992), Childs, Mauer,
and Ott (2005), Hackbarth and Mauer (2012), Goldstein, Ju, and Leland (2001), Strebulaev (2007) and Frank and
Goyal (2009) among many others.
BEE deals introduce *wrong-way risk* into the firm. It is this risk that drives equity holders to become risk averse. To see this effect, consider what happens if the equity of the vendor performs poorly. In this case, the assets supporting the vendor loan underperform and the BEE claimants default on their obligations. As a result, the third party financiers may recover some value, and the vendor loans, typically being junior, recover less. So, in the very states of nature where the vendor firm is performing poorly, they recover little, and in these bad states they are further penalized by losing their BEE credentials. This is *wrong-way risk*, in that it is these states that the vendor would like to insure against. In this regard, the vendor is essentially providing contingent equity to disadvantaged participants where payment for these claims is made through loans collateralized by the equity of the firm. In particular, the form of these contracting arrangements give BEE investors upside potential, since they hold contingent equity. The firm, on the other hand, retains all the downside risk, since it holds a contingent claim (risky loan) that loses most of its value if the firm performance deteriorates.

Such a contract is the exact opposite of contingent convertible bond, or COCO, which resides on the balance sheet of a firm as debt, but if the performance of the firm deteriorates, the security automatically converts to equity. COCOs were first proposed and later reviewed by Flannery (2014) with the goal of reducing the likelihood that banks experience financial distress. They provide a mechanism for automatic capital restructuring that extinguishes debt and replaces it with new common equity when the bank is under stress and fresh capital is hard to raise (see Glasserman and Nouri (2016) and Sundaresan and Wang (2015)). BEE contracts do the exact opposite by increasing risk in bad states of nature, and this, *ex ante*, induces some risk-averse behavior among the equityholders. While BEE deals draw in a more diversified shareholder base, our analysis reveals that a cost associated with this regulatory regime includes the cost associated with shareholder/manager risk-aversion—extinguishing entrepreneurial risk-seeking and impacting investment decision making in the firm.

Finally, our paper should be of interest to financial economists who are interested in mechanisms that might reduce the Gini coefficient in the USA. The Gini coefficient in the USA is over 0.40 and has been steadily increasing over the last decade. Relative to most developed countries the US has one of the highest Gini coefficients. The Gini coefficient in South Africa is near 0.70, making it the highest ranked country in the world (Sulla and Zikhali (2018)). The overall concern about wealth consolidation in South Africa has resulted in actions taken to alter this that would be unprecedented in western democracies. Obtaining a deeper understanding of the economic forces resulting from the regulatory policies used in BEE deals in South Africa could result in fresh insights and new regulatory policy changes that may more efficiently draw broader groups into the economy.

The paper proceeds as follows: Section 1 provides some background on the Black Economic Empowerment environment. Section 2 describes the modeling framework. Section 3 establishes the value of all claims associated with a BEE deal, first in a static environment where there are no opportunities for restructuring the deal and then in a dynamic context, where the firm can take actions near maturity. In all cases we investigate optimal second best capital structures based

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5 There is significant discussion about the percentage share of US income controlled by the top one percent. Popular books, e.g., Piketty (2015) and Conard (2016) debate whether this is a concern.
on maximizing the original equityholders interests. In Section 4 we develop the optimal financing mechanisms that involve a portfolio of bank and vendor financing. Section 5 presents numerical results. Along the way, we demonstrate that in spite of limited liability and the tax advantages of using debt, the original shareholders may act as if they are risk-averse. Section 6 concludes.

1 Background and Literature Review

Many policies could be used to reverse the economic inefficiencies created as a result of apartheid. Obvious first responses are for the government to allocate more resources to education, to implement affirmative action policies, and to focus on providing direct and indirect assistance to small and medium enterprises (SMEs) who contribute up to 60% of total employment and 40% of national income in emerging economies, and, according to the World Bank, create four out of five new employment opportunities. For South Africa, 81% of working people are employed in SMEs with less than 100 employees, and, according to Finscope surveys (Grundling and Kaseke (2006) and Grundling and Kaseke (2010)), SMEs accounted for 90% of jobs created between 1998 and 2005. In spite of their importance, the creation rate of new SMEs in South Africa is one of the lowest in the world (Herrington, Kew, and Mwanga (2017)) with three out of four failing within two years. von Broembsen, Wood, Herrington, Shay, and Scheepers (2005) report that the probability of surviving beyond 42 months is less likely in South Africa than in any other GEM participating country.

The focus in SA has been to provide a big push by primarily focusing on mechanisms that would transfer equity, with its ownership rights, into the hands of previously disadvantaged individuals, who otherwise could not afford the equity. These plans were enacted, first in the rather narrow 1998 Employment Equity Act, followed by the 2003 Broad-Based Black Economic Empowerment Act.

Early empirical evidence suggests that the first set of policies have had a negligible effect on reducing wage gaps and on drawing more blacks into the economy. Studies by Acemoglu, Gelb, and Robinson (2007), Burger and Jafta (2010), and Krüger (2014) demonstrated that after 1998, the overall racial wage gap widened, with one exception, namely the wage gap among the very top earners (Ponte, Roberts, and van Sittert (2007)). Acemoglu, Gelb, and Robinson (2007) map the connections between important black politicians and companies by looking at their board of directors and find a small collection of well connected politicians sit on multiple boards. Robinson (2012) uses SA as a case study to argue that understanding how elites form is key to understanding the persistence of institutions over time. In particular, since the set of institutions in South Africa generated huge rents for the white elite, it could do the same for a new, mostly black, elite. A more recent study by Burger, Jafta, and von Fintel (2016) is more positive about the regulatory shift, concluding that it has resulted in a decline in discrimination, with black wages becoming as responsive to economic growth as white wages, and returns to tertiary education being especially

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7Global Entrepreneurship Monitor (GEM) countries cover 70% of the world’s population and 85% of the world’s GDP. The reported survival rate of new businesses in South Africa that survive for more than 3.5 years is 2.1% compared with Angola (8.6%), Zambia (9.6%), Uganda (27.7%) and Ghana (35.5%).
high for black men.

The purpose of the Broad-Based Black Economic Empowerment Act of 2003 was to remedy some of the shortcomings by providing economic empowerment to black people, women, workers, youth, people with disabilities and people living in rural areas. The Department of Trade and Industry in South Africa developed codes of good practice for Broad-Based Black Economic Empowerment, with a scorecard that forms the basis of assessing a firm’s status when it engages in any form of business with the government. These include licenses, such as mining rights, concessions, partnerships and opportunities to acquire state-owned enterprises. Not only does the latest regulation require changes to ownership and control structures in order for the firm to do business with government, it also requires the firm’s suppliers to become compliant by making these changes.

The scorecard consists of three broad metrics. The first, and most important, measures the extent to which designated groups own and control businesses and assets. The second measures the extent to which the firm provides active training for these designated groups, and the third measures the extent to which the firm interacts with other firms through preferential procurement from black firms, through profit and sharing contracts with black enterprises, as well as local content sourcing. BEE rating agencies provide quantitative empowerment ratings that are used to assess compliance. These ratings have a direct influence on the potential earning stream of the firm. Specifically the higher the rating the easier it is for the firm to gain market share and to reduce costs in their supply chains. As Andrews (2008) points out, “firms could gain or lose private sector business because of BEE status, making BEE status a competitive calling card and a new form of relational currency in the corporate sector.”

In sum, firms with a high proportion of black ownership and supply chains that utilize firms with a high proportion of black ownership are awarded high BEE points. Firms with high BEE points are certified as black economically empowered and rewarded with increased access to government contracts and other investment opportunities.

1.1 The Structuring of BEE Deals

A typical deal is conducted in several phases. First the firm, called vendor, finds a group of black participants, i.e., a BEE partner. Second, a special purpose vehicle (SPV) is formed. The BEE partner provides the SPV with a small amount of cash, and the vendor may also provide some cash or shares. These proceeds are, however, insufficient to purchase the 25% of outstanding equity required for temporary certification. To cover the shortfall, the SPV borrows funds from financial institutions, i.e., banks, and from the vendor using the equity held by the SPV as collateral. Usually the bank financing is the senior claim, with the vendor financing being junior.

The SPV is mandated to hold the shares for a certain period, say five or ten years. The SPV issues claims to the BEE holders and to the financiers in such a way that the BEE partners get full voting rights, together with some stream of vendor dividends, called a trickle dividend, while the financiers get claims that give them some economic rights until the loans are paid off. This can be facilitated in the SPV in a number of ways. For example, the SPV can issue preferred shares to the financiers while the BEE group retain 100% of the ordinary shares. The SPV has to buy
back the preferred shares at the latest by the maturity date of the SPV. Before redemption, the preferred stock holders are entitled to the majority of the dividends. These dividends are used to pay down the loans. Full redemption occurs when the outstanding loans have been paid off. If by maturity of the deal the preferred stock has not been fully redeemed then the remaining outstanding loans are paid from the sale of the ordinary shares in the SPV, with the BEE group keeping the residuals. If the shares in the SPV are not sufficient to cover the obligations, then the SPV declares bankruptcy and the financiers receive the assets in accordance with the predetermined debt priority rules. On the other hand, if, upon paying out all claims, the BEE holders retain an ownership in the empowering firm that exceeds a minimum threshold, then the vendor receives permanent BEE certification.

Figure 2 Here

In the early days of BEE (1992–2003) the most common form of financing BEE transactions was with third party financing. Legal restrictions emanating from Section 38 of the Companies Act prevented vendor financing and derivative structures to be used. Since the financiers did not want to take on huge amounts of credit risk, they typically required the empowering firm to provide a backstop to the loans made. The legal restrictions were removed in 2004, and since then it has become more common for the vendor to provide a more active role in the financing mechanisms that have been adopted.

Table 1 shows 10 of the largest deals conducted in South Africa. The deals are ordered by their initiation date. The deals include 8 of the 10 largest deals by value, as of the end of 2015, as reported by Intellidex (Theobald, Tambo, Makuwerere, and Anthony (2015)), together with two of the largest deals conducted in 2017 and 2018.

Table 1 Here

The table shows that the FirstRand deal, originally structured in 2004, with 49% vendor financing, 38% bank financing, 11% vendor facilitation, and 2% contribution by the BEE participants, was worth over 23 billion rands in 2015, making it the most valuable BEE deal. When the BEE proceeds are high, then it is typically the case that the deal is an expansion of an earlier deal. As an example, the 2018 Vodacom deal, the largest deal viewed from its initiation date, involved a restructuring of the original 2008 deal, done a decade earlier. This deal involved a BEE participation of 30% of total financing, with the proceeds coming from the earlier deal. The table shows that the actual percent of total loans made by the SPV to finance the purchase of equity varies considerably across deals. The proportion of total financing of the BEE shares by the vendor ranges from 0% (Aspen) to 85% (Sanlam). Similarly, the proportion of financing provided by banks ranges from 0% (Naspers) to 80% (Aspen). In the largest individual deal to date, namely the 2018 Vodacom deal, the contributions by third party financiers and vendor financiers were similar.

Table 2 shows a sample of 31 BEE deals, taken in a single industry group, namely mining. Similar to Table 1, it shows a large range of financing, with vendor and bank loans loans ranging from 0% to 100%. In 15 (14) cases vendor (bank) financing was used for more than 50% of total
financing. It appears that even within an industry group, there is a wide array of different financing of BEE deals.

Table 2 Here

BEE investors prefer vendor financing because typically it is heavily subsidized. Usually, vendor loans carry an interest rate with no credit spread, while bank loans are more likely to carry a large positive spread. Since the vendor controls the structure of the financing, the vendor will determine the financing so as to maximize its interests. Table 1 and Table ?? suggest that for the most part the vendor chooses to fund part but not all of the deal.

As a result, in doing a deal, the firm is faced with significant wrong-way risk. If good conditions occur, the loans are paid off through dividends and through the sale of a few shares out of the SPV. Moreover, at time $T$, the BEE group own sufficient shares such that permanent BEE certification is accomplished. At the other extreme, if the firm’s performance is so poor that at time $T$ the liabilities to the financiers exceed the market value of the equity held in the SPV, then the BEE partners have to liquidate all their holdings and the firm not only has to right off its vendor loan, but the BEE certification is lost and the scale of the EBIT stream is reduced.

2 Modeling a Standard BEE Contract

2.1 Capital Structure Before the BEE Deal

Consider a firm that generates a risk-neutralized EBIT process of the form

$$dX_t = (r - q)X_t dt + \sigma X_t dW_t,$$

where $W_t$ is a Weiner process, $r > 0$ and $\sigma > 0$ are the risk-free rate and volatility, respectively, and $0 < q < r$. The capital structure in place consists of senior debt, with face value $F$ and maturity $T$. Let $CB_t$ be the value of this senior corporate bond at time $t$, and let $EQ_t$ be the value of equity of the firm. Let $\tau$ be the effective tax rate, with $\gamma = 1 - \tau$, and let $b$ represent the proportional deadweight bankruptcy cost. At maturity, the after-tax value of the assets of the firm is $V_T = \gamma X_T/q$. So, if the firm defaults the deadweight cost is $bV_T$.

Now consider the situation where no BEE deal is yet concluded—we use the superscript ND to indicate this. If $V_T < F$, the equityholders default, which means that $EQ_{T|ND} = 0$ and $CB_{T|ND} = (1 - b)V_T$. If $V_T \geq F$ the firm remains solvent with $EQ_{T|ND} = V_T - F$ and $CB_{T|ND} = F$. When the coupon is zero we have analytical solutions for the equity and corporate debt. Specifically, following Merton (1974), we obtain

$$EQ_0^{ND} = C_0(F) + D_0,$$

$$CB_0^{ND} = (1 - b)V_0e^{-qt} - (1 - b)C_0(F) + bFB_0(F).$$
Here, \( C_0(K) \) is the time 0 price of a Black-Scholes call option with underlying \( V_0 \), strike \( K \) and maturity \( T \), so that
\[
C_0(K) = V_0e^{-qT}N(d_1) - Ke^{-rT}N(d_2), \quad \text{with} \quad d_1 = \frac{\log(V_0/K) + (r-q+\sigma^2/2)T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T}.
\]
The claim \( B_0(K) \) is a binary call option, with strike \( K \), that pays a unit of currency, with
\[
B_0(K) = e^{-rT}N(d_2),
\]
while \( D_0 \) is the present value of the dividend stream up to time \( T \), given by
\[
D_0 = V_0(1 - e^{-qT}).
\]

Having considered the situation where no deal is in place, we now explore the expansion opportunities that exist due to BEE certification.

### 2.2 The Expansion Effect of Entering a BEE Deal

Consider the situation where a BEE deal is concluded with maturity \( T \). The BEE partners provide the SPV with a subscription price, \( P \), and the vendor provides a fraction, \( f_V \), of shares worth \( f_VEQ_0^D \) as a signing bonus, also known as a facilitation cost. Since it is assumed that the market fairly anticipates the deal, the equity price has been superscripted by D. The SPV borrows BL\_0^D from the bank at a rate of \( r + s_B \), where \( s_B \) is an appropriate (fair) credit spread. The loan is for \( T \) years with principal reduced by a fraction of the dividend flow. In addition, the SPV borrows VL\_0^D from the vendor at a rate of \( r + s_V \), also with maturity \( T \). This loan is subordinate to the bank loan. Typically, the spread charged by the vendor, \( s_V \), is below the fair rate, and is often curtailed to be zero.

The SPV uses the total funds, being the subscription price, the bank loan and the vendor loan, to increase its fractional ownership from \( f_V \) to \( f_0 \geq f_B \). The fraction \( f_B \) is the proportion of black ownership in the firm that is required for it to receive temporary BEE certification (until time \( T \)). The shares worth \( f_0EQ_0^D \) are held in the SPV. The dividends on these shares are split into two streams. A fraction, \( g \), of the dividends received is used to pay down the loans. Typically, these cash flows are first used to service the bank loan, and then, once this is settled, to service the vendor loan. The remainder of the dividends (fraction \( 1 - g \)) is passed through to the BEE holders in the form of a trickle dividend.

Let the amount due to the bank and vendor at time \( T \) be \( K_B \) and \( K_V \), respectively—in the general case, where \( g > 0 \), these are random variables dependent on the equity dynamics and dividend distributions over time. At maturity, the SPV sells as many shares as is necessary to settle the bank and vendor loans. Let \( BL_T^D \) and \( VL_T^D \) be the actual cash received by the bank and vendor. If there is a shortfall, i.e., \( BL_T^D < K_B \) or \( VL_T^D < K_V \), then the SPV liquidates all its shares and declares bankruptcy. The bank has the senior claim and the vendor holds the junior claim. Since the assets of the SPV are traded securities, there is no significant deadweight bankruptcy
cost.

If the SPV, after settling its obligations, is left with a fraction $f_B$, or more, of the total equity in the firm then the vendor is granted permanent BEE certification, and the shares in the SPV are distributed to the BEE partners. On the other hand, if the SPV settles its obligations but retains less than $f_B$ of the outstanding shares BEE certification is lost.

The advantage to the firm of being BEE compliant is that its supply chain is increased and the market for its products is enhanced. With certification, larger investment opportunities arise, and the scale of the EBIT process increases from its pre-BEE scale of 1 to $\pi > 1$.

Now, if a BEE deal is struck with maturity $T$ then, over the period $[0, T]$, the EBIT process is expanded from $X_t$ to $\pi X_t$. Thus, the value of the dividend stream up to time $T$ is expanded from $D_0$ to $\pi D_0$ with certainty. Since BEE certification may not be permanent, the value of the assets of the firm, at time $T$, will be $\pi V_T$ if certification is retained, but will revert to $V_T$ if certification is lost.

Finally, if the equity performs very badly, so that $V_T < F$, then the vendor is unable to continue business and the equityholders default on their own debt. In this case the real assets are redeployed and the deadweight bankruptcy cost equal to a constant proportion of the value of the assets, $b$, is incurred.

2.3 Restructuring the Deal at Maturity to Ensure Certification

At time $T$ the value of the equity will depend on whether permanent BEE status is obtained. If there is a shortfall then the vendor may decide that it is in their interests to contribute further funds, $\delta_T$, to the SPV so as to ensure that permanent BEE status is secured.

The value of equity after this contribution is denoted $\text{EQ}^C_T$, while $\text{EQ}^C_{T-}$ is the value of the equity an instant before $T$, fairly anticipating that the contribution is made, with

$$\text{EQ}^C_T = \text{EQ}^C_{T-} + \delta_T.$$ 

The shareholders will maximize their interests by determining whether to supply funds or not. So, with a BEE deal concluded, the time $T$ value of equity is given by

$$\text{EQ}^D_T = \begin{cases} 
\text{EQ}^{NC}_T & \text{for } \text{EQ}^C_{T-} \leq \text{EQ}^{NC}_T \\
\text{EQ}^C_T & \text{for } \text{EQ}^C_{T-} > \text{EQ}^{NC}_T.
\end{cases}$$

Thus, the shareholders’ decision is based on $\text{EQ}^D_T = \max[\text{EQ}^{NC}_T, \text{EQ}^C_{T-}]$ and will depend on the state variable $X_T$ (correspondingly $V_T$) and the amounts owed on the bank and vendor loans at time $T$, namely $K_B$ and $K_V$. In Section 3.1 we derive expressions for $\text{EQ}^{NC}_T$ and the other associated claims. Section 3.2 considers the derivation of $\text{EQ}^C_{T-}$. From this the equity price $\text{EQ}^D_{T-}$ and all the other claims of the firm are established. With these boundary conditions established, we identify situations where analytical solutions for these claims can be computed at time 0. The
final step will then be to compare the value of equity if a deal is concluded with the value of equity assuming no deal.

3 Modeling the Claims Held by BEE Participants

3.1 Value of Claims with no Time T Contribution

We begin by focusing on the time $T$ values of all claims when no contribution is made at that time. These include the equity, $EQ_T^{NC}$, the corporate bond, $CB_T^{NC}$, the bank loan, $BL_T^{NC}$, the vendor loan, $VL_T^{NC}$ and the BEE claim, $BEE_T^{NC}$. Having established these, we are able to provide closed form prices of the claims at time zero, under the assumption that the proportion of dividends used to pay down the debt is $g = 0$.

At time $T$, if the SPV is unable to settle its obligations in full it will go bankrupt, BEE participants will receive nothing and BEE certification of the vendor will be lost. Starting with this assumption, there are three sub-cases to consider. In the worst case, if the asset value of the firm, $V_T$, is less than its obligation to the bond holders, $F$, the vendor also defaults, the equity held by the SPV is worthless, and the bank loses its full claim, $K_B$. On the other hand, if $V_T \geq F$ the vendor is able to service its obligations and the SPV has valuable assets to pay down some of its obligations. The bank will receive full payment if $f_0 (V_T - F) \geq K_B$, or $V_T \geq \xi_1$, where

$$\xi_1 = F + \frac{1}{f_0} K_B. \quad (1)$$

The vendor will only begin to partially recover its loan if $V_T > \xi_1$, and will receive full payment of $K_V$ if $f_0 (V_T - F + K_V) \geq K_B + K_V$, or equivalently if $V_T \geq \xi_2$, where

$$\xi_2 = F + \frac{1}{f_0} K_B + \frac{1-f_0}{f_0} K_V. \quad (2)$$

Thus, the SPV only survives bankruptcy when $V_T > \xi_2$. BEE certification is, however, retained only if the remaining equity held by the SPV, after all obligations are settled, exceeds the fraction $f_B$ of the outstanding equity in the vendor firm. This situation occurs when $(f_0 - f_B)/\pi (V_T - F + K_V) \geq K_B + K_V$, from which we see that certification is retained only if $V_T \geq \xi_3$, where

$$\xi_3 = \frac{\pi}{\pi} \quad \text{and} \quad c_1 = F + \frac{1}{f_0 - f_B} K_B + \frac{1-f_0+f_B}{f_0 - f_B} K_V. \quad (3)$$

While it is always the case that $F \leq \xi_1 \leq \xi_2$, the location of $\xi_3$ relative to these constants depends on the magnitude of $\pi$. The following lemma specifies the possible orderings.

**Lemma 1.** Let

$$C_1^{NC} = \left[ \frac{c_1}{\xi_1}, \infty \right), \quad C_2^{NC} = \left[ \frac{c_1}{\xi_2}, \frac{c_1}{\xi_1} \right), \quad C_3^{NC} = \left[ \frac{c_1}{\xi_3}, \frac{c_1}{\xi_2} \right) \quad \text{and} \quad C_4^{NC} = \left( 1, \frac{c_1}{\xi_2} \right)$$

be subsets of $\mathbb{R}_+$, then four orderings are possible:
1. If \( \pi \in C_{1}^{NC} \) then \( \xi_{3} \leq F \leq \xi_{1} \leq \xi_{2} \).
2. If \( \pi \in C_{2}^{NC} \) then \( F < \xi_{3} \leq \xi_{1} \leq \xi_{2} \).
3. If \( \pi \in C_{3}^{NC} \) then \( F \leq \xi_{1} < \xi_{3} \leq \xi_{2} \).
4. If \( \pi \in C_{4}^{NC} \) then \( F \leq \xi_{1} \leq \xi_{2} < \xi_{3} \).

**Proof.** See Appendix.

Given the magnitudes of \( \pi \) from the above lemma, there are four possible payouts of the equity at time \( T \). These are shown in Figure 3.

In each segment, the payouts are linear. There are only four possible affine payouts defined on segments. In particular they are represented by \( P_{j}(V_{T}) \) for \( j \in \{1, 2, 3, 4\} \) with

\[
\begin{align*}
P_{1}(V_{T}) & = V_{T} - F \\
P_{2}(V_{T}) & = \frac{1}{1-f_{0}}(V_{T} - F - K_{B}) \\
P_{3}(V_{T}) & = V_{T} - F + K_{V} \\
P_{4}(V_{T}) & = \pi V_{T} - F + K_{V}.
\end{align*}
\]

In reverse order, \( P_{4}(V_{T}) \) comes from the fact that when \( V_{T} \geq \xi_{3} \) the vendor is granted permanent BEE certification and recovers the full vendor loan. In all other cases the vendor loses certification. When \( \xi_{2} \leq V_{T} < \xi_{3} \) the vendor still recovers the full vendor loan, hence \( P_{3}(V_{T}) \). When \( \xi_{1} \leq V_{T} < \min[\xi_{2}, \xi_{3}] \) the vendor partially recovers the loan, which has value \( V_{L_{NC}}^{T} = f_{0}^{EQ_{NC}} - K_{B} \). Thus, \( EQ_{NC}^{T} = V_{T} - F + V_{L_{NC}}^{T} \). Solving this for \( EQ_{NC}^{T} \) gives \( P_{2}(V_{T}) \). Finally, when \( F \leq V_{T} < \min[\xi_{1}, \xi_{3}] \) then the vendor recovers none of the loan, which gives \( P_{1}(V_{T}) \).

**Proposition 1.** The time \( T \) equity value, with no time \( T \) contribution, \( EQ_{NC}^{T}(V_{T}, K_{B}, K_{V}) \), is given for the various cases as follows:

If \( \pi \in C_{1}^{NC} \) then

\[
EQ_{NC}^{T} = \begin{cases} 
0 & \text{for } V_{T} < \xi_{3} \\
P_{4}(V_{T}) & \text{for } V_{T} \geq \xi_{3}.
\end{cases}
\]

If \( \pi \in C_{2}^{NC} \) then

\[
EQ_{NC}^{T} = \begin{cases} 
0 & \text{for } V_{T} < F \\
P_{1}(V_{T}) & \text{for } F \leq V_{T} < \xi_{3} \\
P_{4}(V_{T}) & \text{for } V_{T} \geq \xi_{3}.
\end{cases}
\]
If \( \pi \in \mathcal{C}_3^{NC} \) then

\[
\begin{align*}
\text{EQ}^{NC}_T &= \left\{ 
\begin{array}{ll}
0 & \text{for } V_T < F \\
P_1(V_T) & \text{for } F \leq V_T < \xi_1 \\
P_2(V_T) & \text{for } \xi_1 \leq V_T < \xi_3 \\
P_4(V_T) & \text{for } V_T \geq \xi_3.
\end{array}
\right.
\end{align*}
\]

If \( \pi \in \mathcal{C}_4^{NC} \) then

\[
\begin{align*}
\text{EQ}^{NC}_T &= \left\{ 
\begin{array}{ll}
0 & \text{for } V_T < F \\
P_1(V_T) & \text{for } F \leq V_T < \xi_1 \\
P_2(V_T) & \text{for } \xi_1 \leq V_T < \xi_2 \\
P_3(V_T) & \text{for } \xi_2 \leq V_T < \xi_3 \\
P_4(V_T) & \text{for } V_T \geq \xi_3.
\end{array}
\right.
\end{align*}
\]

**Proof.** See Appendix. ■

Proposition 1 gives the full set of boundary conditions for the price of equity at time \( T \), given the state variable \( V_T \) and the outstanding obligations to the bank and vendor. The value of the equity is contingent on the values of \( K_B \) and \( K_V \), which at this point have not been determined. However, when \( g = 0 \) these amounts are known at time 0. In this case, we obtain analytical solutions for the time 0 value of equity.

**Proposition 2.** Under the assumption that \( g = 0 \), the time 0 equity price is given by

\[
\text{EQ}^{NC}_0(K_B, K_V) = \left\{ 
\begin{array}{ll}
\pi C_0(\xi_3) + P_2(\xi_3) B_0(\xi_3) + \pi D_0 & \text{for } \pi \in \mathcal{C}_1^{NC} \\
C_0(F) + (\pi - 1) C_0(\xi_3) + (P_4(\xi_3) - P_1(\xi_3)) B_0(\xi_3) + \pi D_0 & \text{for } \pi \in \mathcal{C}_2^{NC} \\
C_0(F) + \frac{\xi_1}{1 - f_0} C_0(\xi_1) + \left( \pi - \frac{1}{1 - f_0} \right) C_0(\xi_3) + (P_4(\xi_3) - P_2(\xi_3)) B_0(\xi_3) + \pi D_0 & \text{for } \pi \in \mathcal{C}_3^{NC} \\
C_0(F) + \frac{\xi_1}{1 - f_0} (C_0(\xi_1) - C_0(\xi_2)) + (\pi - 1) C_0(\xi_3) + (P_4(\xi_3) - P_3(\xi_3)) B_0(\xi_3) + \pi D_0 & \text{for } \pi \in \mathcal{C}_4^{NC}.
\end{array}
\right.
\]

**Proof.** See Appendix. ■

Once the decisions have been made by the equityholders, the values of the other claims immediately follow.

**Proposition 3.** Under the assumption that \( g = 0 \), the time 0 prices of the other claims are as follows:

1. The value of the corporate bond is

\[
\text{CB}^{NC}_0(K_B, K_V) = (1 - b)(V_0 e^{-qT} - C_0(\xi_{\text{min}})) + (F - (1 - b)\xi_{\text{min}}) B_0(\xi_{\text{min}}),
\]

where \( \xi_{\text{min}} = \min[F, \xi_3] \).
2. The value of the bank loan is

\[
\text{BL}^\text{NC}_0(K_B, K_V) = \begin{cases} 
K_B B_0(\xi_3) & \text{for } \pi \in \mathcal{C}_1^{\text{NC}} \\
 f_0(C_0(F) - C_0(\xi_3)) + (K_B - f_0 P_1(\xi_3)) B_0(\xi_3) & \text{for } \pi \in \mathcal{C}_2^{\text{NC}} \\
 f_0(C_0(F) - C_0(\xi_1)) & \text{for } \pi \in \{ \mathcal{C}_3^{\text{NC}}, \mathcal{C}_4^{\text{NC}} \}.
\end{cases}
\]

3. The value of the vendor loan is

\[
\text{VL}^\text{NC}_0(K_B, K_V) = \begin{cases} 
K_V B_0(\xi_3) & \text{for } \pi \in \{ \mathcal{C}_1^{\text{NC}}, \mathcal{C}_2^{\text{NC}} \} \\
 \frac{f_0}{1 - f_0} (C_0(\xi_1) - C_0(\xi_3)) + (K_B + K_V - f_0 P_2(\xi_3)) B_0(\xi_3) & \text{for } \pi \in \mathcal{C}_3^{\text{NC}} \\
 \frac{f_0}{1 - f_0} (C_0(\xi_1) - C_0(\xi_2)) & \text{for } \pi \in \mathcal{C}_4^{\text{NC}}.
\end{cases}
\]

4. The value of the BEE claim is

\[
\text{BEE}^\text{NC}_0(K_B, K_V) = f_0 \text{EQ}^\text{NC}_0(K_B, K_V) - \text{BL}^\text{NC}_0(K_B, K_V) - \text{VL}^\text{NC}_0(K_B, K_V).
\]

**Proof.** See Appendix.

### 3.2 Value of Claims with Time \( T \) Contribution

We have seen that if \( V_T > \xi_3 \) then BEE certification is obtained without the SPV receiving any additional support. However, if \( V_T \) falls below this threshold then it may be in the interests of the equityholders to provide funds to the SPV so that permanent certification can be retained. We assume that if the firm is to raise funds for the SPV it will do so through equity financing, which dilutes the shares. All equityholders fairly contribute to the equity financing in proportion to their original holdings. The fraction of shares created in the process, denoted by \( x \), is transferred to the SPV, which then has an expanded fraction \( (f_0 + x)/(1 + x) \) of the total equity. To retain BEE certification the SPV must own \( f_B \text{EQ}^C_T \) after dilution and after paying off their obligation of \( K_B \) and \( K_V \). Hence, we require

\[
f_B \text{EQ}^C_T = \frac{f_0 + x}{1 + x} \text{EQ}^C_T - K_B - K_V,
\]

from which

\[
x = \frac{K_B + K_V - (f_0 - f_B) \text{EQ}^C_T}{(1-f_B)\text{EQ}^C_T - K_B - K_V}.
\]

Let \( \text{EQ}^C_{T-} \) be the equity value an instant before dilution happens, but fairly anticipating that it does, given by

\[
\text{EQ}^C_{T-} = \frac{\text{EQ}^C_T}{(1+x)}.
\]

Substituting (4) into this expression yields

\[
\text{EQ}^C_{T-} = \frac{(1-f_B)\text{EQ}^C_T - K_B - K_V}{(1-f_0)}.
\]
After the transaction, the value of the equity is $\text{EQ}_T^C = \pi V_T - F + K_V$, which means that
the pre-dilution equity may be rewritten as

$$
\text{EQ}_T^C(V_T, K_B, K_V) = \frac{1-f_0}{1-f_B}(\pi V_T - F) - \frac{1}{1-f_B} K_B - \frac{f_0}{1-f_B} K_V.
$$

From this we see that the firm solvency condition, $\text{EQ}_T^C \geq 0$, is true if and only if $V_T \geq \xi_0$, where

$$
\xi_0 = \frac{c_0}{\pi} \quad \text{and} \quad c_0 = F + \frac{1}{1-f_B} K_B + \frac{f_0}{1-f_B} K_V.
$$

For convenience we define $P_3(V_T) = \text{EQ}_T^C(V_T, K_B, K_V)$, in which case we have the following lemma.

**Lemma 2.** When $\xi_0 < F$ then $P_3(\xi_1) > P_1(\xi_1)$.

**Proof.** See Appendix.

When refinancing is allowed, there are a number of possible equity payoffs, at time $T$, depending on the magnitude of $\pi$ and the terminal loan values, $K_B$ and $K_V$. As mentioned in Section 2.3, the shareholders’ decision is based on $\text{EQ}_T^R = \max\{\text{EQ}_T^C, \text{EQ}_T^D\}$. Figure 4 shows the combinations of these payoffs on which the decision is made.

**Figure 4** Here.

As seen previously, when there is no refinancing, there are four possible affine payouts, $P_j(V_T)$ $j \in \{1, 2, 3, 4\}$. Figure 4 shows these as solid lines. The payout with refinancing at time $T$, $P_3(V_T)$, is shown in each panel as a dotted line.

Taking into account Lemma 1 and Lemma 2, there are twelve optimal payoffs depending on how $P_3(V_T)$ intersects with the original four payouts. As can be seen by the ordering of $\xi_1$, $\xi_2$ and $\xi_3$, panel 1 in Figure 4 corresponds to $\mathcal{C}_1^{NC}$ of Lemma 1, panels 2 and 3 correspond to $\mathcal{C}_2^{NC}$, panels 4, 5 and 6 correspond to $\mathcal{C}_3^{NC}$, and the remaining six panels correspond to $\mathcal{C}_4^{NC}$.

Based on this figure we have the following inequalities

\[
\begin{align*}
P_3(\xi_1) &< P_1(\xi_1) & \Leftrightarrow & & \pi < \pi_1, \\
P_3(\xi_2) &< P_2(\xi_2) & \Leftrightarrow & & \pi < \pi_2,
\end{align*}
\]

where

\[
\begin{align*}
\pi_1 = & \frac{1}{\xi_1} \left( F + \frac{K_B + f_0 f_B K_V}{f_0 (1-f_B)} \right), \\
\pi_2 = & \frac{1}{\xi_2} \left( F + \frac{K_B + (1-f_0 + f_0 f_B) K_V}{f_0 (1-f_B)} \right).
\end{align*}
\]

This allows us to determine if $\text{EQ}_T^C(\xi_1) < \text{EQ}_T^{NC}(\xi_1)$ and if $\text{EQ}_T^C(\xi_2) < \text{EQ}_T^{NC}(\xi_2)$, when $\xi_1, \xi_2 < \xi_3$. Moreover, we have the following equalities

\[
\begin{align*}
P_3(\xi_1^I) &= P_3(\xi_1^I), & \text{(possible intersection of } \text{EQ}_T^C \text{ and } \text{EQ}_T^{NC} \text{ when } F \leq \xi_1^I < \xi_1) \\
P_3(\xi_2^I) &= P_2(\xi_2^I), & \text{(possible intersection of } \text{EQ}_T^C \text{ and } \text{EQ}_T^{NC} \text{ when } \xi_1 \leq \xi_2^I < \xi_2) \\
P_3(\xi_3^I) &= P_3(\xi_3^I), & \text{(possible intersection of } \text{EQ}_T^C \text{ and } \text{EQ}_T^{NC} \text{ when } \xi_2 \leq \xi_3^I < \xi_3) 
\end{align*}
\]

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where

$$\xi_1^I = \frac{(f_0 - f_B)F + K_B + f_B k_V}{(1 - f_B) \pi - (1 - f_B)}$$, \hspace{1cm} \xi_2^I = \frac{f_B (F - K_V)}{1 - (1 - f_B) \pi} \hspace{1cm} \text{and} \hspace{1cm} \xi_3^I = \frac{(f_0 - f_B)F + K_B + (1 - f_0 + f_B)k_V}{(1 - f_B) \pi - (1 - f_0)}.$$

The next proposition establishes the equity price at an instant before \(T\), assuming an optimal refinancing policy is used, i.e., \(\mathbf{EQ}^D_{T-}\). This is the quantity that is subsequently used to compute the fair equity price at time 0, by taking an appropriate risk-neutral expectation.

**Proposition 4.** Let

$$\mathbf{c}_1^D = \left\{ [\frac{\pi}{T}, \infty), \max\left(\frac{\pi}{T}, \frac{\pi_1}{T}, \frac{\pi_2}{T}\right), \max\left(\frac{\pi}{T}, \frac{\pi_1}{T}, \frac{\pi_2}{T}\right), \left[\max\left(\frac{\pi}{T}, \frac{\pi_1}{T}, \frac{\pi_2}{T}\right)\right] \right\},$$

$$\mathbf{c}_2^D = \left\{ \left[\frac{\pi}{T}, \frac{\pi_1}{T}\right), \max\left(\frac{\pi_1}{T}, \frac{\pi_2}{T}\right), \max\left(\frac{\pi_1}{T}, \frac{\pi_2}{T}\right), \left[\max\left(\frac{\pi_1}{T}, \frac{\pi_2}{T}\right)\right] \right\},$$

$$\mathbf{c}_3^D = \left\{ \left[\frac{\pi}{T}, \min\left(\frac{\pi_1}{T}, \frac{\pi_2}{T}\right)\right), \left[\pi_2, \min\left(\frac{\pi_1}{T}, \frac{\pi_2}{T}\right)\right) \right\},$$

$$\mathbf{c}_4^D = \left\{ \left[1, \min\left(\pi_1, \frac{\pi_1}{T}, \frac{\pi_2}{T}\right)\right), \left[\pi_1, \min\left(\pi_2, \frac{\pi_1}{T}, \frac{\pi_2}{T}\right)\right) \right\}$$

and $$\mathbf{c}_5^D = \left\{ \frac{\pi}{T}, \min\left(\pi_2, \frac{\pi_1}{T}, \frac{\pi_2}{T}\right) \right\}$$

be subsets of \(\mathbb{R}_+\). Then \(\mathbf{EQ}^D_{T-}(V_T, K_B, K_V)\) is given for the various cases as follows:

If \(\pi \in \mathbf{c}_1^D\) then

$$\mathbf{EQ}^D_{T-} = \begin{cases} 0 & \text{for } V_T < \xi_0 \\ P_3(V_T) & \text{for } \xi_0 \leq V_T < \xi_3 \\ P_4(V_T) & \text{for } V_T \geq \xi_3. \end{cases}$$

If \(\pi \in \mathbf{c}_2^D\) then

$$\mathbf{EQ}^D_{T-} = \begin{cases} 0 & \text{for } V_T < F \\ P_1(V_T) & \text{for } F \leq V_T < \xi_1^I \\ P_5(V_T) & \text{for } \xi_1^I \leq V_T < \xi_3 \\ P_4(V_T) & \text{for } V_T \geq \xi_3. \end{cases}$$

If \(\pi \in \mathbf{c}_3^D\) then

$$\mathbf{EQ}^D_{T-} = \begin{cases} 0 & \text{for } V_T < F \\ P_1(V_T) & \text{for } F \leq V_T < \xi_1 \\ P_2(V_T) & \text{for } \xi_1 \leq V_T < \xi_2 \\ P_3(V_T) & \text{for } \xi_2 \leq V_T < \xi_3 \\ P_4(V_T) & \text{for } V_T \geq \xi_3. \end{cases}$$

If \(\pi \in \mathbf{c}_4^D\) then

$$\mathbf{EQ}^D_{T-} = \begin{cases} 0 & \text{for } V_T < F \\ P_1(V_T) & \text{for } F \leq V_T < \xi_1 \\ P_2(V_T) & \text{for } \xi_1 \leq V_T < \xi_2 \\ P_3(V_T) & \text{for } \xi_2 \leq V_T < \xi_3 \\ P_4(V_T) & \text{for } V_T \geq \xi_3. \end{cases}$$

\(\mathbf{EQ}^D_{T-}\) is given for the various cases as follows:

\(\xi_1^I\) relate to panels 1, 3, 6 and 12 of Figure 5; the subsets of \(\mathbf{c}_2^D\) relate to panels 2, 5 and 10; the subsets of \(\mathbf{c}_3^D\) relate to panels 4 and 8; while the sets \(\mathbf{c}_4^D\), \(\mathbf{c}_5^D\) and \(\mathbf{c}_6^D\) relate to panels 7, 9 and 11, respectively.

\(^{8}\)Some simplification of these sets is possible, but we leave them unsimplified and point out that the subsets of \(\mathbf{c}_1^D\)
Proposition 5. Under the assumption of $g$ and $K_B$ being known at time 0, the time $t = 0$ equity price is given by

$$
\begin{align*}
\text{if } \pi \in \mathcal{C}_5^D \text{ then } & P_1(V_T) \quad \text{for } V_T < F \\
& P_2(V_T) \quad \text{for } F \leq V_T < \xi_1^T \\
& P_3(V_T) \quad \text{for } \xi_1^T \leq V_T < \xi_2^T \\
& P_4(V_T) \quad \text{for } \xi_2^T \leq V_T < \xi_3^T \\
& P_5(V_T) \quad \text{for } \xi_3^T \leq V_T < \xi_3 \\
& P_6(V_T) \quad \text{for } V_T \geq \xi_3.
\end{align*}
$$

Proof. See Appendix.

Proposition 4 is extremely important in that it identifies all possible equity payouts conditional on knowing the state variable, $V_T$, and the decision variables, $K_B$ and $K_V$. Once these decisions have been made, the terminal values of all other contingent claims may be identified.

When $g = 0$, the values of outstanding loans at time $T$ are known at time 0, and, as a result, analytical solutions are available for the equity.

Proposition 5. Under the assumption of $g = 0$, the time $t = 0$ equity price is given by

$$
\text{if } \pi \in \mathcal{C}_6^D \text{ then } \begin{cases}
0 & \text{for } V_T < \xi_0 \\
P_1(V_T) & \text{for } \xi_0 \leq V_T < \xi_1^T \\
P_2(V_T) & \text{for } \xi_1^T \leq V_T < \xi_2^T \\
P_3(V_T) & \text{for } \xi_2^T \leq V_T < \xi_3^T \\
P_4(V_T) & \text{for } V_T \geq \xi_3.
\end{cases}
$$

Proof. See Appendix.

The values of the other claims then follow.

Proposition 6. Under the assumption of $g = 0$, the time 0 prices of the other claims are as follows:

1. The value of the corporate bond is

$$
\text{CB}_0^D(K_B, K_V) = (1 - b)(V_0e^{-qT} - C_0(\xi_{\min})) + (F - (1 - b)\xi_{\min})B_0(\xi_{\min}).
$$
where $\xi_{\text{min}} = \min[F, \xi_0]$. 

2. The value of the bank loan is 

$$
BL^D_{0}(K_B, K_V) = \begin{cases} 
K_B B_0(\xi_0) & \text{for } \pi \in \{\mathcal{E}^D_1, \mathcal{E}^D_2\} \\
 f_0(C_0(F) - C_0(\xi_1^1)) + (K_B - f_0 P_1(\xi_1^1))B_0(\xi_1^1) & \text{for } \pi \in \{\mathcal{E}^D_2, \mathcal{E}^D_3\} \\
 f_0(C_0(F) - C_0(\xi_1)) & \text{for } \pi \in \{\mathcal{E}^D_3, \mathcal{E}^D_4\}.
\end{cases}
$$

3. The value of the vendor loan is 

$$
VL^D_{0}(K_B, K_V) = \begin{cases} 
K_V B_0(\xi_0) & \text{for } \pi \in \mathcal{E}^D_1 \\
 f_0(C_0(\xi_1) - C_0(\xi_2)) + (K_B + K_V - f_0 P_2(\xi_2))B_0(\xi_2) & \text{for } \pi \in \mathcal{E}^D_3 \\
 f_0(C_0(\xi_1) - C_0(\xi_2)) & \text{for } \pi \in \mathcal{E}^D_4 \\
 f_0(C_0(\xi_1) - C_0(\xi_2)) + f_0(C_0(\xi_2) - C_0(\xi_2)) & \text{for } \pi \in \mathcal{E}^D_5 \\
 f_0(C_0(\xi_1) - C_0(\xi_2)) & \text{for } \pi \in \mathcal{E}^D_6.
\end{cases}
$$

4. The value of the BEE claim is 

$$
BEE^D_{0}(K_B, K_V) = \begin{cases} 
B_0(\xi_0)P_3(\xi_0) + f_0\pi(C_0(\xi_0) - C_0(\xi_3)) + f_0\pi C_0(\xi_3) + f_0\pi D_0 & \text{for } \pi \in \mathcal{E}^D_1 \\
B_0(\xi_1^1)\tilde{P}_3(\xi_1^1) + f_0\pi(C_0(\xi_1^1) - C_0(\xi_3)) + f_0\pi C_0(\xi_3) + f_0\pi D_0 & \text{for } \pi \in \mathcal{E}^D_2 \\
B_0(\xi_1^2)\tilde{P}_3(\xi_1^2) + f_0\pi(C_0(\xi_1^2) - C_0(\xi_3)) + f_0\pi C_0(\xi_3) + f_0\pi D_0 & \text{for } \pi \in \mathcal{E}^D_3 \\
f_0(C_0(\xi_2) - C_0(\xi_3)) + B_0(\xi_3)\tilde{P}_3(\xi_3) & \text{for } \pi \in \mathcal{E}^D_4 \\
+ f_0\pi(C_0(\xi_3) - C_0(\xi_3)) + f_0\pi C_0(\xi_3) + f_0\pi D_0 & \text{for } \pi \in \mathcal{E}^D_5 \\
+ f_0\pi(C_0(\xi_3) - C_0(\xi_3)) + f_0\pi C_0(\xi_3) + f_0\pi D_0 & \text{for } \pi \in \mathcal{E}^D_6.
\end{cases}
$$

where 

$$
\tilde{P}_3(V_T) = f_0 P_3(V_T) - K_B - K_V \quad \text{and} \quad \tilde{P}_3(V_T) = f_0 P_4(V_T).
$$

**Proof.** See Appendix. ■

The time 0 value of all claims are impacted by the fact that shareholders may rationally contribute funds at time $T$. For example, the value of the corporate bond will now increase because re-
financing reduces the chances of bankruptcy. The value of $CB_0^D$ depends on whether the bankruptcy point is above or below $\xi_0$. This highlights the fact that the fair credit spread of senior debt is impacted by the possibility of shareholders refinancing at time $T$ and avoiding bankruptcy.

4 Determining the Optimal Financing Mechanisms

The senior debt, together with the bank loan, and the vendor loan, as well as the equity value at time 0 are indeterminate at this point, since both $K_B$ and $K_V$ are not known. Recall, if the loans to the SPV were set at fair market rates, then for given face values, $K_B$ and $K_V$, the funds raised would be $BL_0^D$ and $VL_0^D$. The appropriate spreads on these loans are $s_B$ and $s_V$, as determined by solving $BL_0^D = K_Be^{(r+s_B)T}$ and $VL_0^D = K_Ve^{(r+s_V)T}$ respectively. However, in practice, the vendor loan is provided at a subsidized rate, typically with no spread. In this case the SPV receives $K_Ve^{-rT}$ from the vendor with the vendor taking on an asset of lesser value, $VL_0^D(K_B,K_V)$. So, the total initial direct vendor contributions are

$$f_VEQ_0^D(K_B,K_V) + K_Ve^{-rT} - VL_0^D(K_B,K_V).$$

The SPV purchases the required equity using the funds raised. This implies that

$$(f_0 - f_V)EQ_0^D(K_B,K_V) = P + BL_0^D(K_B,K_V) + K_Ve^{-rT}. \quad (5)$$

The contribution made by the BEE group into the SPV, $P$, is typically small and of the order of 2.5% of the required equity. The vendor finances the loan of $K_Ve^{-rT}$ with equity and provides a facilitation of $f_V$ shares. In return for this funding the asset base of the firm is increased by the addition of the vendor loan. In return the equityholders receive partial ownership, namely $1 - f_V$, of the equity of the new firm, i.e., the total equity less the proportion $f_V$ given to the SPV. Let $EQ_0^D$ be the value of the deal to the original equityholders, anticipating that the deal is done. Then

$$EQ_0^D(K_B,K_V) + K_Ve^{-rT} = (1 - f_V)EQ_0^D(K_B,K_V). \quad (6)$$

Here the original equityholders surrender their equity, pay $K_Ve^{-rT}$, and in return receive a diluted ownership of the equity in the larger firm that has a temporary BEE certificate and a vendor loan as its assets.

In general the third party financier, our bank, will require a pledge of assets to secure the loan—this places restrictions on the maximum amount loaned. The total assets held by the SPV will provide this security. To model this we introduce a cover ratio, $\kappa$, and insist that

$$BL_0^D(K_B,K_V) \leq \frac{1}{\kappa} \left( f_0 EQ_0^D(K_B,K_V) \right),$$

where the cover ratio is typically 2 or higher.

The original equityholders maximize their interests. Thus the decision variables, $K_B$ and $K_V$,
are chosen so as to maximize $\text{EQ}_{D0}^d$ in which case, by (6), the optimization undertaken is

$$\max_{K_B \geq 0, K_V \geq 0} (1 - f_V)\text{EQ}_{0}^d(K_B, K_V) - K Ve^{-rT},$$

subject to

$$(f_0 - f_V)\text{EQ}_{0}^d(K_B, K_V) = P + \text{BL}_{0}^d(K_B, K_V) + K Ve^{-rT}$$

$${\text{BL}_{0}^d(K_B, K_V) \leq \frac{1}{\kappa} \left( f_0\text{EQ}_{0}^d(K_B, K_V) \right)}.$$

The first constraint specifies that the total finance raised, including initial contribution from the BEE consortium, should be equal to the purchase price of the number of shares required to bring the SPV’s total holding in the firm up to the required fraction of $f_0$, as per (5). The second ensures that the maximum cover ratio, required by the bank, is not breached, as per (7).

The decision to do the deal or not is then established by a maximizing equity value. Let $\text{EQ}_{0}^{opt}$ be the optimal value, then

$$\text{EQ}_{0}^{opt} = \max[\text{EQ}_{0}^d, \text{EQ}_{0}^\pi].$$

(8)

To illustrate the range of possible solutions, in Figure 5 we plot the feasible set of $(K_B, K_V)$ values for different growth factors $\pi$ when benchmark parameters are set at $X_0 = 20$, $q = 0.03$, $r = 5\%$, $\sigma = 25\%$, the tax rate and bankruptcy cost are $\tau = 25\%$, and $b = 10\%$, with the BEE contract terms, $T = 5$, $f_B = 25\%$, $f_0 = 26\%$, $f_V = 2.5\%$ and $P = 14$ (based on the other parameters, this value of $P$ equates to about 2.5% of the equity value of the vendor, $\text{EQ}_{0}^d$, which has a value in the range 480–650). The bank requires a cover ratio of $\kappa = 2$ and the vendor loan carries no credit spread. Before the deal, the vendor firm has a capital structure consisting exclusively of equity and debt, with the debt being a loan with face value $F = 100$, due at time $T = 5$.

The left panel shows the feasible sets of $(K_B, K_V)$ values for four different $\pi$ values. For each feasible value, the right-hand plot shows the vendor equity value and identifies the optimal solution. For these parameters, the optimal solutions consist of both bank debt and vendor financing.

Figure 5 Here

5 Properties of BEE Deals

With a model in hand, we explore various properties of BEE deals.

5.1 Optimal Financing for BEE Deals

Table 1 shows the composition of financing for 10 of the largest BEE deals conducted in South Africa. As can be seen the role of vendor and third party financing varies considerably from deal to deal. The Aspen deal had zero vendor financing, the Anglo coal deal was almost exclusively vendor financed, while the Vodacom deal, was financed with an almost equal contribution of vendor
and third party contributions. In this section we explore the primary determinants that affect the sources of funding.

Figure 6 compares the value of equity of the vendor, first for the first case where the BEE deal is funded with as much bank financing as possible (subject to the cover ratio exceeding $\kappa = 2$) with the shortfall of funds secured from the vendor, with a second case, where all funding is provided by the vendor. These two extreme source of funding are also compared with the optimal source of funding. Typically, neither of the two extreme fundings sources are optimal. The results are shown for increasing values of $\pi$ and in each plot for increasing volatilities.

When $\pi = 1.25$, we see that there is no incentive for the firm to do a deal unless $\sigma < 15\%$. In this case, the optimal solution, would be to utilize maximum bank financing. Full vendor financing is never viable. As $\pi$ increases to 1.275 we see that doing a deal dominates not doing a deal for all values of $\sigma$, and, as before, involves maximum bank financing. When $\pi = 1.30$ the optimal deal still involves maximum bank financing. In the lower three panels we see that as $\pi$ increases, vendor financing plays an increasingly more important role. However, bank financing is always a significant component of financing in the optimal SPV portfolio of loans.

The top left panel of Figure 7 shows the proportion of total loans in the BEE deal provided by the bank, for increasing growth factors, $\pi$, and for five different earnings volatilities. When earnings volatility is 10% the bank loan proportion is fairly insensitive to the growth factor, $\pi$. As volatility increases the proportion of bank loans to total borrowing can drop significantly. Specifically, when $\sigma = 30\%$ the bank loan proportion drops below 50% for $\pi$ values exceeding 1.4. When $\sigma = 50\%$ the bank loan proportion drops further to 30% for these $\pi$ values.

The top right panel shows the behavior of fair bank loan credit spreads as $\pi$ increases. The non-monotonic behavior of these two top panels results from the fact that equityholders are making decisions to maximize their own benefits and their decisions account for the specific regime (out of the possible 6 regimes) that they determine. The equity regimes are shown in the bottom right panel. For these case parameters the equityholders choose regime 4 for low $\pi$ values. The specific $\pi$ value at which equityholders switch into regime 5, varies with volatility. As $\pi$ increases further, the equityholders switch into regime 2 and this regime is optimal thereafter. Indeed, in regime 2, the equityholders can avoid bankruptcy by making a larger contribution at time $T$. The sensitivity of bank loan credit spreads changes with the regime according to Proposition 3. So while the credit spreads are continuous in $\pi$, when the regime switches the sensitivities change.

The vendor provides three tangible forms of subsidy. The first is a direct grant of $P$, the second takes the form of a loan with no credit spread and the third takes the form of providing contingent time $T$ refinancing. Note that the vendor loan is provided to the SPV with no credit spread. However, the fair credit spread, $s_V$ say, can be established from $V_{L_0} = K_V e^{-(r+s_V)T}$. The bottom left panel shows how the spread behaves as the growth factor, $\pi$, increases. While the proportion of vendor funding increases with $\pi$, the required fair credit spread actually declines.
In sum, our results indicate that the vendor’s design of SPV financing is sensitive to the magnitude of $\pi$ and $\sigma$. As these values increase, the vendor finds it optimal to grant larger subsidies by providing a larger proportion of vendor financing to the BEE group. Conversely, relatively more bank loans might be preferable if both volatility and growth factors are low.

The mining and telecommunications sectors are heavily reliant on government contracts and licenses. In these sectors, both the growth factor, $\pi$, and the earnings volatility is high.

Table 2 here

Our model therefore would predict that in these sectors, vendor financing should dominate third party financing. Table 2 shows a sample of deals in the mining sector. Of the 31 deals bank financing exceeded 50% of financing in 14 of the deals, while vendor financing exceeded 50% of financing in 15 cases. However, since 2009, all deals have been financed with a majority of vendor loans.

### 5.2 Sensitivity to Risk

Typically, corporations, with limited liability, are established to ensure owners take on risk. Indeed, most countries give tax advantages to debt, so that firms have strong entrepreneurial incentives to lever their risk. This is good for society, since if limited liability was removed, then risk-averse behavior would be the norm, and many projects would never be initiated. This risk-seeking behavior is well understood by bondholders, who, *ex ante*, either incorporate this risk-seeking behavior into their bond valuation, or negotiate bond covenants to mitigate excessive risk-taking, that might arise when the firm is close to bankruptcy. In this section we explore how the impact of a BEE deal affects risk-seeking behavior of a vendor with debt on its balance sheet.

The two panels in Figure 8 show how the equity value before and after the deal, $EQ_0$ and $EQ_0$, respectively, are affected by increasing volatility. We perform this analysis for two different $\pi$ values.

Figure 8 Here

Notice that regardless of $\pi$, the value to the equityholders before the deal is done is not increasing in volatility, and indeed, for most reasonable volatility values is decreasing. That is, given a deal will take place, the original equityholders become risk-averse, preferring lower risk than higher risk.

This risk-averse behavior stems from the fact that in future bad states of nature, not only does the firm do poorly, but in these bad states, the SPV is likely to go bankrupt, in which case the vendor is a junior claimant on the salvage value of the SPV. On top of this, the vendor loses BEE certification, and this adversely affects the future EBIT flow. This is wrong-way risk, and the rational response to wrong-way risk, is to avoid it by reducing the likelihood of it happening, which in this case implies preferring investments with low volatility. The degree of risk-aversion towards volatility flows over to the equity price $EQ_0$ after the deal is done. The panel shows that
for a large range of volatilities equity prices after the deal decline as volatility increases. Beyond a critical value of volatility, that increases with $\pi$, risk-seeking behavior returns.

The exact magnitude of the decline in equity, as a function of volatility, can be small or fairly large. What is important, is that once a BEE deal is struck, the existence of wrong-way risk rationally leads to firms adopting policies that avoid large risk exposures.

Figure 8 assumes the SPV is financed optimally. Figure 6 already shows the results when the SPV is financed with the maximum use of third party financing and also exclusively with vendor financing. For mining firms where $\pi$ is very large, and where $\sigma$ is perhaps in the interval $[0.05, 0.30]$, we see that bank financing introduces the greatest risk-averse behavior response, while pure vendor financing avoids this response. However, it is not the case that vendor financing always results in equityholders maintaining risk-seeking behavior. Specifically, the top panels in Figure 6 show equity prices declining in volatility over a rather large range of earnings volatilities.

COCOs attempt to ensure a bank from wrong-way risk, by converting debt into equity in bad states of nature. BEE deals have the opposite property. In the very states where the firm is doing poorly, they are hit with loosing certification, unless they are able to raise significant funds for the SPV. In the presence of a BEE deal, a natural response is to attempt to avoid these bad states by reducing earnings volatility.

5.3 Impact of Capital Structure on BEE Deals

How does the leverage of the vendor firm impact the structuring of a BEE deal? With no BEE deal in place, the equity of a levered firm is a call option on the assets of the firm, and increasing volatility of earnings increases the value of equity. To investigate this we increase the face value of the corporate bond from $F = 100$ to $F = 400$. For each setting of the face value we investigate the sensitivity of the equity price to volatility. Since the equity price decreases as more corporate debt is issued, we report the percentage change in equity, where the base equity values are taken to be the values when $\sigma = 5\%$.

When the face value of corporate debt is $F = 100$, we see, as before, that as volatility increases the equity price decreases over the range $[0.05, 0.40]$. When the firm is more levered, the range where equity prices decrease as $\sigma$ increases, narrows. Indeed, when $F = 400$, the range of risk-averse behavior is reduced to $[0.05, 0.175]$. Leverage induces risk-seeking behavior and competes against the risk-averse behavior associated with the BEE deal.

5.4 The Value of BEE Deals for the BEE Partners

The BEE partners provide an initial contribution to the SPV. In return they receive a stream of dividends up to the maturity date, and then they have a claim on the stock of the vendor firm. Indeed, since the vendor has strong incentives to get permanent BEE credentials, there is a strong likelihood that if the vendor is only doing moderately well, further subsidies will be forthcoming
and the chance of the claim being exercised is high. With a levered bet on the vendor stock, it is possible that the BEE partners would like to see the firm take on significant risk.

However, an offsetting argument is the fact that the equity underlying their claim is not increasing in volatility, and the choice of financing of the SPV is essentially controlled by the vendor who chooses policies to maximize the original shareholders interests. So it is unclear how the BEE claim will respond to increased volatility.

Figure 10 shows numerical results. The top right panel shows the value of their claim increases with the growth option that they bring to the deal. For our parameters, the regime shifts from 4 to 5 to 2 as $\pi$ increases. It is these shifts in regime that cause the sensitivity to $\pi$ to change. However, the sensitivity of these values are very dependent on the level of volatility. In general, as volatility increases, the value of the BEE claim decreases. This is seen more clearly in the left panel where the value of the BEE claim is plotted against volatility for a few $\pi$ values, in the range where doing the deal is superior to not doing the deal.

Figure 10 Here

When $\pi$ equals or exceeds 1.4, the BEE claim is somewhat insensitive to increasing volatility. It is in these cases that most of the funding comes from the vendor. In contrast, when $\pi = 1.30$ and the optimal funding includes more bank borrowing, then wrong-way risk is exacerbated, and the BEE claim decreases rapidly with volatility. Indeed, when $\pi = 1.30$, increasing volatility from 10% to 30%, causes a significant drop of over 30% in the BEE claim.

So, in general, the BEE holders are aligned with the equity holders in that their behavior is consistent with risk-aversion. This is important, since the representatives of the BEE holders usually occupy board seats of the vendor firm.

6 Conclusion

We have developed a dynamic capital structure model of the firm that incorporates the salient features of BEE contracts. On the one hand, the resulting models provide guidance for vendors, who ultimately are responsible for structuring the BEE deals, to financiers, who have to determine the fair credit spread to charge for loans, and to the BEE participants, who want to value their claim in the SPV. Current modeling approaches used in the majority of deals completely ignore the growth option that is brought to the deal by the BEE group and take the BEE claim to be a rather simple claim on the exogenous equity. By pushing uncertainty from the stock dynamics to the earning stream, and acknowledging the frictions created by the BEE regulations, our approach adds realism, and has the potential not only to provide more accurate pricing, but also guidance for how best to structure the BEE deals themselves. On the other hand, and more importantly, the model developed here, provides information for ongoing policy changes by providing insights into the economic ramifications to firms that engage in BEE deals. In following this approach, it is revealed that wrong-way risk creates distortions whereby rational equityholders might forgo positive NPV projects with large volatilities. Finally, and perhaps somewhat surprisingly, the BEE
holders, who essentially have an option on the equity, are aligned with the equityholders in that their claim is less valuable if volatility of earnings is expanded. This risk-aversion could have effects on firm behavior in the South African context.

Our model has identified important properties that lead directly to empirical hypotheses. First, the major determinants of the BEE structure put in place by the vendor depends heavily on the magnitude of the growth factor, $\pi$, and on the volatility of the earnings stream, $\sigma$. For firms in the telecommunications or mining industries, where close cooperation with government for licenses is important and where $\pi$ is large, our results suggest high vendor financing is appropriate. In contrast, for firms where both $\pi$ and $\sigma$ are smaller, then we should see a higher participation by third party financial institutions. A very preliminary look at data seems to support this, but a more detailed examination is called for. Second, given that a BEE deal is in place, and especially for firms that make significant use of bank debt, we could expect wrong-way risk to play a significant role in managerial decision making and especially in capital budgeting decisions. Specifically, such firms may be more cautious, may delay investments, and in general, may choose to reduce their volatility of earnings. Such firms may also use less executive stock option plans and compensate executives more heavily with fixed wages. Alternatively, they may more actively use derivative contracts to hedge against this wrong-way risk. Third, the capital structure of the vendor does play an important role. An increasing debt level in the capital structure of the vendor can offset the risk-aversion inherent in the BEE deal. It would be interesting to investigate if, after BEE deals are established, the vendor alters its capital structure. It remains for future research to empirically examine these effects.

On the methodological front some natural extensions are possible. In our analysis the value of the claims at the termination date are completely general, in that the equations depend explicitly on the terminal values of the loans due to the bank and vendor. However, to derive analytical solutions at time 0, we assumed that all dividend flows into the SPV were passed through as trickle dividends to the BEE participants. This assumption removed path dependence issues associated with changing loan amounts over the time to maturity. Using Monte Carlo simulation, it is possible to extend our analysis to the more general path dependent case where some dividends flowing into the SPV are used to pay down the SPV loans. We leave this extension for future work.

Finally, it remains for future research to investigate how BEE regulations can be improved upon, ensuring firms continue to empower previously disadvantaged groups, while at the same time overcoming the issues associated with wrong-way risk and accompanying risk-aversion. One interesting mechanism for reducing wrong-way risk is to automatically reduce the threshold level, $f_B$, if the economy falls into recession. This would be akin to COCOs, where market triggers are sometimes used to force conversion of debt into equity. Here, in bad states of the economy, the pressure to force a high degree of black ownership at the maturity date, could be temporarily suspended.
References


Piketty, T., 2015, The Economics of Inequality, Harvard University Press.


Appendix

Proof of Lemma 1

It is obvious from (1) and (2) that $F \leq \xi_1 \leq \xi_2$. From (3), it is also the case that $F \geq \xi_3 = \frac{c_1}{\pi}$ iff $\pi \geq \frac{\xi_1}{F}$. This defines the first ordering and the subset $\mathcal{C}_{1}^{NC}$. Similarly, $\xi_1 \geq \xi_3$ iff $\pi \geq \frac{c_1}{\xi_1}$. This means that $F < \xi_3 \leq \xi_1$ iff $\pi > \frac{c_1}{\xi_1}$, which defines the second ordering and the subset $\mathcal{C}_{2}^{NC}$. The other two cases follow similarly.

Proof of Proposition 1

In terms of the four orderings of $F$, $\xi_1$, $\xi_2$ and $\xi_3$ from Lemma 1, Figure 3 shows the terminal equity payouts in terms of $P_j(V_T)$, for $j \in \{1, 2, 3, 4\}$. The result follows by writing down the payouts directly.

Proof of Proposition 2

The result follows by a direct risk-neutral replication of the payouts in Proposition 1 with the addition of the expected dividends earned up to time $T$. For example, consider $\pi \in \mathcal{C}_{2}^{NC}$, corresponding to the top right panel in Figure 3. One long call, struck at $F$, combined with one short call struck at $\xi_3$ replicates the payout for $F \leq V_T < \xi_3$, which has slope one. Adding a long binary struck, at $\xi_3$, paying $(P_4(\xi_3) - P_1(\xi_3))$, gives the jump at $V_T = \xi_3$ from $P_1(\xi_3)$ to $P_4(\xi_3)$. Finally, adding $\pi$ calls struck at $\xi_3$ replicates the payout for $V_T \geq \xi_3$, which has slope $\pi$. The other cases follow using similar replication.

Proof of Proposition 3

In a manner analogous to Proposition 1, Table A.1 shows the terminal payouts applicable for the corporate bond, bank loan and vendor loan at time $T$. With these terminal payouts it is possible to use risk-neutral replication to provide time 0 values for the claims in a manner similar to that used in the proof of Proposition 2.

Proof of Lemma 2

For $F \leq x \leq \xi_1$, we have $P_1(x) = x - F$, and

$$P_5(x) = \frac{1-f_0}{1-f_B} (\pi x - F) - \frac{1}{1-f_0} K_B - \frac{f_0}{1-f_0} K_V$$

$$= \frac{1-f_0}{1-f_B} (\pi P_1(x) + (\pi - 1) F) - \frac{1}{1-f_0} K_B - \frac{f_0}{1-f_0} K_V,$$

which means that

$$\frac{1-f_0}{1-f_B} P_5(x) - \pi P_1(x) = (\pi - 1) F - \frac{1}{1-f_0} K_B - \frac{f_0}{1-f_0} K_V = \pi F - c_0.$$
Now, $\xi_0 < F$ implies that $\pi F - c_0 > 0$, while $f_0 \geq f_B$ and $\pi > 1$ imply that $\frac{1-f_0}{\pi(1-f_B)} \leq 1$. Therefore

$$P_1(x) < \frac{1-f_0}{\pi(1-f_B)} P_3(x) < P_3(x).$$

**Proof of Proposition 4**

Firstly note that, in the light of Lemma 1 and Lemma 2, the payout combinations as depicted in Figure 4 are exhaustive. Note also that there are only six distinct payoff profiles, since panels one, three, six and twelve are the same, panels two, five and ten are the same, panels four and eight are same, while the remainder of the panels, seven, nine and eleven are distinct. We now treat each of these six cases in order.

For case one, consider the four panels:

**Panel 1:** Since $\xi_3 \leq F \leq \xi_1 \leq \xi_2$, we require $\pi \in \left[ \frac{c_0}{F}, \infty \right)$.

**Panel 3:** Since $F < \xi_3 \leq \xi_1 \leq \xi_2$, we require $\pi \in \left[ \frac{c_0}{\xi_3}, \frac{c_0}{\xi_1} \right]$. We also require $\xi_0 = \frac{c_0}{\pi} \leq F$, which means $\pi \geq \frac{c_0}{F}$. Thus we require $\pi \in \left[ \max \left[ \frac{c_0}{F}, \frac{c_1}{\xi_1} \right], \frac{c_0}{F} \right]$.  

**Panel 6:** Since $F \leq \xi_1 < \xi_3 \leq \xi_2$, we require $\pi \in \left[ \frac{c_0}{\xi_2}, \frac{c_1}{\xi_1} \right]$. We also require $\xi_0 = \frac{c_0}{\pi} \leq F$,  

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<td>if $V_T \geq \xi_3$</td>
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<td>$K_B$</td>
<td>$f_0P_2(V_T) - K_B$</td>
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**Table A.1:** Table of payout for corporate bond, bank loan and vendor loan.
which means $\pi \geq \frac{p}{P}$. Thus we require $\pi \in \left[ \max\left\{ \frac{p}{P}, \frac{c_i}{c_j}\right\}, \frac{c_i}{c_j}\right]$. Note that Lemma 2 guarantees $P_3(\xi_1) > P_1(\xi_1)$.

Panel 12: Since $F \leq \xi_1 \leq \xi_2 < \xi_3$, we require $\pi \in \left(1, \frac{c_i}{c_j}\right)$. We also require $\xi_0 = \frac{p}{p} \leq F$, which means $\pi \geq \frac{c_i}{c_j} > 1$, and that $P_3(\xi_2) \geq P_2(\xi_2)$, which means $\pi \geq \pi_2$. Thus we require $\pi \in \left[ \max\left\{ \pi_2, \frac{p}{P}, \frac{c_i}{c_j}\right\}, \frac{c_i}{c_j}\right]$. Note that Lemma 2 guarantees $P_3(\xi_1) > P_1(\xi_1)$.

These four sets, when combined, define $\mathcal{E}_1^D$ and provide the payout for equity of 0 for $V_T < \xi_0$, $P_3(V_T)$ for $\xi_0 \leq V_T < \xi_3$ and $P_4(V_T)$ for $V_T \geq \xi_3$.

For case two, consider the three panels:

Panel 2: Since $F < \xi_1 \leq \xi_2$, we require $\pi \in \left(\frac{c_i}{c_j}, \frac{p}{P}\right)$. We also require $\xi_0 = \frac{p}{p} > F$, which means $\pi < \frac{p}{P} = \frac{c_i}{c_j}$ (for last inequality, see below). Thus we require $\pi \in \left(\frac{c_i}{c_j}, \frac{p}{P}\right)$.

Panel 5: Since $F \leq \xi_1 < \xi_2 \leq \xi_3$, we require $\pi \in \left(\frac{c_i}{c_j}, \frac{p}{P}\right)$. We also require $\xi_0 = \frac{p}{p} > F$, which means $\pi < \frac{p}{P}$, and that $P_3(\xi_1) \geq P_1(\xi_1)$, which means $\pi \geq \pi_1$. Thus we require $\pi \in \left[ \max\left\{ \pi_1, \frac{p}{P}, \frac{c_i}{c_j}\right\}, \frac{c_i}{c_j}\right]$.

Panel 10: Since $F \leq \xi_1 < \xi_2 \leq \xi_3$, we require $\pi \in \left(1, \frac{c_i}{c_j}\right)$. We also require $\xi_0 = \frac{p}{p} > F$, which means $\pi < \frac{p}{P}$, that $P_3(\xi_1) \geq P_1(\xi_1)$, which means $\pi \geq \pi_1 > 1$ and that $P_3(\xi_2) \geq P_2(\xi_2)$, which means $\pi \geq \pi_2 > 1$. Thus we require $\pi \in \left[ \max\{\pi_1, \pi_2\}, \frac{p}{P}\right]$).

These three sets, when combined, define $\mathcal{E}_2^D$ and provide the payout for equity of 0 for $V_T < F$, $P_1(V_T)$ for $F \leq V_T < \xi_1$, $P_3(V_T)$ for $\xi_1 \leq V_T < \xi_2$, $P_4(V_T)$ for $\xi_2 \leq V_T < \xi_3$ and $P_1(V_T)$ for $V_T \geq \xi_3$.

For Panel 2 above, we require $\frac{p}{p} < \frac{c_i}{c_j}$. This follows from

\[
c_0 = F + \frac{1}{1-f_B} K_B + \frac{f_B}{1-f_B} K_V \quad \text{and} \quad c_1 = F + \frac{1}{f_0-f_B} K_B + \frac{1-f_0+f_B}{f_0-f_B} K_V,
\]

and that $\frac{1}{1-f_B} < \frac{f_B}{1-f_B} \quad \text{and} \quad \frac{f_B}{1-f_B} < \frac{1-f_0+f_B}{1-f_B}$, which are straightforward to show.

For case three, consider the two panels:

Panel 4: Since $F \leq \xi_1 < \xi_3 \leq \xi_2$, we require $\pi \in \left(\frac{p}{p}, \frac{c_i}{c_j}\right)$. We also require $\xi_0 = \frac{p}{p} > F$, which means $\pi < \frac{p}{p}$, and that $P_3(\xi_1) < P_1(\xi_1)$, which means $\pi < \pi_1$. Thus we require $\pi \in \left(\frac{p}{p}, \min\left\{ \pi_1, \frac{p}{P}, \frac{c_i}{c_j}\right\}\right)$.

Panel 8: Since $F \leq \xi_1 < \xi_2 \leq \xi_3$, we require $\pi \in \left(1, \frac{c_i}{c_j}\right)$. We also require $\xi_0 = \frac{p}{p} > F$, which means $\pi < \frac{p}{p}$, that $P_3(\xi_1) < P_1(\xi_1)$, which means $\pi < \pi_1$ and that $P_3(\xi_2) \geq P_2(\xi_2)$, which means $\pi \geq \pi_2 > 1$. Thus we require $\pi \in \left(\pi_2, \min\left\{ \pi_1, \frac{p}{P}, \frac{c_i}{c_j}\right\}\right)$.

These three sets, when combined, define $\mathcal{E}_3^D$ and provide the payout for equity of 0 for $V_T < F$, $P_1(V_T)$ for $F \leq V_T < \xi_1$, $P_2(V_T)$ for $\xi_1 \leq V_T < \xi_2$, $P_3(V_T)$ for $\xi_2 \leq V_T < \xi_3$ and $P_4(V_T)$ for $V_T \geq \xi_3$. 

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For case four, consider:

Panel 7: Since $F \leq \xi_1 \leq \xi_2 < \xi_3$, we require $\pi \in \left(1, \frac{c_0}{c_2} \right)$. We also require $\xi_0 = \frac{c_0}{F} > F$, which means $\pi < \frac{c_0}{F}$, that $P_5(\xi_1) < P_1(\xi_1)$, which means $\pi < \pi_1$ and that $P_5(\xi_2) < P_2(\xi_2)$, which means $\pi < \pi_2$. Thus we require $\pi \in \left(1, \min\left[\pi_1, \pi_2, \frac{c_0}{F}, \frac{c_1}{c_2} \right] \right)$.

This set defines $\mathcal{C}_5^D$ and provides the payout for equity of 0 for $V_T < F$, $P_1(V_T)$ for $F \leq V_T < \xi_1$, $P_2(V_T)$ for $\xi_1 \leq V_T < \xi_2$, $P_3(V_T)$ for $\xi_2 \leq V_T < \xi_3$, $P_3(V_T)$ for $\xi_3 \leq V_T < \xi_3$ and $P_4(V_T)$ for $V_T \geq \xi_3$.

For case five, consider:

Panel 9: Since $F \leq \xi_1 \leq \xi_2 < \xi_3$, we require $\pi \in \left(1, \frac{c_0}{c_2} \right)$. We also require $\xi_0 = \frac{c_0}{F} > F$, which means $\pi < \frac{c_0}{F}$, that $P_5(\xi_1) \geq P_1(\xi_1)$, which means $\pi \geq \pi_1 > 1$ and that $P_5(\xi_2) < P_2(\xi_2)$, which means $\pi < \pi_2$. Thus we require $\pi \in \left[\pi_1, \min\left[\pi_2, \frac{c_0}{F}, \frac{c_1}{c_2} \right] \right)$.

This set defines $\mathcal{C}_5^D$ and provides the payout for equity of 0 for $V_T < F$, $P_1(V_T)$ for $F \leq V_T < \xi_1^l$, $P_5(V_T)$ for $\xi_1^l \leq V_T < \xi_2$, $P_2(V_T)$ for $\xi_2 \leq V_T < \xi_3$, $P_3(V_T)$ for $\xi_3 \leq V_T < \xi_3$ and $P_4(V_T)$ for $V_T \geq \xi_3$.

For case six, consider:

Panel 11: Since $F \leq \xi_1 \leq \xi_2 < \xi_3$, we require $\pi \in \left(1, \frac{c_0}{c_2} \right)$. We also require $\xi_0 = \frac{c_0}{F} \leq F$, which means $\pi \geq \frac{c_0}{F} > 1$, and that $P_5(\xi_2) < P_2(\xi_2)$, which means $\pi < \pi_2$. Thus we require $\pi \in \left[\frac{c_0}{F}, \min\left[\pi_2, \frac{c_0}{c_2} \right] \right)$. Note that Lemma 2 guarantees $P_5(\xi_1) > P_1(\xi_1)$.

This set defines $\mathcal{C}_5^D$ and provides the payout for equity of 0 for $V_T < \xi_0$, $P_5(V_T)$ for $\xi_0 \leq V_T < \xi_2^l$, $P_5(V_T)$ for $\xi_2^l \leq V_T < \xi_2$, $P_3(V_T)$ for $\xi_2 \leq V_T < \xi_3$, $P_5(V_T)$ for $\xi_3 \leq V_T < \xi_3$ and $P_4(V_T)$ for $V_T \geq \xi_3$.

**Proof of Proposition 5**

With the terminal payouts described in Proposition 4 it is possible to use risk-neutral replication to provide the time 0 value for the claim in a manner similar to that used in the proof of Proposition 2.

**Proof of Proposition 6**

Table A.2 shows the terminal payouts applicable for the corporate bond, bank loan and vendor loan at time $T$ for the six possible cases described by Proposition 4 With these terminal payouts it is possible to use risk-neutral replication to provide time 0 values for the claims in a manner similar to that used in the proof of Proposition 2.
For the BEE claim, consider the payouts that the BEE group receive at time $T$.

If $\pi \in C_D^1$ then

$$EQ_{T_-}^D = \begin{cases} 
0 & \text{for } V_T < \xi_0 \\
\hat{P}_3(V_T) & \text{for } \xi_0 \leq V_T < \xi_3 \\
\hat{P}_4(V_T) & \text{for } V_T \geq \xi_3.
\end{cases}$$

If $\pi \in C_D^2$ then

$$EQ_{T_-}^D = \begin{cases} 
0 & \text{for } V_T < \xi_1' \\
\hat{P}_3(V_T) & \text{for } \xi_1' \leq V_T < \xi_3 \\
\hat{P}_4(V_T) & \text{for } V_T \geq \xi_3.
\end{cases}$$

If $\pi \in C_D^3$ then

$$EQ_{T_-}^D = \begin{cases} 
0 & \text{for } V_T < \xi_2' \\
\hat{P}_3(V_T) & \text{for } \xi_2' \leq V_T < \xi_3 \\
\hat{P}_4(V_T) & \text{for } V_T \geq \xi_3.
\end{cases}$$

If $\pi \in C_D^4$ then

$$EQ_{T_-}^D = \begin{cases} 
0 & \text{for } V_T < \xi_1 \\
\hat{P}_3(V_T) & \text{for } \xi_1 \leq V_T < \xi_2 \\
\hat{P}_4(V_T) & \text{for } \xi_2 \leq V_T < \xi_3 \\
\hat{P}_5(V_T) & \text{for } \xi_3 \leq V_T < \xi_3 \\
\hat{P}_4(V_T) & \text{for } V_T \geq \xi_3.
\end{cases}$$

If $\pi \in C_D^5$ then

$$EQ_{T_-}^D = \begin{cases} 
0 & \text{for } V_T < \xi_1 \\
\hat{P}_3(V_T) & \text{for } \xi_1 \leq V_T < \xi_2 \\
\hat{P}_4(V_T) & \text{for } \xi_2 \leq V_T < \xi_3 \\
\hat{P}_5(V_T) & \text{for } \xi_3 \leq V_T < \xi_3 \\
\hat{P}_4(V_T) & \text{for } V_T \geq \xi_3.
\end{cases}$$

In the above the payouts are defined as follows:

$$\hat{P}_3(V_T) = f_0 P_3(V_T) - K_B - K_V$$
$$\hat{P}_4(V_T) = f_0 P_4(V_T) - K_B - K_V$$
$$\hat{P}_5(V_T) = f_B P_4(V_T).$$

To price the claim at time 0, it is sufficient to note that $\hat{P}_3(\xi_0) = 0$, $\hat{P}_3(\xi_2) = 0$ and $\hat{P}_5(\xi_3) = \hat{P}_4(\xi_3)$, which means there are no jumps in value at these points, and replicate using risk-neutral valuation.
1. If $\pi \in \mathcal{C}_1^D$ then
   
<table>
<thead>
<tr>
<th>Condition</th>
<th>CB$_T^D(K_B, K_V)$</th>
<th>BL$_T^D(K_B, K_V)$</th>
<th>VL$_T^D(K_B, K_V)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_T &lt; \xi_0$</td>
<td>$(1-b)V_T$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$V_T \geq \xi_0$</td>
<td>$F$</td>
<td>$K_B$</td>
<td>$K_V$</td>
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2. If $\pi \in \mathcal{C}_2^D$ then
   
<table>
<thead>
<tr>
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<th>CB$_T^D(K_B, K_V)$</th>
<th>BL$_T^D(K_B, K_V)$</th>
<th>VL$_T^D(K_B, K_V)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_T &lt; F$</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$F \leq V_T &lt; \xi_1^f$</td>
<td>$F$</td>
<td>$f_0P_1(V_T)$</td>
<td>0</td>
</tr>
<tr>
<td>$V_T \geq \xi_1^f$</td>
<td>$F$</td>
<td>$K_B$</td>
<td>$f_0P_2(V_T) - K_B$</td>
</tr>
</tbody>
</table>

3. If $\pi \in \mathcal{C}_3^D$ then
   
<table>
<thead>
<tr>
<th>Condition</th>
<th>CB$_T^D(K_B, K_V)$</th>
<th>BL$_T^D(K_B, K_V)$</th>
<th>VL$_T^D(K_B, K_V)$</th>
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</thead>
<tbody>
<tr>
<td>$V_T &lt; F$</td>
<td>$(1-b)V_T$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$F \leq V_T &lt; \xi_1$</td>
<td>$F$</td>
<td>$f_0P_1(V_T)$</td>
<td>0</td>
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<tr>
<td>$\xi_1 \leq V_T &lt; \xi_2$</td>
<td>$F$</td>
<td>$K_B$</td>
<td>$f_0P_2(V_T) - K_B$</td>
</tr>
<tr>
<td>$V_T \geq \xi_2$</td>
<td>$F$</td>
<td>$K_B$</td>
<td>$K_V$</td>
</tr>
</tbody>
</table>

4. If $\pi \in \mathcal{C}_4^D$ then
   
<table>
<thead>
<tr>
<th>Condition</th>
<th>CB$_T^D(K_B, K_V)$</th>
<th>BL$_T^D(K_B, K_V)$</th>
<th>VL$_T^D(K_B, K_V)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_T &lt; F$</td>
<td>$(1-b)V_T$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$F \leq V_T &lt; \xi_1$</td>
<td>$F$</td>
<td>$f_0P_1(V_T)$</td>
<td>0</td>
</tr>
<tr>
<td>$\xi_1 \leq V_T &lt; \xi_2$</td>
<td>$F$</td>
<td>$K_B$</td>
<td>$f_0P_2(V_T) - K_B$</td>
</tr>
<tr>
<td>$V_T \geq \xi_2$</td>
<td>$F$</td>
<td>$K_B$</td>
<td>$K_V$</td>
</tr>
</tbody>
</table>

5. If $\pi \in \mathcal{C}_5^D$ then
   
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<tr>
<th>Condition</th>
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<tr>
<td>$V_T &lt; F$</td>
<td>$(1-b)V_T$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$F \leq V_T &lt; \xi_1^f$</td>
<td>$F$</td>
<td>$f_0P_1(V_T)$</td>
<td>0</td>
</tr>
<tr>
<td>$\xi_1^f \leq V_T &lt; \xi_2^f$</td>
<td>$F$</td>
<td>$K_B$</td>
<td>$f_0P_2(V_T) - K_B$</td>
</tr>
<tr>
<td>$V_T \geq \xi_2^f$</td>
<td>$F$</td>
<td>$K_B$</td>
<td>$K_V$</td>
</tr>
</tbody>
</table>

6. If $\pi \in \mathcal{C}_6^D$ then
   
<table>
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<th>Condition</th>
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<th>BL$_T^D(K_B, K_V)$</th>
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<tr>
<td>$V_T &lt; \xi_0$</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\xi_0 \leq V_T &lt; \xi_2^f$</td>
<td>$F$</td>
<td>$K_B$</td>
<td>$K_V$</td>
</tr>
<tr>
<td>$\xi_2^f \leq V_T &lt; \xi_2$</td>
<td>$F$</td>
<td>$K_B$</td>
<td>$f_0P_2(V_T) - K_B$</td>
</tr>
<tr>
<td>$V_T \geq \xi_2$</td>
<td>$F$</td>
<td>$K_B$</td>
<td>$K_V$</td>
</tr>
</tbody>
</table>

Table A.2: Table of payout for corporate bond, bank loan and vendor loan.
Figure 1: Distribution of BEE Deal Values

The histogram shows the distribution of values of BEE deals in millions of U.S. dollars. The raw data comes from Theobald, Tambo, Makuwerere, and Anthony (2015), who provide the details of most deals done by the top 100 firms listed on the Johannesburg Stock Exchange over the time period 2000 through 2014. The total dollar value to BEE beneficiaries of these deals is approximately 30 billion dollars.
Figure 2: Summary of a BEE Transaction.

The figure shows the inception of a BEE deal (top). The bulk of funds used to purchase shares come from financiers. The equity purchased is retained in an SPV. During the life of the deal (middle), some fraction of the dividends received into the SPV are used to pay down interest and principal payments on the loans. At maturity (bottom), the SPV is liquidated and the outstanding loans are paid off, with any residual equity paid out to the BEE holders. If the BEE holders net equity stake exceeds $f_B$ then permanent BEE status is achieved.
Figure 3: Possible Payout Values for Equity.

The figure shows the four possible payouts of equity at maturity, $T$ under the assumption that the equity-holders do not make a contribution at maturity.
Figure 4: Possible Payout Values for Equity with Refinancing.

The figure shows all possible payouts of equity at maturity. The solid line assumes that no refinancing takes place. The dashed line shows the equity value at time $T-$ assuming the equityholders make a contribution at $T$.  

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Figure 5: Feasible Regions for Policy Variables

The left graph shows the feasible set of \((K_B, K_V)\) variables for an increasing set of BEE growth factors, \(\pi\). The feasible set is constrained by the requirement that SPV borrowing needs are restricted to their requirement of purchasing fraction \(f_0\) of outstanding equity, and the banks requirement of meeting a minimum cover ratio (see equations (6) and (7)). The right graph shows the value of the equity for all these feasible values, for each BEE growth factor, \(\pi\). The equity maximizing point is indicated with a solid dot. The case parameters are \(X_0 = 20\), \(q = 0.03\), \(r = 5\%\), \(\sigma = 25\%\). The tax rate is assumed to be \(\tau = 25\%\), and the deadweight bankruptcy cost for the vendor is assumed to be \(b = 10\%\) of the value of assets at bankruptcy. The contract has maturity \(T = 5\), \(f_B = 25\%\), \(f_0 = 26\%\) and \(f_V = 2.5\%\), with the BEE contribution \(P = 14\). The face value of the corporate bond is \(F = 100\) and the cover ratio required by the bank is \(\kappa = 2\).
Figure 6: Comparison of Equity Prices Under Different SPV Financing Choices

Each graph contains four plots of equity under different financing structures for the SPV. The first utilizes as much bank financing as is feasible. The second uses no bank financing and relies completely on vendor financing. The third is the optimal mix of bank and vendor financing that satisfies the two constraints (see equations (6) and (7)). These three values are compared to the value of equity when no deal is done. The sensitivity of equity to increasing earnings volatility is shown for six different growth factors, $\pi$. The case parameters are $X_0 = 20$, $q = 0.03$, $r = 5\%$, $\sigma = 25\%$. The tax rate is assumed to be $\tau = 25\%$, and the deadweight bankruptcy cost for the vendor is assumed to be $b = 10\%$ of the value of assets at bankruptcy. The contract has maturity $T = 5$, $f_B = 25\%$, $f_0 = 26\%$ and $f_V = 2.5\%$, with the BEE contribution $P = 14$. The face value of the corporate bond is $F = 100$ and the cover ratio required by the bank is $\kappa = 2$. 

[Graphs showing different financing options for various $\pi$ values]
The different lines on each curve represent different EBIT volatility levels. The top left panel shows the percentage of total loans obtained by the SPV that are made by the bank when the vendor maximizes its interests. The top right panel shows the fair credit spreads charged by the bank for funds lent to the SPV. The bottom left panel shows the sensitivity of the value of the total vendor subsidy to the growth factor, $\pi$. The bottom right graph shows how the regimes change as the growth factor, $\pi$ increases. The case parameters are $X_0 = 20$, $q = 0.03$, $r = 5\%$, $\sigma = 25\%$. The tax rate is assumed to be $\tau = 25\%$, and the deadweight bankruptcy cost for the vendor is assumed to be $b = 10\%$ of the value of assets at bankruptcy. The contract has maturity $T = 5$, $f_B = 25\%$, $f_0 = 26\%$ and $f_V = 2.5\%$, with the BEE contribution $P = 14$. The face value of the corporate bond is $F = 100$ and the cover ratio required by the bank is $\kappa = 2$. 

Figure 7: SPV Loan Compositions at Optimal Equity: Sensitivity to Growth Factors
Figure 8: The impact of volatility on equity

The left panel shows the behavior of equity to volatility before the deal is consummated, and compares the equity price to the price just before and after a deal is done. When the value of $\pi$ is smaller than 1.2 no deal is optimal. The right panel shows the same pattern for $\pi = 1.6$. The case parameters are $X_0 = 20$, $q = 0.03$, $r = 5\%$. The tax rate is assumed to be $\tau = 25\%$, and the deadweight bankruptcy cost for the vendor is assumed to be $b = 10\%$ of the value of assets at bankruptcy. The contract has maturity $T = 5$, $f_B = 25\%$, $f_0 = 26\%$ and $f_V = 2.5\%$, with the BEE contribution $P = 14$. The cover ratio required by the bank is $\kappa = 2$. 
Figure 9: Leverage Effects on BEE Deals

For given corporate bond face values, we compute the percentage change in equity value, as volatility of earnings increases. The benchmark value for volatility is taken to be $\sigma = 5\%$. The face value of corporate debt, $F$, ranges from 100 to 400, and volatility, $\sigma$ ranges from 5% to 50%. The left panel shows the percentage change in equity when $\pi = 1.3$ while the right panel is for $\pi = 1.6$. The graphs show that as face value of debt increases, the leverage effect eventually overwhelms the BEE wrong-way risk effect. The case parameters are $X_0 = 20, q = 0.03, r = 5\%$. The tax rate is assumed to be $\tau = 25\%$, and the deadweight bankruptcy cost for the vendor is assumed to be $b = 10\%$ of the value of assets at bankruptcy. The contract has maturity $T = 5, f_B = 25\%, f_0 = 26\%$ and $f_V = 2.5\%$. The cover ratio required by the bank is $\kappa = 2$. 
Figure 10: The Value of the BEE Claim

The left panel shows the value of the BEE claim as a function of volatility for different growth factors, $\pi$. The right panel shows the sensitivity of the BEE claim to the growth factor $\pi$ for different earnings volatility, $\sigma$. The case parameters are $X_0 = 20$, $q = 0.03$, $r = 5\%$. The tax rate is assumed to be $\tau = 25\%$, and the deadweight bankruptcy cost for the vendor is assumed to be $b = 10\%$ of the value of assets at bankruptcy. The contract has maturity $T = 5$, $f_B = 25\%$, $f_0 = 26\%$ and $f_V = 2.5\%$. The cover ratio required by the bank is $\kappa = 2$. 
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<tr>
<th>Vendor</th>
<th>Main BEE Partner</th>
<th>Year</th>
<th>Deal Val. Rands (m)</th>
<th>2015 Val. Rands (m)</th>
<th>Vendor %</th>
<th>Bank %</th>
<th>BEE %</th>
<th>Facil. %</th>
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<td>First Rand</td>
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Table 1: Larger BEE Deals

The table shows ten of the largest BEE deals conducted in South Africa. The deal value at the initiation date, together with the composition of financing is reported. The value of the deals, either at conclusion, or at 2015, whichever came first, are indicated. These 2015 values are taken from the Intellidex report (Theobald, Tambo, Makuwerere, and Anthony (2015)). We chose the firms, based on the largest deals identified in this report. We dropped Impala Platinum, which was sixth on their list, because the deal was based on replacing a royalty arrangement with an ownership stake. We also dropped RMH, which was eighth on their list, because we could not identify the financing composition. We replaced these two deals with the 2005 Mediclinic deal and the largest deal done to date, namely the 2018 Vodacom deal. We also included the restructured Exxaro deal of 2017. The financing compositions of the 2016 MTN, Exxaro, and Vodacom deals, are all based on the restructured deals where the BEE contributions were based on using proceeds from their original deal. The exchange rate over this period ranged from approximately 7 to 15 Rands per U.S. dollar.
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<th>Vendor</th>
<th>Main BEE Partner</th>
<th>Year</th>
<th>Value Rands (m)</th>
<th>Vendor %</th>
<th>Bank %</th>
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Table 2: BEE Deals in the Mining Sector

The table shows a sample of individual BEE deals that have been done in the mining sector in South Africa, ordered by year, over the time period starting in 2004 and ending in 2012. The value of the full financing is referenced in local currency. The percentage contributions by the vendor, third parties (banks) and BEE groups are provided. The source of the data is Nhasengo (2016). We only report deals larger than 200 million Rands. The exchange rate over this period ranged from approximately 8 to 10 Rands per U.S. dollar. QUESTION: Perhaps Equity consists of Facilitation as well???? Need to check