Risk Free Interest Rates

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Preliminary and Incomplete

Abstract

We document differing risk-free rates in a range of asset classes, providing a uniquely clean measure of segmentation between markets. The asset markets we consider are the government bond market, commodity markets for precious metals, exchange rate markets and option markets. We find that risk-free rates across markets can deviate for prolonged periods of time and we characterize market segmentation through the speed of convergence. We analyze how shocks propagate across rate spreads and develop an aggregate arbitrage index which captures the common variation of these spreads across markets. We further present a novel high-frequency measure of the convenience yield on government bonds, which equals 38 basis points on average and grows substantially during periods of financial distress. We argue that option-market-implied risk-free rates provide a convenience-yield-free and effectively credit-risk-free measure of time preference measured accurately at a minutely frequency. This makes such rates a strong candidate for the risk free benchmark rate and we explore a range of empirical asset pricing applications.

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1. Introduction

Arguably the most important variable in financial economics is the return on risk free investing. In frictionless asset pricing models, it is determined by the investors' time preference. However, a recent literature has questioned whether the time preference of money is the only determinant of this rate, providing evidence that a scarcity of safe assets drives up their price and lowers the corresponding interest rate.[1] Under this alternative interpretation, the time preference of investors can only be inferred by measuring the risk-free rate implied by the prices of risky assets, where the spread between this implied risk-free rate and the observed return on safe assets measures the convenience yield that these safe assets provide.

In this paper, we provide evidence that risk free interest rates can vary substantially across different asset markets, in contrast to the unique rate implied by the neo-classical asset pricing literature. This raises two important questions. First, what empirical measure of the risk-free rate is most appropriate to use in empirical asset pricing applications? Second, what are the characteristics of this persistent cross-sectional dispersion in rates and what variables does it correlate with?

To make progress on the first question, we use a large panel of risky assets to estimate a convenience-yield and essentially credit-risk-free measure of risk free interest rates. This is motivated by the characteristic of several recent asset pricing models with frictions that all risky assets are priced in an internally consistent way. Moreover, the risk-free rate implied by the pricing of risky assets lies strictly above the rate earned on safe assets, where the difference is often interpreted as a measure of the severity of financial frictions. In any such model, our empirical approach provides a natural and robust way to estimate this risk-free rate implied by risky asset prices. As a result, the spread between our estimated rate and that observed on government bonds is a measure of financial frictions implied by such models.

With respect to the second question, our paper is unique in the literature in that it evaluates jointly a large panel of precisely estimated arbitrage opportunities. This allows us to estimate at a daily frequency level the speed of convergence of the spread between markets as well as implied spreads between assets within markets. One could arguably interpret these

[1] See for instance Krishnamurthy and Vissing-Jorgensen (2012)

speeds of convergence as a novel measure of slow-moving capital that accounts for the relative difficulty that intermediaries face in shifting capital between or even within markets.

To the extent possible, we evaluate asset prices within and across asset classes to a minute-level (in some cases hourly) precision. That is, we restrict ourselves to arbitrage opportunities that are so precisely measured that the deviation we document are not driven by asynchronicities in closing prices or other approximation errors. We avoid approximate arbitrages such as the CDS basis, put-call parity relations with American options, and futures with embedded (cheapest-to-deliver) options.

Measuring the relative mispricing of assets has proven to be challenging in the asset pricing literature. The reason is that risky assets demand a potentially time-varying compensation for risk, which therefore makes the efficiency of markets an elusive and untestable concept (Fama (1970)). Because we focus on a large cross-section of assets that all provide a risk free payoff, which by their very nature do not require a compensation for risk, we do not have to formulate a model for risk in our study.

Our paper contributes to several related literatures. First, several existing papers (Amihud and Mendelson (1991), Krishnamurthy (2002), Musto, Nini, and Schwarz (2018), Du, Tepper, and Verdelhan (2018), Daves and Ehrhardt (1993)) respectively study individual arbitrages that we consider in our analysis across multiple asset classes. While these papers cannot make statements about the relative size and speed of convergence of different arbitrage spreads that our multi-market analysis allows for, they do provide additional institutional details about the frictions related to each arbitrage opportunity. The first such paper, Amihud and Mendelson (1991) documents a spread between maturity matched treasury notes and bills and relates it to measures of relative illiquidity. Krishnamurthy shows that spreads between repo rates makes it difficult for a levered investor to profit from the spread between on and off the run bonds. Musto, Nini, and Schwarz (2018) shows how the relative liquidity (measured using bid ask spreads and other proxies from the microstructure literature) of notes and bonds contributes to the spread between their yields. Du, Tepper, and Verdelhan (2018) shows how the size of the covered interest parity spread seems to be influenced by quarterly regulatory reviews of European bankings, suggesting that arbitrage spreads can be influenced by bank regulation as well as showing that they are correlated with some other arbitrage spreads. Daves and Ehrhardt (1993) shows that the spread between interest and principal STRIPS seems related to measures of their degree of illiquidity. Finally, Golez, Jackwerth and Slavutskaya (2018) use a combination of 3-month option and futures data on the S&P500 index to construct a funding illiquidity measure and find that this measure significantly affects the returns of leveraged managed portfolios by hedge funds.

Second, our paper relates to the literature on intermediary asset pricing, particularly the subset of the literature relating arbitrage spreads to financial frictions. He and Krisnamurthy (2013) presents the canonical intermediary asset pricing model, showing theoretically and quantitatively that the capitalization of financial intermediaries is a key state variable for the dynamics of asset prices. A related theoretical and empirical paper by Frazzini and Pedersen (2014) presents a model in which the spread between the return on a zero-beta security and the risk-free rate measures the tightness of leverage constraints for levered investors and shows that this zero-beta rate is very high in a large range of asset classes. Their results raise the question of whether such a zero-beta rate loads on other risk factors. The risk-free rate we estimate from options markets circumvents this problem and implies a considerably smaller spread than the estimates in their paper, so the spread we estimate measures the tightness of leverage constraints in any multi-factor generalization of their model. Also close to our work is Hébert (2018), which presents a theoretical model in which arbitrage spreads are due to constraints on the trading of financial intermediaries. In his model, all constraints are assumed to be due to government policy, and he shows how to infer the government's normative objectives by observing multiple arbitrage spreads. Like our paper, he considers data on multiple arbitrage opportunities though with a more limited empirical scope. Finally, Krishnamurthy and Vissing-Jorgensen (2012) estimate the spread between AAA bonds and U.S. treasuries and provide a conceptual framework through which to interpret this as measuring a convenience yield on government debt. We estimate a smaller convenience yield using our risk-free rate inferred from options markets, suggesting that part but not all of their spread may reflect credit risk.

The paper proceeds as follows. In Section 2 we compute a range of known government bond arbitrages. In Section 3 we use high-frequency option trade and quote data across several underlying assets to infer option-implied risk-free rates and compare them with LIBOR rates and government bond implied rates. In Section 4 we use the covered interest parity relationship to construct exchange rate related arbitrages. In Section 5 we use the no arbitrage relationship between futures and spot prices to infer risk free interest rates from commodities whose physical cost-of-carry is very small relative to the value of the underlying: precious metals. In Section 6 we study how shocks to different interest rates propagate to other markets. Section 7 concludes.

2. Government Bond Markets

2.1. Constructing Risk Free Arbitrages

We consider four distinct categories of arbitrage spreads using government bond data. Two of them relate to zero coupon bond arbitrages, which can be computed without estimating (and interpolating) a yield curve, and two of them involve bonds with coupon payments that do require an estimated yield curve.

2.1.1. Zero Coupon Bond Arbitrages

6 month Spread

First, we consider the spread between notes/bonds that mature within the next 6 months and yields on treasury bills that mature on the exact same date. Treasury bills are more liquid and therefore tend to have lower yields (Amihud and Mendelson (1991)). Because treasury securities pay coupons every 6 months, there are no intermediate coupon payments for either security used in constructing this spread. For each day, we compute the median of the continuously compounded yields to construct a daily time series.

STRIP Spread

Second, we consider the spread between two types of STRIPS (Separate Trading of Registered Interest and Principal of Securities) constructed respectively from interest and principal payments on U.S. government debt. These securities pay identical cash flows and are backed by the full faith of the U.S. government, so any difference between the yields on coupon vs principal STRIPS identifies an arbitrage. In general, whichever of the principal or interest STRIP that has a higher supply outstanding tends to have a lower yield. At short maturities, interest STRIPS are in larger supply while at long maturities principal STRIPS are.[2] Because all principal and interest payments happen on a regular 6 month schedule, there are enough overlapping bonds to consider only spreads between interest and principal strips that mature on exactly the same day. We present averages of both the level as well as the value of this spread across all maturity matched pairs of coupon and principal STRIPS below.

2.1.2. Coupon Bond Arbitrages

Because the two spreads we study in the previous section are between pairs of zero coupon securities with the exact same maturity, no assumptions were required regarding the shape of the yield curve to construct them. This is not true for the two arbitrage spreads that we consider next. The reason is that these next two measures relate to government bonds which make coupon payments for which no exact matching security may exist. As a result, we compute these spreads by comparing a bond's true yield to the yield implied by fitting a yield curve to all treasury bonds. To estimate this yield curve, we estimate a parametric model following Svensson (1994), and Gürkaynak, Sack, and Wright (2007). A Nelson-Siegel-Svensson (NSS) instantaneous forward rate τ periods in the future is assumed to have the functional form

$$f(\tau) = \beta_0 + \left(\beta_1 + \beta_2 \frac{\tau}{\tau_1}\right) \exp\left(-\frac{\tau}{\tau_1}\right) + \beta_3 \frac{\tau}{\tau_2} \exp\left(-\frac{\tau}{\tau_2}\right).$$

Given parameters $(\beta_0, \beta_1, \beta_2, \tau_1, \tau_2)$, this forward rate function uniquely implies a zero coupon yield curve that can be used to price any risk free bond. To estimate the parameters, we use data from GovPX between 3pm and 4pm of each day and consider the price of all off-the-run notes and bonds. Let y_i be the yield to maturity of bond i, D_i be the duration of bond i, and $y_i(\beta_0, \beta_1, \beta_2, \tau_1, \tau_2)$ be its yield to maturity implied by the NSS yield curve. We estimate the parameters of the yield curve for each day by minimizing

$$\sum_i \frac{1}{D_i} \left(y_i - y_i(\beta_0, \beta_1, \beta_2, \tau_1, \tau_2)\right)^2$$

[2] The reason is that all bonds of all maturities pay coupon payments every 6 months and contribute to the coupon-related supply.

where the sum i goes over all bonds quotes between 3 and 4pm that day.

On the Run Spread

We use the NSS yield curve to compute an implied yield for the most recently issued bond of each maturity, called the on-the-run bond, and take its difference from the true yield on that bond. On-the-run bonds tend to be more liquid than off-the-run bonds and therefore trade at a lower yield as shown below. The spread between on- and off-the-run bonds is related to the timing of the treasury auction cycle. This is particularly true for the yield on the on-the-run 10-year bond. We plot the spread between this on-the-run 10-year bond yield and yield implied by the NSS yield curve in Figure I.

Notes vs Bonds

We also use the NSS yield curve to consider the relative spread between treasury notes (which by definition have a maturity less than 10 years after issuance) and bonds (which mature more than 10 years after issuance), that have less than 10 years of maturity left, following (Musto, Nini, and Schwarz (2018)). For each note and bond that mature between 3 and 10 years from the day on which the security is traded, we compute the spread between the security's actual yield to maturity and the yield implied by the estimated NSS yield curve. We then take the median of this spread across all notes, and the median of the spread across all bonds on each day and compute a daily difference between these two medians. As the above authors show, this spread is small in normal times but spikes during the financial crisis.

2.2. Data

Our U.S. treasury security prices come from the GovPX database, which reports trades and quotes from the inter-dealer market for U.S. treasuries. The database provides prices for bills, notes, and bonds at an intraday frequency. We use indicative quotes, which provide the most frequent measure of bond prices on GovPX from 3 to 4pm on each day. In addition, we have data from Tradeweb on the prices of STRIPS, which are zero coupon bonds created by separating the principal and interest payments on treasury securities. This database provides quotes 2 times a day, and we restrict ourselves to quotes at 3pm. Whenever using quote data, we take the midpoint of the bid and ask as the price measure.

Table 1
Summary Statistics of Government Bond Arbitrages 2004-2018

	Mean (in bp)	St. Dev.
6 month Spread	6.4388	6.9544
STRIP Spread	3.8696	8.8144
On the Run Spread		
All Bonds	0.4945	1.9194
10 year Bond	2.1587	2.4044
Notes vs Bonds	0.3576	0.9573

2.3. Results

First, we present summary statistics on the four above-mentioned government bond arbitrages in Table 1 and we plot them in Figures I, II and III.

Several patterns appear across all of these arbitrage spreads. First, both their level and volatility generally increases during the financial crisis period of late 2008 and early 2009. Second, most spreads are smallest in the later part of our sample, suggesting that government bond markets are now even more integrated than they were before the crisis. Third, some spreads (such as the 10-year on the run spread) seem to be driven in part by idiosyncratic factors such as the treasury auction cycle. That is, the regular spikes in Figure I correspond to auction cycle dates. Finally and most importantly, the magnitude of these spreads are uniformly smaller than the spreads we estimate in this paper from several other asset classes. This last fact is consistent with the idea that all government bonds benefit from some degree of specialness due to their safety and liquidity, so that all of their yields lie strictly below the risk-free rate that is implied by risky asset prices.

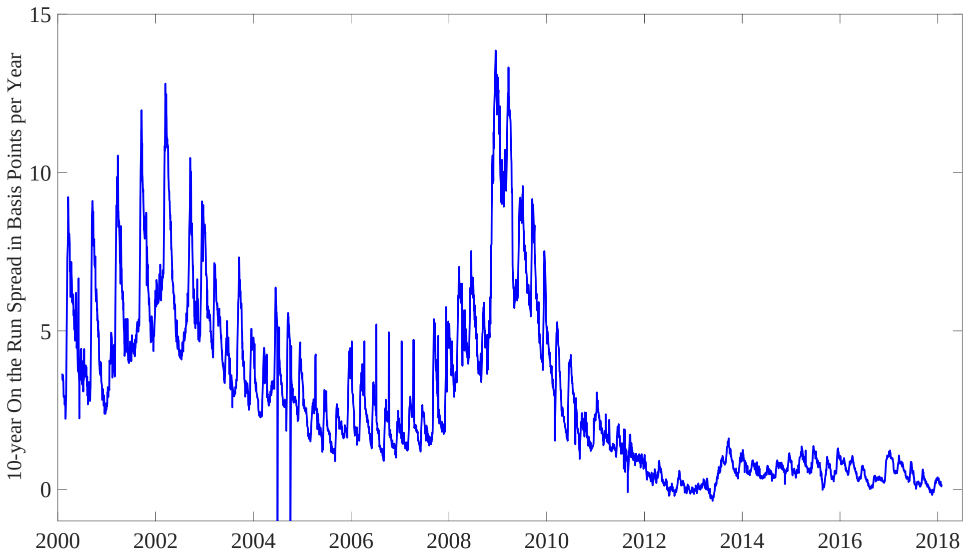

FIGURE I

Ten Year On-the-Run Spread in Basis Points per Year.

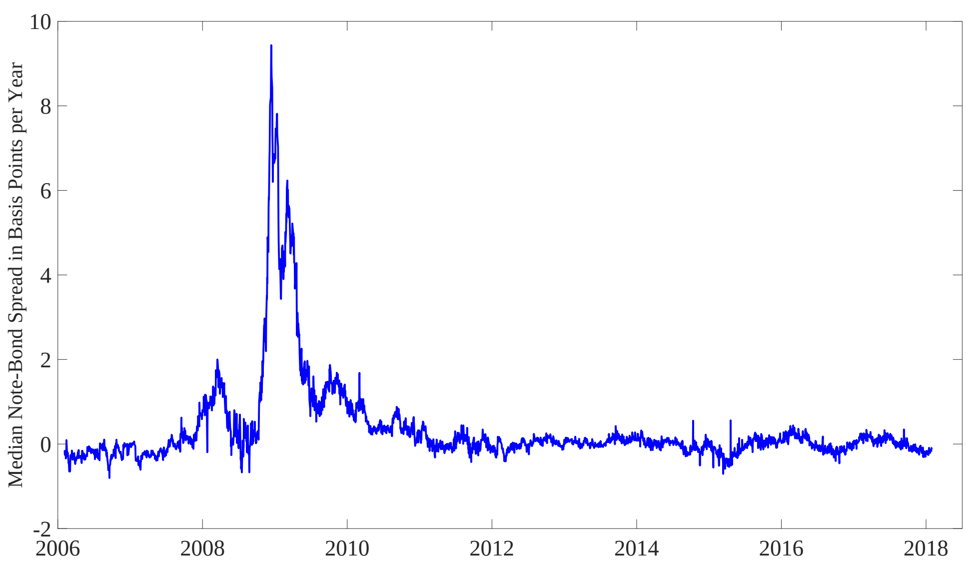

FIGURE II

Notes/Bonds Spread in Basis Points per Year.

FIGURE III
Median Absolute STRIP Spread.

3. Option Markets

Next we infer the risk-free rate from a number of option markets.

3.1. Constructing Risk Free Assets

The starting point of our analysis here is the put-call parity relationship for European options. At each time t, for each time to maturity T, option price quotes are available for a large cross-section of different strike prices indexed by $i = 1, ...N$. The put-call parity relationship then states that at time t, for each time to maturity T, and each strike price K_i, the difference between the put price $p_{i,t,T}$ and call price $c_{i,t,T}$ equals the discounted value of the strike K_i minus the value of the underlying S_t, where we need to adjust the latter for the present value of the cash flow (or convenience) that the security delivers.[3] Denote this

[3]For dividend paying stock indices this price is the present value of the dividends paid out between time t and T, also called the dividend strip price (van Binsbergen, Brandt, and Koijen (2012).

present value of the cash flow (or convenience) by $\mathcal{P}_{t,T}$, then the relationship is given by:

$$p_{i,t,T} - c_{i,t,T} = (\mathcal{P}_{t,T} - S_t) + \exp(-r_{t,T}T)K_i. \tag{1}$$

This relationship provides two ways of obtaining the risk free interest rate $r_{t,T}$ implied by these markets.

Estimator 1: At each time t and for each maturity T, we run the following cross-sectional regression:

$$p_i - c_i = \alpha + \beta K_i + \varepsilon_i \tag{2}$$

where the slope of the line is equal to:

$$\beta = \exp(-r_{t,T}T), \tag{3}$$

and where the intercept is equal to:

$$\alpha = \mathcal{P}_{t,T} - S_t. \tag{4}$$

The continuously compounded risk free interest rate at time t for maturity T can then be computed from the slope coefficient as follows:

$$r_{t,T} = -\frac{1}{T}\ln(\beta). \tag{5}$$

The estimated β of this regression can also be interpreted as the realized risk free return that is earned on a particular trading strategy. To see this, consider the Ordinary Least Squares (OLS) estimator of the slope:

$$\beta_{OLS} = \frac{\sum_i \left((p_i - c_i - \overline{p-c})(K_i - \overline{K})\right)}{\sum_i (K_i - \overline{K})^2} \tag{6}$$

where

$$\overline{p-c} = \frac{\sum_i (p_i - c_i)}{N} \tag{7}$$

and

$$\overline{K} = \frac{\sum_i K_i}{N} \tag{8}$$

So the strategy involves buying (writing) a total of $K_i - \bar{K}$ put options for which the strike is above (below) average and writing (buying) a total of $K_i - \bar{K}$ calls for which the strike is above (below) average for each $i \in 1, ..., N$. This strategy will deliver the continuously compounded realized risk-free rate equal to $-\frac{1}{T}\ln(\beta_{OLS})$.

Estimator 2: At each time t and for each maturity T take all possible combinations of strikes, indexed by $1, ..., A$ where $A = \frac{N(N-1)}{2}$ and compute an implied risk-free rate for that strike pair. That is, $\forall i \in i = 1, ..., N$ and $\forall j \in i = 1, ..., -i, ..., N$ for which $K_i > K_j$, we compute:

$$r_{t,T,a} = -\frac{1}{T}\ln\left(\frac{(p_{i,t,T} - c_{i,t,T}) - (p_{j,t,T} - c_{j,t,T})}{K_i - K_j}\right), \tag{9}$$

with $a \in 1, ..., A$. We then compute the estimate for the risk free as the median over all these implied rates:

$$r_{t,T} = \text{median}_{a \in A}\left(r_{t,T,a}\right). \tag{10}$$

This estimator, which is also known as the Theil–Sen estimator (Theil (1950)) allows for robust estimation of the slope of the regression line even when there are large outliers in the underlying data. It also corresponds to a trading strategy, which is to invest in the strike pair i and j that deliver the median risk-free rate observation. That is, buying the put of strike K_i and the call of strike K_j while writing the call of strike K_i and the put of strike K_j. If one holds these positions till maturity, then the payoff is risk free and equal to $K_i - K_j > 0$. Because buying and writing these puts and calls costs a total of $(p_{i,t,T} - c_{i,t,T}) - (p_{j,t,T} - c_{j,t,T})$, this trading strategy earns exactly the risk-free rate corresponding to the Theil-Sen estimator.

3.2. Data

Our data set contains all option trades and quotes from the Chicago Board Options Exchange (CBOE) on two underlying assets: the S&P 500 index (SPX), the Dow Jones Index (DJX), between 2004 and 2018.[4] The traded options on these underlying assets are European, implying that the put-call parity relationship should hold exactly (for American options it only holds with an inequality). The data set contains the bid price, the ask price,

[4]The VIX is another underlying asset that we studied. The results will be added in the next draft of this paper.

the strike and the maturity date for a large range of strike prices for each minute. We compute risk-free rate estimates at the minute level using the mid prices using all strike prices for puts and call with a particular maturity. To compute daily estimates, we then take a median over the minute-level estimates in the day.

3.3. Results

We now describe the results for each of the underlying assets that we study.

3.3.1. Results S&P 500 Index (SPX)

We start with the results from the S&P 500 index. In Table 1 we provide summary statistics for SPX implied yields for three maturities: 6 months, 12 months and 18 months, and we compare them with the corresponding yields on government bonds as implied by the NSS parameters estimated by Gürkaynak, Sack, and Wright, as well as the continuously compounded version of the LIBOR rate.

The table shows that for all maturities the average yield on the SPX implied interest rates are above those of the corresponding government bonds and below those of the LIBOR rate. The average difference between the SPX implied rate and the government bond rate is 35-37 basis points per year, with very little variation across maturities. The average difference between the LIBOR rate and the SPX implied rate is positive for both the 6-month and 12-month maturities, equal to 7 basis points and 24 basis points respectively. For the 18-month maturity, a LIBOR rate is not available. Further, the LIBOR rate has the lowest volatility, and the SPX implied rate the highest. The autocorrelation of the spreads are high and typically above 0.9.

To better understand the variation and comovement in the three rates, we plot in Figures IV, V and VI the three interest rates for all three maturities.

The three graphs shows a consistent pattern. Before 2008 the SPX implied yields are above the corresponding government bond yield, and closely follow LIBOR. Between 2008 and 2017 a substantial deviation from LIBOR occurs and the SPX implied yields are in

Table 2
Summary Statistics of SPX Option Implied Interest Rates 2004-2018

Zero Coupon Yields: 6 month maturity			
	Mean	St. Dev.	AR(1) (daily)
Option Implied SPX	0.0178	0.0174	0.9995
LIBOR Implied	0.0185	0.0173	0.9999
Government Bond	0.0142	0.0167	0.9998
LIBOR Implied - Option Implied SPX	0.0007	0.0021	0.9638
Option Implied SPX - Government Bond	0.0035	0.0022	0.9607

Zero Coupon Yields: 12 month maturity			
	Mean	St. Dev.	AR(1) (daily)
Option Implied SPX	0.0185	0.0171	0.9980
LIBOR Implied	0.0210	0.0160	0.9998
Government Bond	0.0148	0.0164	0.9997
LIBOR Implied - Option Implied SPX	0.0024	0.0026	0.9148
Option Implied SPX - Government Bond	0.0037	0.0021	0.8738

Zero Coupon Yields: 18 month maturity			
	Mean	St. Dev.	AR(1) (daily)
Option Implied SPX	0.0194	0.0167	0.9996
Government Bond	0.0157	0.0159	0.9996
Option Implied SPX - Government Bond	0.0037	0.0021	0.9774

between the LIBOR rate and the government bond yield. This suggests that between 2009 and 2017 banks faced substantial credit risk, as measured by the spread between LIBOR and the SPX implied zero coupon yield.

To further study the dynamics of this credit risk measure and the convenience yield of government bonds, we plot in Figures VII, VIII and IX the spreads between the SPX implied yield and the government bond yield, as well as the spread between LIBOR and the SPX implied yield with maturities of 6 months, 12 months and 18 months. As LIBOR rates only have maturities up to 12 months, we only plot the spread between the SPX implied yield and the government bond yield for that maturity.

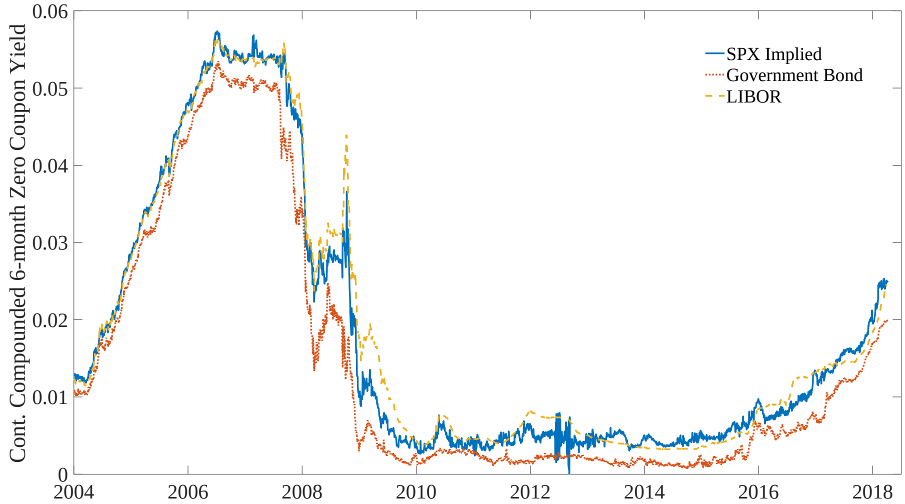

FIGURE IV

Comparison of 6-month zero coupon interest rates implied from SPX options with government bond rates and LIBOR rates. All rates are continuously compounded.

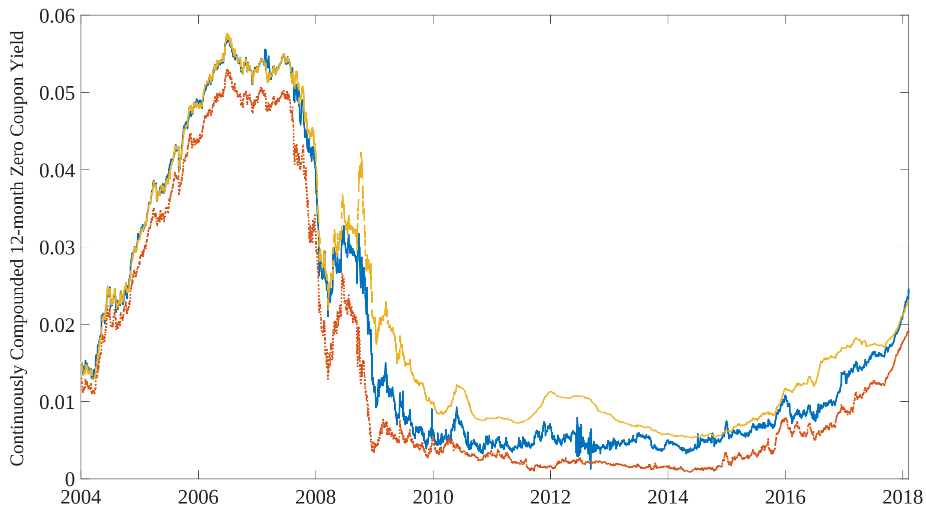

FIGURE V

Comparison of 12-month zero coupon interest rates implied from SPX options with government bond rates and LIBOR rates. All rates are continuously compounded.

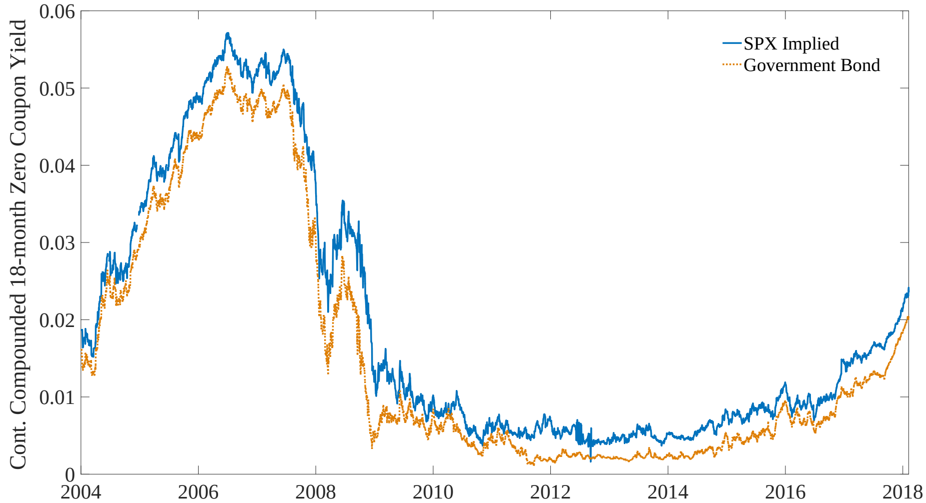

FIGURE VI

Comparison of 18-month zero coupon interest rates implied from SPX options with government bond rates and LIBOR rates. All rates are continuously compounded.

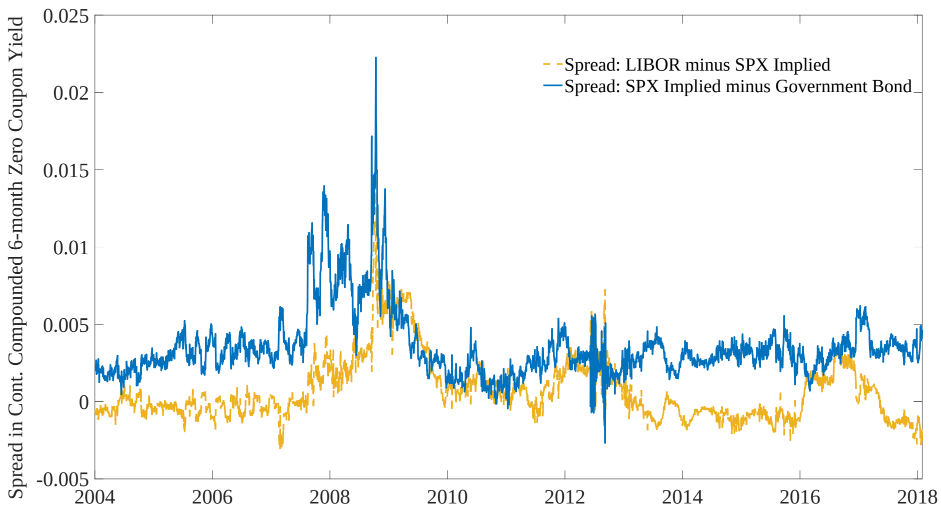

FIGURE VII

Spreads of 6-month zero coupon interest rates implied from SPX options with government bond rates and LIBOR rates. All rates are continuously compounded.

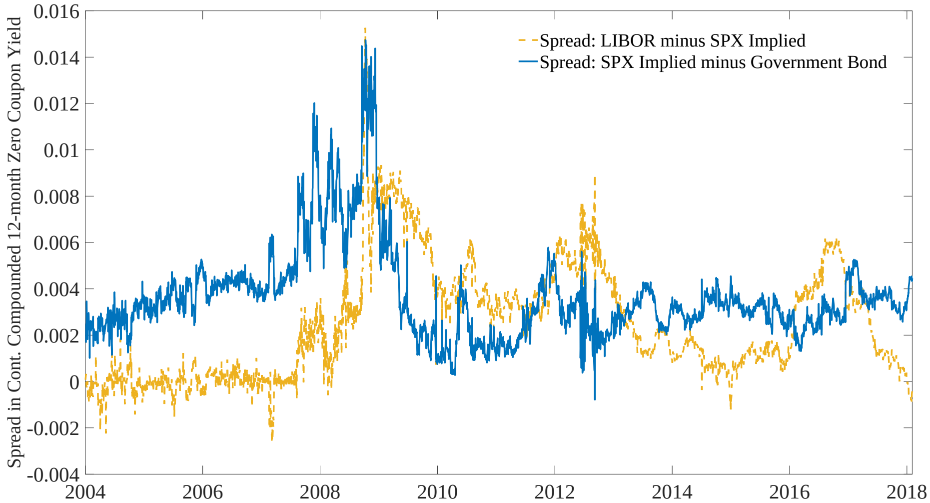

FIGURE VIII

Spreads of 12-month zero coupon interest rates implied from SPX options with government bond rates and LIBOR rates. All rates are continuously compounded.

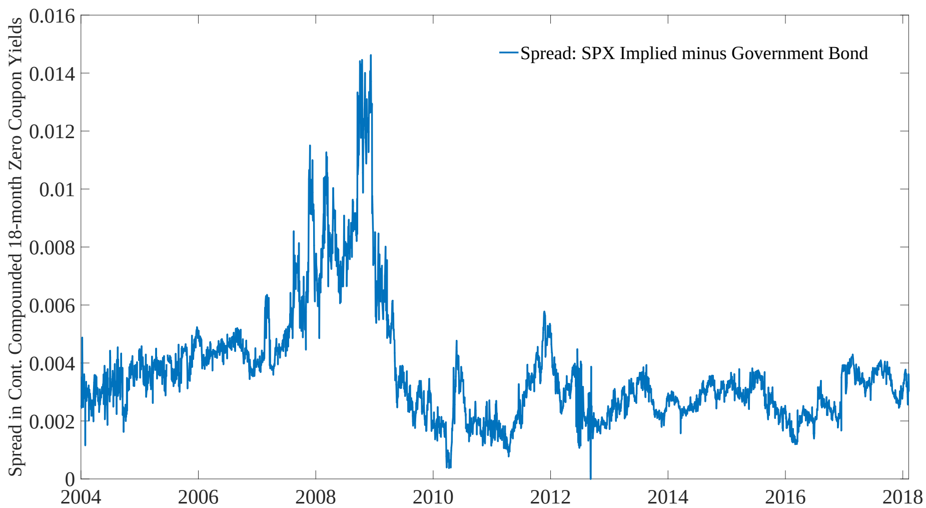

FIGURE IX

Spreads of 18-month zero coupon interest rates implied from SPX options with government bond rates and LIBOR rates. All rates are continuously compounded.

Note that for all three maturities, both spreads exhibit large variation, and they both go up during the crisis and (just as with the results for government bond arbitrages in the previous section) have since been reduced to levels closer to zero.

Finally, if no-arbitrage conditions hold perfectly, the R-squared of the regression in equation 2 equals 1. As such, this measure of fit can be interpreted as a measure of efficiency within the market for options for this particular underlying asset. Because the slope of the regression is so close to 1, we can also easily map this measure of market efficiency to variation in estimated (non-annualized) rates ($Tr_{t,T}$) across the strikes. To see this note that the population R-squared of the regression in equation 2 is given by:

$$
\begin{aligned}
R^2 &= \frac{\text{var}(\beta K_i)}{\text{var}(\beta K_i) + \text{var}(\varepsilon)} && (11)\\
&= \frac{\beta^2 \text{var}(K_i)}{\beta^2 \text{var}(K_i) + \text{var}(\varepsilon)} && (12)\\
&= \frac{1}{1 + \frac{\text{var}(\varepsilon)}{\beta^2 \text{var}(K_i)}} && (13)\\
& && (14)
\end{aligned}
$$

Rewriting this equation, we find:

$$
\frac{1}{R^2} - 1 = \frac{\text{var}(\varepsilon)}{\beta^2 \text{var}(K_i)} \approx \frac{\text{var}(\varepsilon)}{\text{var}(K_i)}. \quad (15)
$$

Assuming uncorrelated error terms, the asymptotic variance of the univariate OLS estimator equals the variance of the error term scaled by N times the variance of the right-hand side variable, that is, the variance across the strike prices. This then implies that the variance of the OLS estimated interest rates can be approximated by (using the approximation that β is close to 1 and that the log-linearized regression coefficient uncovers the interest rate):

$$
\sigma(\hat{r}_{t,T}) \equiv \sigma\left(\frac{1}{T}\ln(\beta_{OLS})\right) \approx \sqrt{\frac{1}{NT^2}(\frac{1}{R^2} - 1)} \quad (16)
$$

As a consequence, for a regression for maturity $T = 1$, with $N = 20$ strike prices and an R-squared of 0.999999, the standard error of the estimate at each time t (i.e. each minute) is in the order of magnitude of 2 basis points. For 100 strikes, this number is 1 basis point. Given that our daily estimates are computed by taking a median over the minutely observations, the standard error of the daily estimate is even smaller than that. As an illustration, we plot in Figure X a daily series of the standard error of the minute-level risk free zero coupon yield estimate for the 18 month maturity. We use the actual standard error implied by the regression, which is approximately equal to the non-linear transform of the R-squared as explained in 16, and as such can be interpreted as a measure of market efficiency. To arrive at a daily series for this minute level standard deviation, we take the median standard error across all minutes within a day. The graph shows that the typical standard error is in the

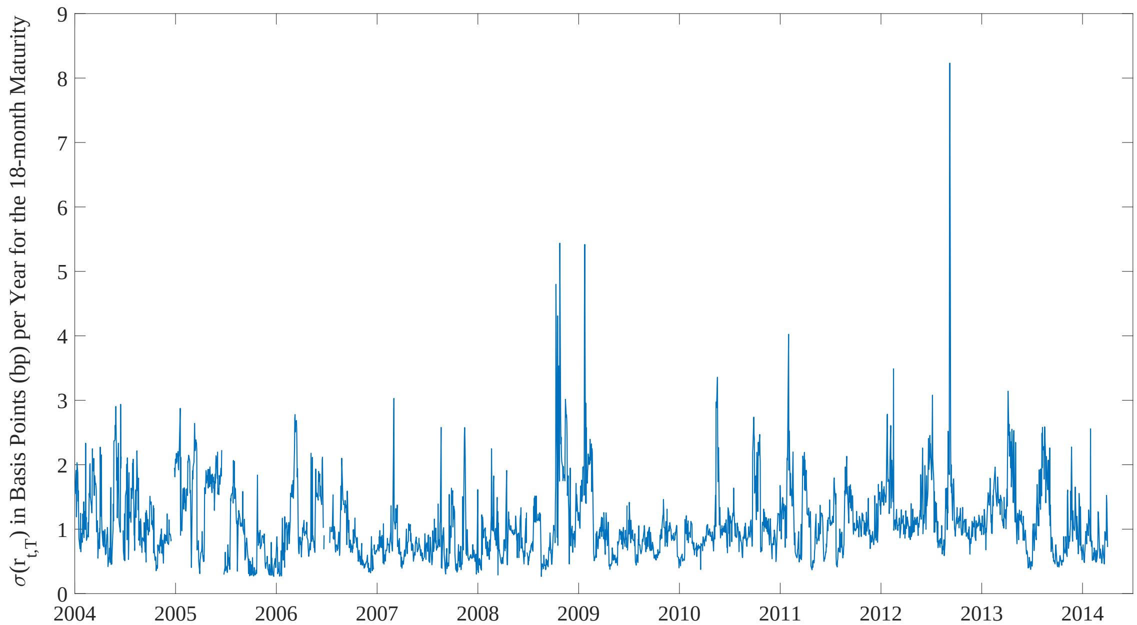

FIGURE X

Efficiency in the SPX option market expressed as the standard error around the implied risk-free rate.

order of magnitude of 1 basis point, but it can occasionally spike. The maximum over our sample period is 8 basis points.

3.3.2. Results Dow Jones Industrial Index (DJX)

Next, we repeat the analysis that we did for the S&P 500 index for options on the Dow Jones industrial index (DJX). In Table 3 we summarize the results for the median estimator (Estimator 2). The regression-based estimator gives highly comparable results. As for the SPX, we find that the implied interest rate is on average higher than the government bond yield by about 40 basis points, which is invariant to maturity. Also, it is generally lower than the Libor rate.

Table 3
Summary Statistics of DJX Option Implied Interest Rates 2004-2018

Zero Coupon Yields: 6 month maturity			
	Mean	St. Dev.	AR(1) (daily)
Option Implied DJX	0.0184	0.0171	0.9906
LIBOR Implied	0.0186	0.0171	0.9999
Government Bond	0.0144	0.0166	0.9998
LIBOR Implied - Option Implied DJX	0.0002	0.0029	0.6816
Option Implied DJX - Government Bond	0.0040	0.0029	0.6756

Zero Coupon Yields: 12 month maturity			
	Mean	St. Dev.	AR(1) (daily)
Option Implied DJX	0.0190	0.0168	0.9961
LIBOR Implied	0.0211	0.0159	0.9998
Government Bond	0.0150	0.0163	0.9997
LIBOR Implied - Option Implied DJX	0.0021	0.0028	0.8623
Option Implied DJX - Government Bond	0.0040	0.0023	0.7787

Zero Coupon Yields: 18 month maturity			
	Mean	St. Dev.	AR(1) (daily)
Option Implied DJX	0.0197	0.0164	0.9982
Government Bond	0.0159	0.0158	0.9996
Option Implied DJX - Government Bond	0.0039	0.0021	0.8875

Given how comparable the results for the DJX are to the SPX we only plot the implied continuously compounded interest rate the 1-year maturity as an illustration in Figure XI. The graphs very much the same pattern, though the implied rates are somewhat noisier than the ones implied by the SPX. Next we repeat the efficiency analysis of Figure X but

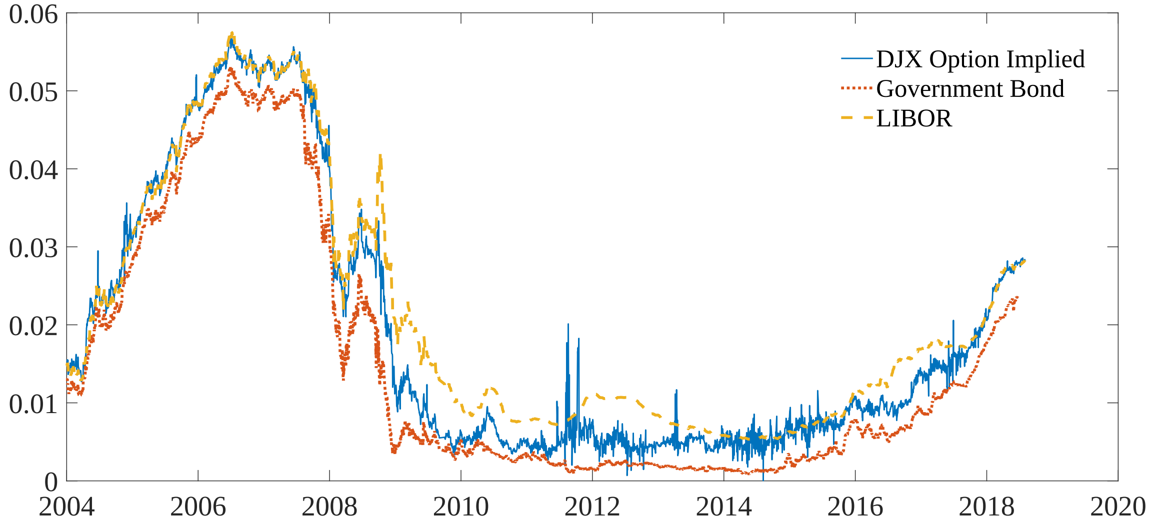

FIGURE XI

Comparison of 1-year zero coupon interest rates implied from DJX options with government bond rates and LIBOR rates. All rates are continuously compounded.

now for DJX. The results are summarized in Figure XII where we plot the standard error of the OLS estimate of equatio 2. The results are comparable to the SPX though the average level of efficiency is substantially lower, with an average standard error of the minutely level estimated rate equal to 3.4 basis points, and spikes that occasionally go as high as 38 basis points. Because our daily estimates are computed by taking a median over all the minute-level observations, those estimates have smaller standard errors.

Finally, we study how the interest rates implied by the DJX differ from those implied by the SPX. For each maturity, we compute a difference between the DJX and the SPX rate and we report the characteristics of that series in the table below.

Table 4
Summary Statistics of DJX Option Implied Interest Rates 2004-2018

Maturity	6-month	12-month	18-month
Mean	0.00046	0.00023	0.00021
Stdev	0.00224	0.00121	0.00103
AR(1) (daily)	0.4302	0.49587	0.5710

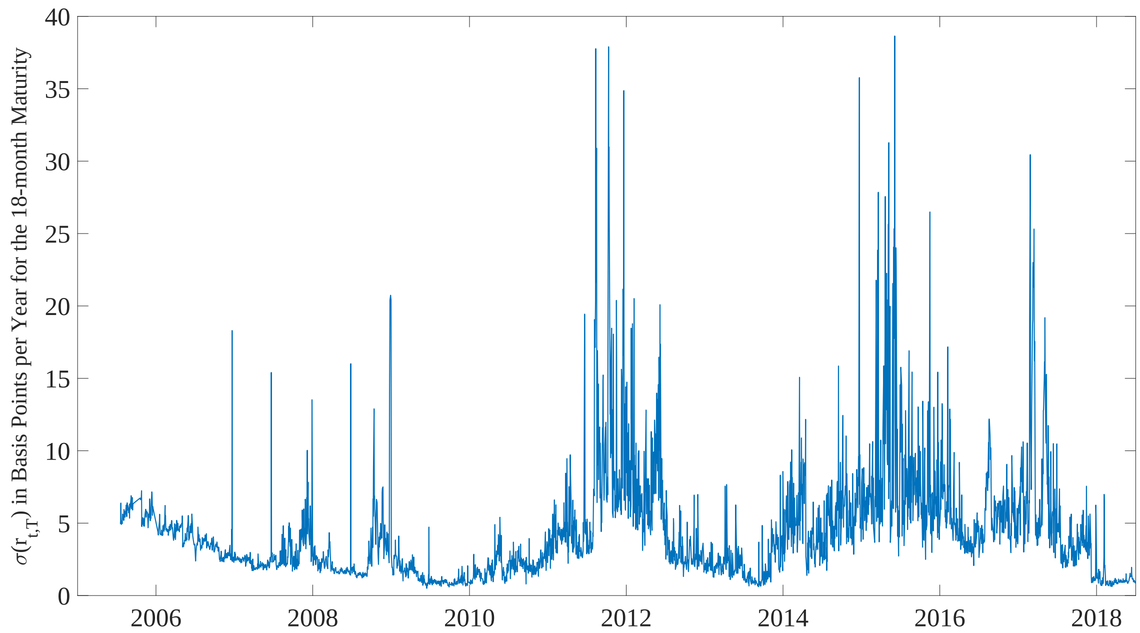

FIGURE XII

Efficiency in the DJX option market expressed as the standard error around the implied risk-free rate.

The table shows that while on average the rates are very close, substantial persistent daily deviations occur. As an illustration, Figure XIII plots the differences between the two yields for the 12 month maturity.

4. Exchange Rate Markets

4.1. The No-Arbitrage Relation

We now test the no-arbitrage hypothesis in the FX markets. To construct a risk-free strategy for this market, we use the well-known covered interest parity relationship. We compare for a US-based agent at time t two alternative strategies. The first alternative is to invest in a riskless asset denominated in dollars with a time to maturity T. The other is to exchange money into foreign currency, invest into the riskless asset denominated in that

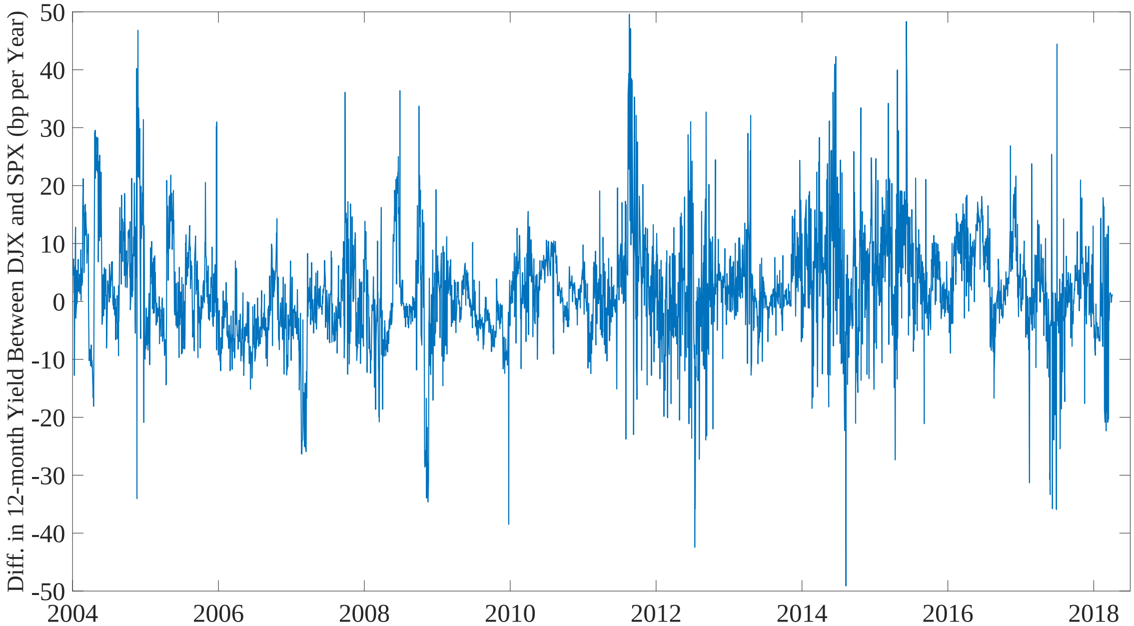

FIGURE XIII

Difference in Daily Continuously Compounded Implied 12-month Zero Coupon Yield Between the DJX and the SPX in Basis Points per Year.

foreign currency (with the same time to maturity T) and buy a promise to exchange the money back into dollars at a predetermined rate at T. Covered Interest Parity (CIP) is a no-arbitrage relationship that states that both strategies should earn the same return. More formally, denote by $r_{t,T}$ and $r^*_{t,T}$ the continuously-compounded riskless interest rate at time t which matures at date T in dollars and foreign currency. CIP predicts that

$$e^{Tr_{t,T}} = e^{Tr^*_{t,T}} \frac{S_t}{F_{t,T}}, \tag{17}$$

where S_t is the time-t spot exchange rate between dollars and foreign currency and $F_{t,T}$ the forward rate of exchange, set at time t with maturity in T years.

Following Du, Im, and Schreger (2018), we construct the *cross-currency basis*, in logs, as

$$x_{t,T} = r_{t,T} - r^*_{t,T} - \frac{1}{T}\ln(S_t/F_{t,T}) \tag{18}$$

$\frac{1}{T}\ln(S_t/F_{t,T})$ is the continuously-compounded “forward premium”.

4.2. Data

Our data set is made of all futures trades made between January 2010 and January 2018 on the Chicago Mercantile Exchange (CME) regarding six major currency pairs: British pound (GBP), Euro (EUR), Canadian dollar (CAD), Japanese yen (JPY), Swiss franc (CHF), and New Zealand dollar (NZD). Every day, more than $100 billion are traded over the CME FX markets, which makes the CME the world’s largest regulated marketplace for foreign currency trading.[5] Daily spot exchange-rate quotes are from the TrueFX dataset, which offers historical tick-by-tick market data for dealable interbank foreign exchange rates for each millisecond. For spot exchange rate quotes, we take the mid-point between the bid and the ask rates.

We exclude futures with less than 30 days to maturity. We match daily and forward rates at the closest millisecond and keep only trades which happen in a range of 3 seconds one from the other. We construct the forward premium, i.e. $\frac{1}{T}\ln(S_t/F_{t,T})$, and take a daily median of it.

We then merge the forward premia with daily curves of interest rates in different countries. For the US, we use the parameters estimated by Jonathan Wright for the Nielson-Siegel-Svensson (NSS) model fitted on US government-bond yields. Similarly, for the Euro area, we use the NSS parameters estimated by the Bundesbank on German bonds. Those parameters characterize the entire yield curve for all maturities.

With respect to the other countries, data on constant-maturity zero-coupon yield curves are downloaded from local central banks’ websites, except from Japan whose rates are from Bloomberg.We then linearly interpolate constant-maturity zero-coupon yields to match the maturity of the forward contract.

[5]More information can be found at https://www.cmegroup.com/trading/fx/why-trade-fx-futures-and-options.html

4.3. Results

On average the *cross-currency basis* with respect to the US dollar is close to zero. Table 5 reports key summary statistics for the annualized cross-currency basis of the 6 currencies mentioned above. Cross-country heterogeneity is visible. Canadian dollar, Euro, and British pound have an average CIP deviation between 0.7 and 2.1 basis points. Their volatility is also very low. On the other hand, Japanese yen, Swiss Franc, and New Zealand dollar have a cross-currency basis which is both larger on average (from 10 to 19 basis points) and more volatile.

Table 5
Summary Statistics – Covered Interest Parity

Statistic	Mean	St. Dev.	25th Percentile	75th Percentile	Autocorrelation (1)
GBP	0.68	12.98	−6.82	10.29	0.94
EUR	−1.58	14.63	−10.76	7.61	0.90
CAD	2.14	11.05	−4.30	9.68	0.90
JPY	−15.68	17.64	−24.34	−4.20	0.93
CHF	−18.64	25.27	−31.07	−2.16	0.89
NZD	−10.36	12.33	−17.87	−3.02	0.72

Notes: This table reports key summary statistics for the annualized cross-currency basis, measured in basis points (1 hundreth of a percentage point), for the 6 major currencies. According to Covered Interest Parity, the cross-currency basis must be zero. We construct the *cross-currency basis*, in logs, as

$$x_{t,T} = r_{t,T} - r^*_{t,T} - \frac{1}{T}\ln(S_t/F_{t,T})$$

using high-frequency futures data from the CME between 2010 and 2018.

Figure XIV plots the cross-country mean of the CIP deviation for the period post financial crisis. The series exhibit a strong autocorrelation, as also evidenced by the statistics in Table 5.

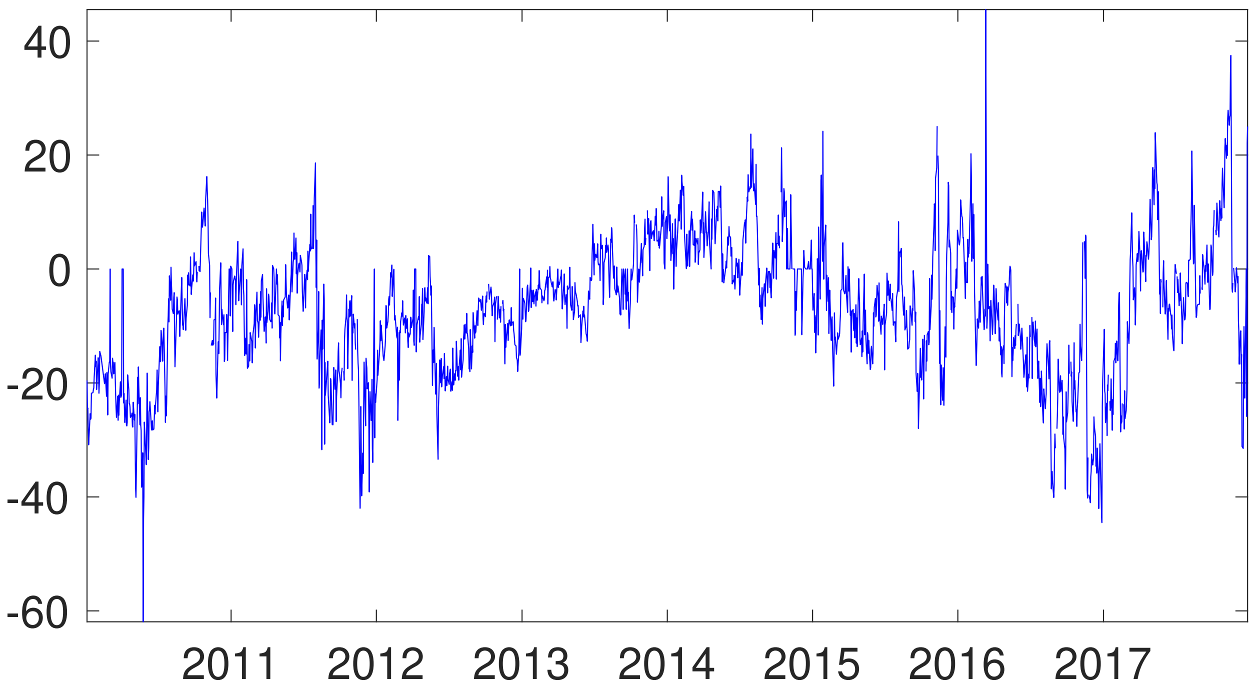

FIGURE XIV

Deviation from Covered Interest Rate Parity. The figure plots the time series for the annualized cross-currency basis, measured in basis points (1 hundreth of a percentage point), for six major currencies. The *cross-currency basis*, in logs, is $x_{t,T} = r_{t,T} - r^*_{t,T} - \frac{1}{T}\ln(S_t/F_{t,T})$.

5. Commodity Markets

5.1. Constructing Risk Free Assets

To construct a risk free asset in commodity markets we use the no-arbitrage relationship between the futures price and the spot price ($F_{t,T}$ and the spot price S_t):

$$F_{t,T} = S_t \exp\left((r_{t,T} + c_{t,T})T\right). \tag{19}$$

where $r_{t,T}$ is the continuously compounded risk free interest rate and $c_{t,T}$ is the net storage cost of the commodity. To derive estimates of the risk free interest rate, we focus on futures contracts on underlying assets that are very cheap to store relative to their underlying value, implying that the term $c_{t,T}$ is essentially zero. As such we focus on precious metals: gold, silver and platinum. The risk-free rate is then computed as:

$$r_{t,T} = \frac{1}{T} \ln \left(\frac{F_t}{S_t} \right). \tag{20}$$

5.2. Data

Our data set contains all futures trades made between May 2007 and January 2018 on the Chicago Mercantile Exchange (CME) regarding three precious metals: gold, silver and platinum. Unlike the CBOE data, which also contains quotes, the database we purchased only contains trades. In a future draft of this paper we intend to extend this evidence to quotes as well.

5.3. Results

In Table 6 we summarize the key statistics for gold and silver implied interest rates. We compare these rates to the government bond yields as implied by the NSS parameters estimated by Jonathan Wright. The table shows that the estimated convenience yield for

Table 6
Risk-free rates and convenience yields implied by precious metal prices

Zero Coupon Yield Curve	Gold		Silver	
	Mean	St. Dev	Mean	St. Dev
Metal implied 6m	0.0118	0.0123	0.0133	0.0174
Metal implied 12m	0.0120	0.0117	0.0116	0.0126
Metal implied 18m	0.0127	0.0112	0.0116	0.0124
Metal Implied - Gov. Bond 6m	0.0043	0.0035	0.0054	0.0117
Metal Implied - Gov. Bond 12m	0.0040	0.0027	0.0036	0.0049
Metal Implied - Gov. Bond 18m	0.0037	0.0027	0.0024	0.0050

government bonds relative to metal-implied interest rates is the same as for our previous estimates and equal to about 40 basis points for gold with no apparent relation to maturity.

For silver the order of magnitude is the same, but there now seems to be a maturity dependence of the estimate, with the convenience yield decreasing with maturity. For platinum, the data is not sufficiently rich to obtain (interpolated) term structure data. However, the average convenience yield across all available maturities is 50 basis points. The volatility of the daily estimates is large, partly due to the fact that we only have trade data and not quote data.

Finally, as an illustration of the comovement, we plot in figure XV the implied 12 month gold rate and compare it to corresponding government bond yield (other maturities follow a similar pattern). To reduce estimation noise induced by using trades instead of quotes, we plot the monthly median rate for both series. The plot confirms that the spread between the two rates is positive, and that this spread varies substantially over time, with movements that are not shared by several of the other markets.

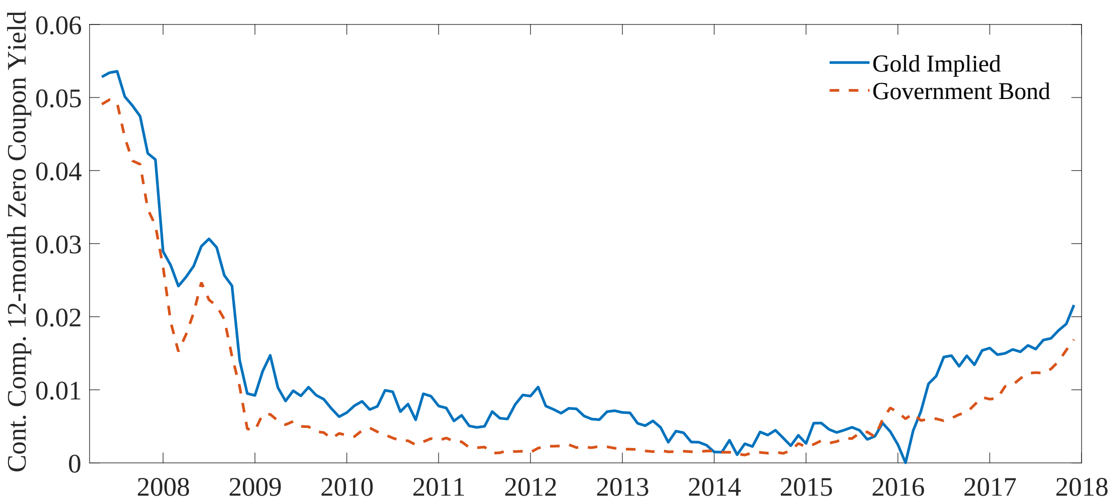

FIGURE XV

Efficiency in the DJX option market expressed as the standard error around the implied risk-free rate.

6. A Multivariate Analysis and an Aggregate Index

This section presents a multivariate analysis of multiple arbitrage spreads analyzed. This allows us to make progress on several questions that previous studies focusing on individual arbitrages cannot answer. First, we are able to show a relatively strong common component in the size of all arbitrage opportunities by principal component analysis (pca). The first principal component of the arbitrages we study (plotted in Figure XVI explains 33.62% of the variance in the cross-section of spreads we consider. We interpret this first principal component as a summary of the overall degree of mispricing in financial markets and can therefore be used as an aggregate arbitrage index. Second, we estimate a first order Vector Auto Regression (VAR) to study how each spread evolves after shocks to other spreads occur. We find that the SPX implied rate is our cleanest, least noisy measure of the risk-free rate of return, since all of our other arbitrage spreads seem to feature greater degrees of idiosyncratic risk and volatility.

One key finding in our analysis is that our overall measure of mispricing increases greatly during the U.S. financial crisis and to a lesser degree during the European financial crisis. This reflects the fact that all of the spreads we consider are large during the U.S. crisis while only the option implied rates we consider seem to be seriously exposed to the European crisis. This is reflected in the loadings we report below of each individual series on our principal components. The SPX and DJX implied rates load more heavily on the first principal component than other series, and our higher principal components seem to be exposed to the U.S. crisis but not the European crisis. One interpretation of this finding is that European banks play a more important role in derivatives markets than they do in the market for other asset classes, perhaps due to less stringent capital requirements on derivatives trades (consistent with findings in Du, Tepper, and Verdelhan (2018)).

The VAR analysis also shows that the option implied series are subject to considerably less idiosyncratic risk than others, while the metal convenience yields are subject to more.[6] As a result, if we are interested in using data on risky asset prices to infer the risk-free rate consistent with investors' time preference, the option implied rates seem to be an ideal

[6]This is also due to the fact that the convenience yields on precious metals are estimated with trade data only leading to noisier estimates.

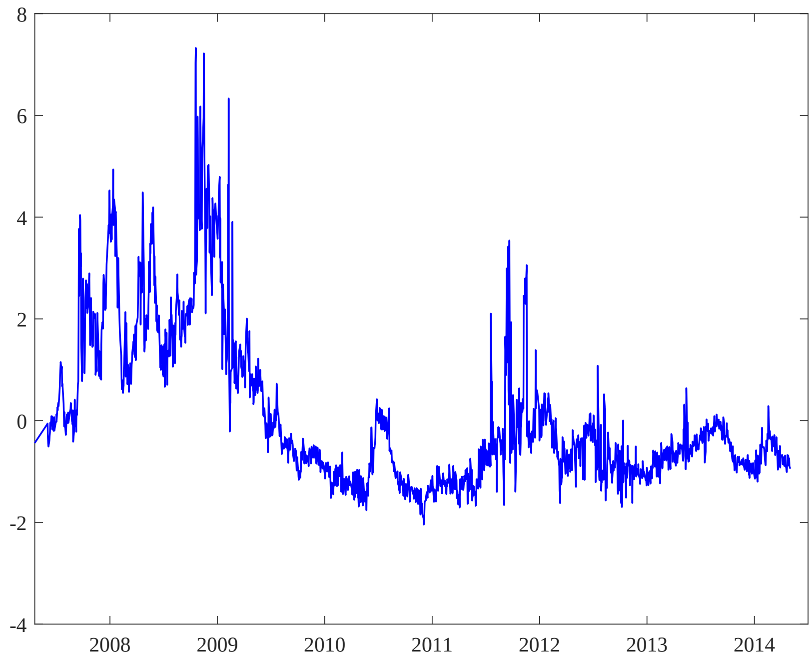

FIGURE XVI

Aggregate Arbitrage Index. We compute the first principal component across all the arbitrage spreads we have computed across the asset classes including the bond arbitrages, the option arbitrages and commodities.

Table 7

VAR(1) Analysis of Arbitrage Spreads

	djx	spx	lessthan6	metal	notesbonds	ontherun
djx	0.6798	0.0634	0.0032	-0.0585	0.0001	-0.0002
spx	0.2662	0.9103	-0.0400	0.4142	-0.0002	0.0001
lessthan6	-0.0066	-0.0323	0.5319	1.4711	-0.0018	-0.0015
metal	0.0005	-0.0002	0.0031	0.0223	0.0000	0.0000
notesbonds	1.6658	0.4304	0.2773	4.7072	0.7044	-0.0094
ontherun	0.0671	0.1230	-0.4992	-8.0279	-0.0037	0.4580
constant	2.7326	0.6319	-1.8320	41.4707	0.0078	-0.0281
R$\hat{2}$	0.8332	0.9403	0.3494	0.0126	0.5004	0.2147

candidate. The R-squareds of predicting the in-sample SPX implied rate is extremely high, while that for the DJX is quite high as well. Government bond arbitrages have intermediate

R-squareds, while the metal series have by far the lowest. This implies that there does not seem to be a high degree of unpredictable, non-persistent noise in the SPX implied rates. If we tend to think that investors have a relatively stable time preference that does not fluctuate daily, then this series seems to be a very good empirical proxy for it. Another important finding in the vector autoregression is that the option implied rates seem to predict each other, while many series seem to respond to the notes/bonds spread. This seems to be due to the fact that the notes/bond spread spikes dramatically during the U.S. financial crisis but is otherwise very small and that the Box rates are responsive to both the U.S. and European financial crises. The other series we consider seem to have more idiosyncratic variation in their spreads, suggesting that these markets may be relatively more segmented from overall financial conditions.

7. Conclusion

We have constructed and analyzed a large panel of different riskless rates of return. The rich heterogeneity in risk-free rates that we document are inconsistent with the standard no arbitrage assumption that underlies most asset pricing models. While puzzling features of rates of return on risky assets may be explained away by (often ad hoc) additional assumptions on the compensation for risk, no such explanation can be applied to the risk free returns that we study.

Because our paper provides an accurate high-frequency measure of the convenience yield on U.S. government debt, we are able to study its dynamics and determinants more precisely than previous work. In particular, we show that this convenience yield spikes during the Lehman bankruptcy event as well as the Euro crisis, suggesting that in addition to the level of nominal rates and bond supply, the health of the intermediary sector is an important determinant of this convenience yield. While all of our arbitrage spreads are large during the U.S. financial crisis, some but not all spike during the European crisis, consistent with the view that European banks are the marginal investor in a subset of convergence trades.

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