Wage bargaining with direct competition and heterogeneous access to vacancies[1]

Adalbert Mayer

Department of Economics

Washington College

Chestertown MD 21620

United States

Theodore L. Turocy[2]

School of Economics and CBESS

University of East Anglia

Norwich NR4 7TJ

United Kingdom

December 16, 2013

[2]Corresponding author: T.Turocy@uea.ac.uk

Abstract

Agents with a richer set of opportunities to trade should be able to demand better terms of trade. For instance, workers who are otherwise equally-qualified may differ in their access to vacancies, e.g. because their social networks are larger or smaller. We present a model of search and matching in which multiple workers may be matched to the same vacancy, and workers compete directly in the wage bargining process. Workers with greater access have a higher dynamic outside option and demand higher wages. They are therefore unsuccessful candidates in some matches; this latter outcome is not possible in existing models based on Nash bargaining to determine wages. In particular, when markets are tight and the expected length of a position is short, workers with better access to opportunities will remain unemployed longer than those with less access.

Keywords: matching, wage dispersion, labor markets, social networks.

JEL Classifications: C78, J31, J64.

1   Introduction

It has been widely recognized that information transmission, for example through social connections, plays an important role in many labor markets. Rees (1966) is an early articulation of this idea, which has been developed subsequently both theoretically and empirically; see the surveys by Ioannides and Loury (2004) and Topa (2011) for summaries. One role such information transmission plays is to help alleviate search frictions. Agents in a large market may face the problem of discovery of opportunities to trade with other agents. Modeling this search and matching process is part of the standard toolbox of economic theory; Rogerson et al. (2005) survey these types of models in labor market settings.

Within these frameworks, the standard in the literature is that, conditional on a match occurring, Nash bargaining is used to determine the split of the surplus between firm and worker. Models featuring Nash bargaining include those of Calvó-Armengol and Zenou (2005), Ioannides and Soetevent (2006), Fontaine (2008), and Galenianos (2011). The use of Nash bargaining as the negotiation technology implies that the negotiation never breaks down; each match of a worker to a firm results in the worker filling the vacancy. In such a model, if workers differ in the frequency in which they are matched, it follows immediately that workers who are matched more frequently (e.g., those with better informational networks) necessarily experience shorter unemployment stints. In addition, Nash bargaining makes a strong informational assumption, that both players know the outside option available to the other player.

We investigate the implications of using a different matching and negotiation stage game within the search framework. We construct a model in which, in each period, an unemployed worker is matched at random with at most one vacant position, but multiple workers may be matched with the same vacant position. Bargaining is therefore multilateral, and workers compete for the vacancy directly, via a process modeled as an auction game.[1] As a result, being a candidate for a vacancy does not imply the worker will be successful in obtaining a job in that period. In our model, firms do not need to know anything about the private characteristic of the worker, and workers need only to have correct beliefs about the distribution of types of workers in the market, not specific information about the workers they might be competing with for a given current vacancy.

We show our model has a unique steady-state equilibrium. Workers with better access to information about vacancies indeed demand higher wages. However, this heterogeneity among workers does not feed through to greater variance in wages among employed workers in equilibrium.

A novel implication of our model is that, when the market is tight and positions have short durations, workers with better access may remain unemployed longer on average than those with

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[1]The matching and bargaining approach we take also appears in the price formation model of Satterthwaite and Shneyerov (2007).

less access. This prediction distinguishes our model from those based on Nash bargaining. Examples of markets where these conditions might plausibly obtain are in contract work for specialist services, such as website design, architecture, actors for movies or plays, and so on. Many of these jobs are focused on a specific project, with an expected duration measured in months rather than years. While the price formation literature sometimes uses an auction as a metaphor for direct price competition (Kultti, 2000; Satterthwaite and Shneyerov, 2007), in these contract work settings, employment is sometimes secured in a literal auction, with the lowest bidder winning the contract. Therefore, our auction-based model differs from the Nash bargaining model precisely in the settings in which the use of the auction device as a modelling tool is most compelling.

In our model, workers are distinguished by a one-dimensional parameter which determines the likelihood of hearing about a vacancy in a period. The model therefore abstracts away from details about how, for instance, a social network might operate procedurally in passing information. Models with explicit representations of networks are employed by Calvó-Armengol (2004) to characterize equilibrium in a game of strategic network link formation, by Calvó-Armengol and Zenou (2005) to derive an aggregate matching function and labor market equilibrium, and by Calvó-Armengol and Jackson (2004, 2007) to show that differential initial conditions in subpopulations can lead to persistent differences in unemployment levels. Our assumptions on the matching technology are sufficiently flexible to accommodate the operation of these more explicitly procedural formulations of information transmission.

The paper is organized as follows. Section 2 presents the model, including details of the matching and wage negotiation technologies. Section 3 shows that a steady-state equilibrium exists and is unique in our setting, and provides equations characterizing the steady-state distributions of unemployed workers and the equilibrium wage demand functions. Section 4 investigates the comparative statics of the model, and draws out the conditions under which the model makes distinct predictions from Nash bargaining approaches. Section 5 concludes by placing the results in the broader theoretical and empirical literatures. To facilitate the flow of the exposition, we defer the details of proofs to Appendix A.

2   The model

Parameters.   The model operates in discrete time $t = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$. There is a continuum of workers with measure W. There is also a continuum of firms, each with demand to hire up to one worker; the measure of firms is J. In each period t, each worker is either employed or unemployed, and each job is either filled or vacant; the pairing of employed workers with filled jobs is one-to-one. A worker is paid a per-period wage by the firm for each period the firm employs the worker. The wage is set at the time the worker is hired, and does not change during the duration

of the match. The processes by which workers and firms are matched, wages are determined, and matches are terminated will be described later.

All firms and jobs are identical, and all workers are equally productive, with no special firm-worker synergies. Without loss of generality, we normalize the maximum per-period wage a firm is willing to pay a worker to be unity. Workers are distinguished only by a type parameter ν, which indexes the chance that the worker, when unemployed, hears about a vacancy. We refer to ν as the worker's access to vacancies. One interpretation of ν consistent with its operation in our model is an index of the size of the worker's social or informational network, similar to Galenianos (2011). The model is sufficiently abstract that ν can also encompass other worker-specific characteristics that influence the frequency with which a worker becomes a candidate for a vacancy.

Types ν are distributed among the population of workers according to a distribution function G, with a corresponding density g satisfying $g(\nu) > \gamma > 0$. The support of g is $[\nu_L, \nu_H]$, with $0 < \nu_L < \nu_H \leq 1$. They maximize the expected discounted present value of their future stream of wages, with per-period discount factor $\delta \in (0, 1)$. We normalize the utility of spending a period in unemployment to zero.

Notation. We will characterize the steady-state distribution of types among employed and unemployed workers, respectively, under the assumption that workers adopt stationary wage bidding strategies. Write g_U for the density of unemployed workers, and g_E for the density of employed workers. Of necessity, $g_U(\nu) + g_E(\nu) = g(\nu)$ for all $\nu \in [\nu_L, \nu_H]$. The mass of employed workers is $E = \int_{\nu_L}^{\nu_H} g_E(\nu) d\nu$ and the mass of unemployed workers is $U = \int_{\nu_L}^{\nu_H} g_U(\nu) d\nu$. The mass of vacancies is denoted $V \equiv J - E$.

Matching technology. Each discrete period proceeds as follows. At the beginning of each period, each unemployed worker may hear about up to one vacancy.[2] We refer to an unemployed worker who learns about a vacancy as a candidate. The probability an unemployed worker becomes a candidate is given by the function $\alpha(\nu; V, E)$. This function is assumed to be continuously differentiable in its parameters. For notational compactness, in places we will suppress the dependency on V and E, and where we write $\alpha'(\nu)$, we refer to the (partial) derivative with respect to ν.

Assumption 1. *The probability a worker learns about some vacancy, $\alpha(\nu; V, E)$, satisfies the following:*

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[2]We imagine the temporal duration of a period to be short; therefore, if the arrival of news of vacancies follows a Poisson process, in a sufficiently short time period, we can neglect the possibility of one worker hearing about two or more vacancies.

1. $\frac{\partial \alpha(\nu;V,E)}{\partial \nu} > 0$: *Larger values of the access parameter result in increasing chances of becoming a candidate;*

2. $\frac{\partial \alpha(\nu;V,E)}{\partial V} \geq 0$: *The chance of becoming a candidate does not decrease when there are more vacancies;*

3. $\frac{\partial}{\partial V}\left(\frac{\alpha(\nu;V,E)}{V}\right) \leq 0$: *a small increase in the number of vacancies does not increase the chance of becoming a candidate disproportionately;*

4. $\frac{\partial}{\partial V}\left(\frac{\alpha'(\nu;V,E)}{\alpha(\nu;V,E)}\right) \geq 0$: *an increase in the number of vacancies does not decrease the marginal effectiveness of access in percentage terms.*

5. $\frac{\partial \alpha(\nu;V,E)}{\partial E} \geq 0$: *The chance of becoming a candidate does not decrease when there are more employed workers.*

The main intuition of our results does not depend on the details of the mechanism of how a worker is matched with a vacancy. In particular, the last assumption allows us to consider mechanisms in which a worker learns about a vacancy in part because another worker in their social network, who is already employed, learns about a vacancy and passes on news of the vacancy.

Assumptions 1, 2, and 5 are natural assumptions regarding the process of learning about vacancies. Assumptions 3 and 4 are primarily technical, and are sufficient to ensure existence and uniqueness of a steady-state equilibrium in the model.

Each worker's random chance of becoming a candidate is realized independently across workers and across periods, and a worker is equally likely to hear about any of the current vacancies. Therefore, for any ν, the measure of workers with type less than ν who become candidates is

$$C(\nu) = \int_{\nu_L}^{\nu} \kappa g_U(\kappa)d\kappa.$$

It follows that $C(\nu_H)$ is the total measure of candidates in a period.

Random matching of candidates to vacancies implies that the number of candidates matched to one specific vacancy is distributed Poisson with mean $\frac{C(\nu_H)}{V}$. Furthermore, a consequence of the Poisson property is that the fact a given candidate is matched to a given vacancy is independent of the number of other candidates also matched to the same vacancy. This implies that the number of candidates with type less than ν who are matched to a given vacancy is also distributed Poisson, with mean $\frac{C(\nu)}{V}$.

Hiring and wage determination. A key characteristic of the model is that there is incomplete information. A firm with a vacancy does not know the types of the candidates it receives in a

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given period. Candidates matched with a vacancy do not know how many other candidates also are competing for the same vacancy, nor what the types of those candidates are. We do assume that in the steady state, a candidate does have correct beliefs about the distribution of how many other candidates will be drawn for the same vacancy by the matching process, and about the distribution of the types of those candidates.

Bargaining between firm and candidates takes the form of a first-price auction.[3] The bids in the auction specify the per-period wage at which the candidate is willing to accept the position. We write the worker's wage demand as a function of their type as $w(\nu)$. Because workers are identical except for ν, and the candidate's type does not affect the value of his labor to the firm, the firm will hire the candidate who submits the lowest per-period wage bid, subject to the wage bid being lower than unity, the firm's maximum willingness to pay. The successful candidate is paid the wage he bids each period for the duration of his employment in the position, starting in the subsequent period. Unsuccessful candidates remain unemployed for the period. In the event of a tie between two or more candidates for the lowest wage bid, one of the tied candidates is chosen at random to get the job.[4]

In parallel to the hiring process, each existing match between a worker and a firm is terminated with probability λ. Separation is realized independently in each period and for each match, and does not depend on the length of the match. Termination occurs effective at the end of the period, so workers receive their contracted wage during the final period of the match. Terminated workers enter the pool of unemployed workers for the next period $t + 1$, and the jobs become available for matching in period $t + 1$.[5]

3 Steady-state equilibrium

3.1 Wage demands are monotonic

Competition among bidders takes place as a first-price auction. We will show that the degree of access parameter ν functions analogously to a private value, sorting agents monotonically by their dynamic opportunity cost of foregoing a position to which they are currently matched. In doing so, we can convert the wage demand bid choice to a static optimization problem.

Let $\rho(w)$ be the probability that a worker who makes wage demand w gets a job, conditional on having been matched with a vacancy.

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[3]Kultti (2000) also analyzes a model of price formation in which agents do know how many others are matched with the same potential trading partner. In that setting, if the match is one-to-one, bargaining occurs, with an auction occurring only if the match is many-to-one.

[4]In equilibrium, ties are a zero-probability event.

[5]Timing matches and dissolutions to be simultaneous keeps the notation simple. The main theoretical results still hold in a model in which dissolutions happen before matches in a period.

Lemma 2. *In any stationary equilibrium, $\rho(w)$ is continuous and strictly decreasing in w.*

We next develop expressions for the discounted expected utility of workers in both the employed and unemployed states. Consider a worker with type ν who adopts wage demand w. Let $\pi_E(w; \nu)$ be his discounted expected utility conditional on being employed, and $\pi_U(w; \nu)$ be his discounted expected utility when unemployed.

When he is employed, he earns a wage of w this period, and faces a probability λ of being terminated at the end of the period. Therefore,

$$
\begin{aligned}
\pi_E(w; \nu) &= w + (1 - \lambda)\delta\pi_E(w; \nu) + \lambda\delta\pi_U(w; \nu) \\
&= \frac{w}{1 - \delta + \lambda\delta} + \frac{\lambda\delta}{1 - \delta + \lambda\delta}\pi_U(w; \nu) \\
&= \kappa_0 w + \lambda\delta\kappa_0\pi_U(w; \nu),
\end{aligned}
$$

where the constant $\kappa_0 \equiv \frac{1}{1 - \delta + \lambda\delta}$ is introduced for notational compactness. When employed at a wage of w, the expected discounted sum of future wages from the current position is $\kappa_0 w$.

When unemployed, he becomes a candidate and subsequently wins the auction with probability $\alpha(\nu)\rho(w)$, and moves into the employed state next period; otherwise, he remains unemployed. Therefore,

$$
\begin{aligned}
\pi_U(w; \nu) &= \alpha(\nu)\rho(w)\delta\pi_E(w; \nu) + [1 - \alpha(\nu)\rho(w)]\,\delta\pi_U(w; \nu) \\
&= \alpha(\nu)\rho(w)\delta\,[\kappa_0 w + \lambda\delta\kappa_0\pi_U(w; \nu)] + [1 - \alpha(\nu)\rho(w)]\,\delta\pi_U(w; \nu) \\
&= \frac{1}{1 - \delta} \times \frac{\alpha(\nu)\rho(w)\delta\kappa_0}{1 + \alpha(\nu)\rho(w)\delta\kappa_0}w
\end{aligned}
\tag{1}
$$

Again for notational compactness, we define[6]

$$
P(w, \nu) = \frac{1}{1 - \delta} \times \frac{\alpha(\nu)\rho(w)\delta\kappa_0}{1 + \alpha(\nu)\rho(w)\delta\kappa_0}
\tag{2}
$$

and so can write the discounted expected utility of the unemployed worker succinctly as

$$
\pi_U(w; \nu) = P(w, \nu)w.
\tag{3}
$$

The development through (3) now expresses the worker's wage bid formulation problem as a static optimization problem. As the function $P(w, \nu)$ is continuous and possible optimal wage bids are on the closed and bounded interval $[0, 1]$, for each ν, a utility-maximizing wage bid exists. Let

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[6]P is the analog to the "discounted ultimate probability of trade" in Satterthwaite and Shneyerov (2007). Our expression differs from theirs because we must take into account that in our model, workers eventually re-enter the market in the future after their employment stint ends, while in their model, agents leave the market after trading and do not return.

$w(\nu)$ be the optimal wage demand as a function of ν. Then, the interim utility for a worker in the unemployed state with type ν is

$$\Pi(\nu) = \max_{w} P(w, \nu)w = P(w(\nu), \nu)w(\nu). \tag{4}$$

It is enough to focus on the unemployed state, as it is only in the unemployed state that the worker makes a strategic choice.

Proposition 3. *The wage demand function $w(\nu)$ is strictly increasing in ν.*

3.2 Steady-state distributions of workers and jobs

The monotonicity of wage demands implies that, for a given vacancy, the candidate with the lowest type is the one who is hired. This fact allows us to characterize the steady-state distributions of types among unemployed and employed workers, respectively, without needing as yet to compute the optimal wage demands.

We define $\phi(\nu) \equiv \frac{g_E(\nu)}{g_U(\nu)}$. This ratio captures the relationship between the type and the proportion of time spent in unemployment in the long run. Because transitions from employment to unemployment are determined by the separation rate λ and do not depend on ν, the expected duration of a given employment stint is identical for all workers. Therefore, comparisons of ϕ across different values of ν pick up variation in expected duration of unemployment stints, with unemployment stints being longer the smaller the value of $\phi(\nu)$.

Proposition 4. *A steady-state equilibrium distribution of types consists of a function $\phi(\nu)$ and a real number V satisfying the differential equation*

$$\phi'(\nu) = \left[\frac{\alpha'(\nu)}{\alpha(\nu)} - \frac{\alpha(\nu)}{V} \cdot \frac{g(\nu)}{1 + \phi(\nu)} \right] \phi(\nu). \tag{5}$$

along with the boundary conditions

$$\phi(\nu_L) = \frac{\alpha(\nu_L)}{\lambda}. \tag{6}$$

$$\phi(\nu_H) = \frac{\alpha(\nu_H)}{\lambda} \left\{ 1 - \lambda \left[\frac{J}{V} - 1 \right] \right\}. \tag{7}$$

Proposition 5. *If α satisfies Assumption 1, a solution to the two-point boundary problem defined by the differential equation (5) with boundary conditions (6) and (7) exists and is unique.*

3.3 Equilibrium wage demand strategies

We can now turn to the characterization of the wage demand function. Conditional on being a candidate for a vacancy, the wage bid of an unemployed worker of type ν solves the static problem

$$\max_w P(w, \nu)w. \tag{8}$$

Proposition 6. *A wage demand function $w(\nu)$ which solves the problem (8) exists and is unique, and satisfies the differential equation*

$$w'(\nu) = \frac{1}{1 + \lambda\phi(\nu)\delta\kappa_0} \cdot \frac{\alpha(\nu)}{V} \cdot \frac{g(\nu)}{1 + \phi(\nu)} \cdot w, \tag{9}$$

with boundary condition $w(\nu_H) = 1$.

To relate the shape of the wage demand function more closely with unemployment rates, define $\beta \equiv \lambda\delta\kappa_0 = \frac{\lambda\delta}{1-\delta+\lambda\delta}$. Then the wage demand equation can be written

$$\frac{w'(\nu)}{w(\nu)} = \frac{1}{1 + \beta\phi(\nu)} \cdot \frac{\alpha(\nu)}{V} \cdot \frac{g(\nu)}{1 + \phi(\nu)} \tag{10}$$

$$= \frac{g_U(\nu)}{(1-\beta)g_U(\nu) + \beta g(\nu)} \cdot \frac{\alpha(\nu)}{V} g_U(\nu). \tag{11}$$

Note that as $\lambda \to 0$, $\beta \to 0$, and as $\delta \to 1$, $\beta \to 1$. So, for markets with a small separation rate ($\lambda \approx 0$),

$$\frac{w'(\nu)}{w(\nu)} \approx \frac{\alpha(\nu)}{V} g_U(\nu),$$

and in markets with very patient workers or very short periods ($\delta \approx 1$),

$$\frac{w'(\nu)}{w(\nu)} \approx \frac{\alpha(\nu)}{V} g_U(\nu) \cdot \frac{g_U(\nu)}{g(\nu)}.$$

In general, (11) indicates that the wage demand function will be steeper (1) in markets which are tighter (smaller V); (2) in ranges of ν where there are many unemployed workers (larger g_U) and (3) among high-access workers (larger α).

4 Comparative statics of the model

The equations characterizing the steady-state distribution of unemployed workers do not have closed-form solutions. Nor is there is a closed-form solution for the wage demand function; this is not surprising given that solving standard first-price auctions for equilibrium bidding functions

generally requires resorting to numerical approaches for all but the simplest of cases. We therefore turn to some numerical explorations of our model.

We are interested in the model's predictions about the effects of heterogeneity in ν on the following:

1. The distribution of wages among employed workers. Empirical measurements of heterogeneity in wages necessarily measure the wages of currently-employed workers.

2. The long-run welfare of workers. To compute this, for each type, we take the expectation weighted by the employment and unemployment probabilities in steady state,

$$\pi^\star(\nu) = \frac{g_E(\nu)\pi_E(w^\star(\nu);\nu) + g_U(\nu)\pi_U(w^\star(\nu);\nu)}{g(\nu)},$$

and then compute the distribution of $\pi^\star(\nu)$ in equilibrium.

3. The duration of unemployment stints. Equation (5) offers the tantalizing possibility that the duration of unemployment might be non-decreasing in access, at least over some ranges of ν, depending on the sign of the term in square brackets.

To address these questions, we adopt a computational methodology which aims to isolate the implications of heterogeneity. We observe that there are (infinitely) many distributions $g(\nu)$ which are consistent with an aggregate unemployment rate of U in a market with J positions and a separation rate of λ. Roughly speaking, requiring an unemployment rate U in a market with parameters J and λ constrains the position of the support of types, but the variance of the distribution can be chosen freely within limits. Therefore, our exercise in comparative statics is to fix the values of J, λ, and U, and relate changes in the variance of types ν to changes in wage distributions, unemployment stints, and long-run welfare.

We normalize the size of the labor force to have measure one, $W = 1$. We choose the duration of a period to be one week, as a plausible match for the pace at which labor markets operate. In the U.S., many quantities of flow in the labor market are reported weekly, including the number of new jobless claims, and duration of unemployment. In addition, in our model, when a firm-worker match dissolves, the worker and firm do not re-enter the search and matching process until the subsequent period. This implies a period should correspond to a relatively short interval of time; it seems reasonable that a newly-unemployed worker would take at least a week to become a candidate for a new post. With the week as the duration of the period, we choose $\delta = 0.9991$ as the per-period discount factor, implying an annual discount factor of about $\delta = 0.954$.

Suppose J, λ, and U are given, and consider the degenerate case of our model in which all workers have the same type $\nu^\star$. In this case, by symmetry, we assume that each candidate has an

equal chance of filling a vacancy, and therefore a vacancy is unfilled in a period if and only if there are no candidates for that vacancy in a period. In this setting, balancing the flows into and out of employment requires that

$$
\left\{ 1 - \exp\left[-\frac{\alpha(\nu^\star; V, E)U}{V} \right] \right\} V = \lambda(J - V)
$$

$$
\alpha^\star \equiv \alpha(\nu^\star; V, E) = -\frac{V}{U} \log\left[1 - \lambda\left(\frac{J - V}{V} \right) \right].
$$

That is, choices for J, λ, and U pin down exactly the required probability $\alpha^\star$ with which un-employed workers become candidates in the degenerate model. It is therefore necessary that $\alpha(\nu_L) \le \alpha^\star \le \alpha(\nu_H)$ in order for an equilibrium to exist with the required U in a market with parameters J and λ.

We then proceed to consider nondegenerate distributions g of types. We consider the case of the uniform distribution on intervals $[\nu_L, \nu_H]$. We then vary the lower boundary of the support over $\nu_L \in [0, \nu^\star]$, and, for each ν_L, determine the corresponding ν_H such that the required level of U obtains. We therefore construct a family of markets which are the same in terms of the aggregate-level quantities, but which vary in the extent to which access types are heterogeneous across the population of workers. Finally, for simplicity of exposition, we take $\alpha(\nu) = \nu$.

We organise our analysis in two phases. In the first, we choose parameters inspired by aggregate national labor markets. Subsequently, we focus in on a parameterization plausibly capturing a specialist services contract-worker market.

For the aggregate national market treatments, we use a 2×2 factorial design. We independently vary the tightness of the market and the separation rate λ. We consider treatments in which unemployment is relatively low, $U = 0.055$,[7] and relatively high, $U = 0.10$. Hall (2005) estimates a typical ratio of vacancies to unemployed workers of about $\frac{V}{U} = 0.54$. For the case of $U = 0.055$, this implies that $V = 0.0297$ and $J = 0.9747$. Taking the Beveridge curve to be of the hyperbolic form $U \times V = $ const, for $U = 0.10$ the corresponding parameterization has $V = 0.01617$ and $J = 0.91617$.

Estimation of separation rates vary across markets; Shimer (2005) estimates a monthly separation rate of about 0.034 in U.S. markets, while Hobijn and Şahin (2009) survey twenty-seven countries in the OECD, estimating monthly separation rates between 0.007 and 0.023. We pick our high separation rate treatment to have the weekly separation rate $\lambda = 0.00765$ to match Shimer (2005), and the low separation rate treatment to have $\lambda = 0.002$, roughly the lower end of the interval from Hobijn and Şahin (2009).

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[7]We pick this as the typical expansionary U.S. unemployment rate from Shimer (2005).

(a) $U = 0.055$, $\frac{V}{U} = 0.0297$, $\lambda = 0.002$

(b) $U = 0.055$, $\frac{V}{U} = 0.0297$, $\lambda = 0.00765$

(c) $U = 0.100$, $\frac{V}{U} = 0.0163$, $\lambda = 0.002$

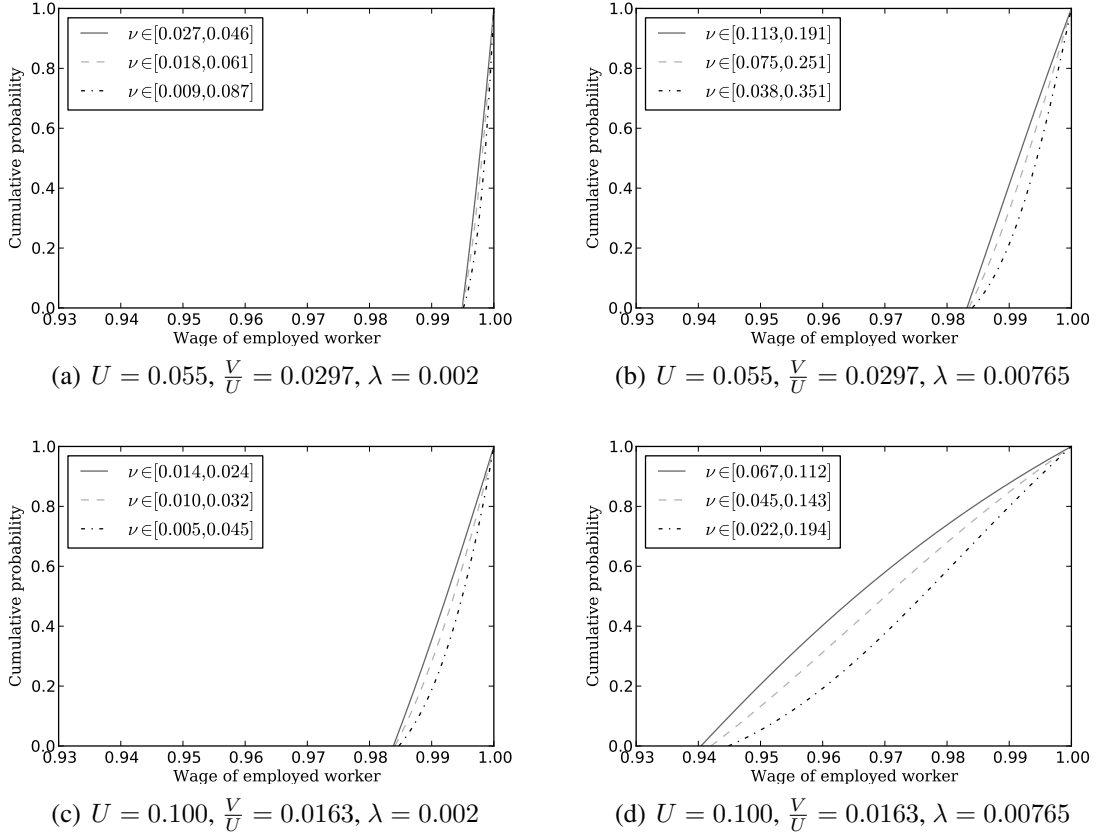(d) $U = 0.100$, $\frac{V}{U} = 0.0163$, $\lambda = 0.00765$

Figure 1: Equilibrium wage distributions as a function of heterogeneity in access.

4.1 Wage distributions

Figure 1 presents equilibrium distributions of employed worker wages for each treatment, for se-
lected supports of the type distribution. In each figure, the solid curve corresponds to the dis-
tribution from the parameterization with the smallest variation in access types. In this and the
subsequent figures, individual curves are not directly comparable across subfigures, as different
ranges of ν are required to obtain the desired parameterizations. What can be compared across
subfigures are the general qualitative features of the variability and shape of the distribution of the
quantity being examined.

We organize the main observations as a series of results.

Result 1. *Heterogeneity in employed worker wages increases in the tightness of the market and in
the frequency of turnover. Conditional on the tightness of the market and the frequency of turnover,
the effect of greater variability in access types on employed worker wages is small.*

Support. In the baseline case of low unemployment and low turnover, workers with the lowest
access earn a wage only about 0.5% below that of those with the highest. Increasing either the

tightness of the market or the turnover frequency separately increases the gap to about 1.7% of wages. In the tight market with high turnover, the gap can be up to 6.0%. The effect of tight markets is qualitatively in line with the earlier analysis of the wage demand equation (11).

However, relative to the magnitude of the effect of market tightness and turnover frequency, varying the heterogeneity in access has little effect. Even in the tight market with high turnover, the size of the wage gap between the lowest-paid and highest-paid workers ranges only from 5.5% to 6.0%. □

Result 2. *Variability in wages is inversely related to variability in access.*

Support. In all treatments, the CDFs of the employed worker wage distribution are ordered, with the CDF arising from a distribution with greater variability in access dominating that arising from a distribution with smaller variability in access. That is to say, if faced with two markets which are identical in the aggregate parameters δ, λ, U, and V, the market with greater wage dispersion among employed workers will be the one with a smaller underlying heterogeneity in worker access. □
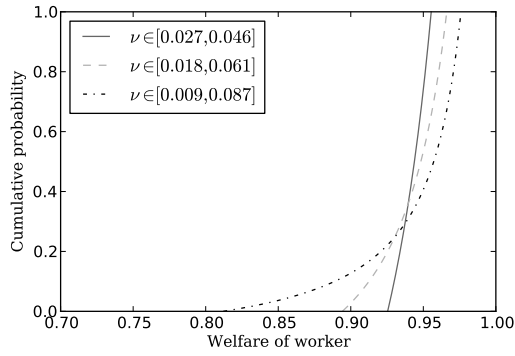
4.2   Value distributions

Heterogeneity in access accounts for a relatively small amount of variation in wages. Nevertheless, the variability in long-run welfare, measured by the present discounted sum of future wages, is substantial. Figure 2 plots distributions of the welfare measure across the population of workers, for the same parameterizations displayed previously.

Result 3. *Long-run welfare of workers in the market is quite sensitive to degree of access.*

Support. When the amount of heteorgeneity in access is small, the distribution of long-run welfare is approximately uniform. In markets with greater heterogeneity, the distribution exhibits a substantial left tail. Greater heterogeneity in the market entails that there are some workers with very small values of ν, who therefore become candidates rather infrequently and spend significant amounts of time unemployed. □

There are two ways in which hearing about more opportunities can be beneficial to long-run welfare. One is the direct effect of being a candidate more often; the other is the strategic effect of having a higher dynamic reservation value due to being a candidate more often. Although we have direct wage bidding competition in our model, the variation in dynamic reservation values does not drive much heterogeneity in observed wages. In most environments, the direct effect dominates. Increasing the heterogeneity among workers has the effect of softening competition, leading to less aggressive bidding and higher, less dispersed wages among employed workers.
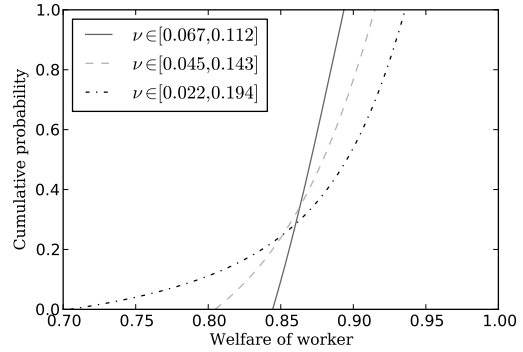
(a) $U = 0.055$, $\frac{V}{U} = 0.0297$, $\lambda = 0.002$
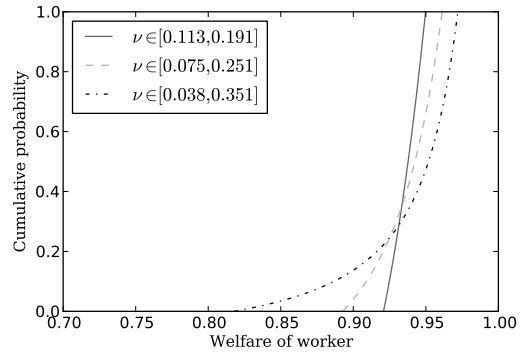
(b) $U = 0.055$, $\frac{V}{U} = 0.0297$, $\lambda = 0.00765$
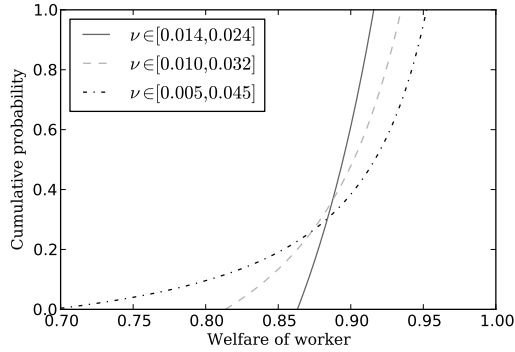
(c) $U = 0.100$, $\frac{V}{U} = 0.0163$, $\lambda = 0.002$

(d) $U = 0.100$, $\frac{V}{U} = 0.0163$, $\lambda = 0.00765$

Figure 2: Equilibrium value distributions as a function of heterogeneity in access.

4.3 Unemployment stint duration

Equation (5) implies the possibility that the duration of unemployment stints may not be decreasing in ν; workers with greater access may nevertheless spend a greater proportion of time in unemployment than some other workers who have lesser access. A sufficient condition for this to obtain is the extreme case where the highest-access workers are unemployed more than the lowest-access ones, which occurs if $\phi(\nu_H) < \phi(\nu_L)$. Combining (6) and (7) and rearranging, this occurs when

$$\frac{\alpha(\nu_L)}{\alpha(\nu_H)} > 1 - \lambda \left[\frac{W}{V} - \frac{U}{V} \right]. \tag{12}$$

For a fixed support $[\nu_L, \nu_H]$ of types, this condition can obtain if (1) the market has a sufficiently high turnover rate (large λ), or (2) the market is sufficiently tight and the ratio of *employed* workers to vacancies is large.

Equation (12) is a sufficient condition for ϕ to have a decreasing region. Decreasing regions may still exist even when (12) is not satisfied. For example, the shape of ϕ when the distribution of types is uniform is necessarily single-peaked.

Proposition 7. *Suppose that $\frac{\alpha'(\nu)}{\alpha(\nu)}$ is decreasing in ν and $g(\nu)$ is uniform. If there exists some $\hat{\nu}$ such that $\phi'(\hat{\nu}) < 0$, then $\phi'(\nu) < 0$ for all $\nu \in [\hat{\nu}, \nu_H]$.*

Result 4. *Nonmonotonicity in the unemployment rate by type is possible given reasonable parameters.*

Support. Figure 3 displays unemployment rates by worker type for each of the parameterizations. In general these are monotonically decreasing in type, and are flatter the less heterogeneous the population is. In the tight, high-turnover market, we obtain a nonmonotonic unemployment rate curve, with the workers with the best access actually having a slightly higher unemployment rate than those with the worst access. □
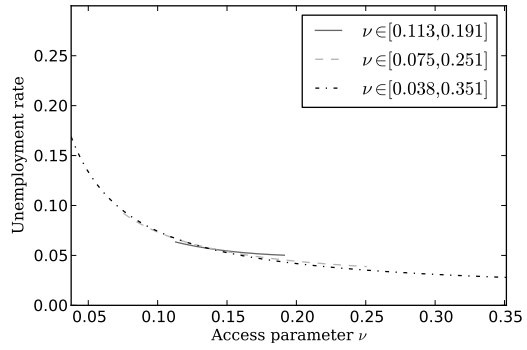
4.4 A "contract-worker" market

The distinctive prediction of our model is that better access and lower unemployment need not go hand- in-hand. As seen above in tight labor markets with high turnover the connection between the two may be weak, or even non-monotonic: high-access workers may both earn high wages when employed and be unemployed for longer stints.
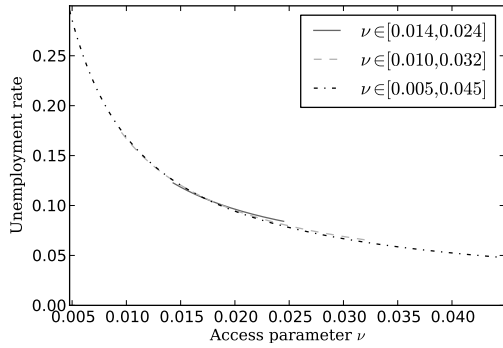
We illustrate this feature of our model for a relatively tight market with higher turnover rates than found in the aggregate market data underlying our preceding calculations. We consider a market in which $\lambda = 0.02$, which corresponds to an expected duration of a job of just under a year.
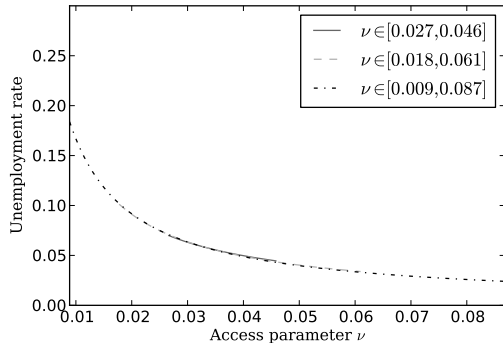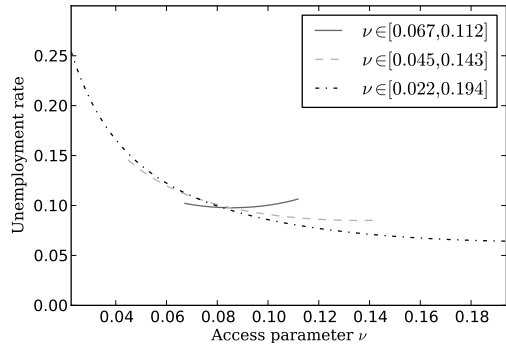
(a) $U = 0.055$, $\frac{V}{U} = 0.0297$, $\lambda = 0.002$

(b) $U = 0.055$, $\frac{V}{U} = 0.0297$, $\lambda = 0.00765$

(c) $U = 0.100$, $\frac{V}{U} = 0.0163$, $\lambda = 0.002$

(d) $U = 0.100$, $\frac{V}{U} = 0.0163$, $\lambda = 0.00765$

Figure 3: Equilibrium unemployment rates by worker type, as a function of heterogeneity in access.
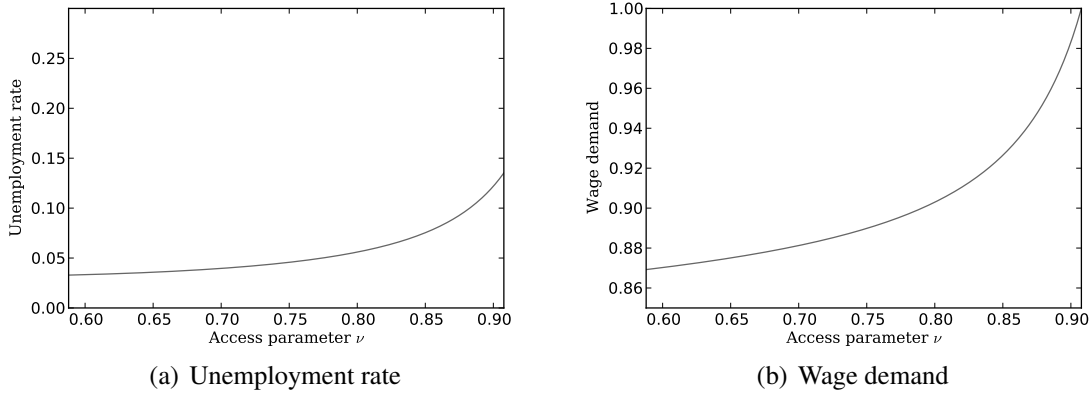
(a) Unemployment rate   (b) Wage demand

Figure 4: Equilibrium characteristics for tight contract-worker market parameterization

We keep the unemployment rate at $U = 0.055$, and pick $V = 0.022$, so $J = 0.967$. The interval of types is set to $[0.588, 0.908]$. Parameters in this range might describe a market in which workers are employed on contracts of relatively short duration. In such a contractor-oriented market, the bargaining model featuring direct competition among candidates in a bidding mechanism may match the real world in which potential contractors may submit formal bids for a particular piece of work.

Figure 4 plots the unemployment rate and wage demand curves for this market. The highest access workers become candidates roughly half again as often as the lowest-access workers, yet remain unemployed about three times as long. At the same time, the highest-access workers enjoy about a 10% wage premium relative to the lowest-access workers. Taken together, this would imply a positive correlation between unemployment stint duration and wages.

5 Conclusion

The model in this paper analyzes the effect of introducing direct competition among workers in a search-and-matching model of a labor market where workers receive news of job vacancies at different rates. The assumptions on how workers vary in the frequency of hearing about vacancies admit a variety of interpretations of the source of the heterogeneity, including but not limited to social networks.

The main results are that greater heterogeneity in access does not lead to greater heterogeneity in employed worker wages, and, under certain parameters, workers with the best access to vacancies may nevertheless go unemployed for the longest. Both results intuitively derive from our use of a bargaining mechanism, specifically a first-price auction, which implements direct competition among workers. In the Nash bargaining approach, competition among workers is indirect, feeding

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into the bargaining process only via influencing the dynamic disagreement payoff of the firm.

There are two channels through which the advantage of hearing about vacancies can be transmitted into better long-run outcomes: the worker can be unemployed for shorter periods of time, or the worker can earn higher wages when employed. The relative amount of the advantage transferred through these channels depends on the rules of the game. In Nash bargaining, disagreement does not occur along the equilibrium path, and a match always results in a hire for the worker. Consequently high-access workers necessarily obtain higher wages and experience shorter unemployment stints. The first-price auction admits in some sense a richer strategy space for high-access workers, who, especially in settings where turnover is high and vacancies are scarce, can choose a strategy of demanding high wages despite remaining unemployed for extended periods.

For most parameterizations, when the ratio of candidates to vacancies is low, our matching technology is approximately similar to the one-to-one matching of Nash bargaining-based models, because when the number of candidates is small, the probability of a given worker being the only candidate for a position is large. In that sense, our results agree with both the Nash bargaining-based models and the empirical literature to date, in that more effective social networks lead to shorter unemployment stints.

We show numerically that equilibria in which high-access workers remain unemployed for relatively long stints can be generated by unemployment and turnover rates are at the high end of what has been observed for aggregate labor markets. A more promising strategy to test this prediction of the model empirically is to examine more specialized labor markets, in which the length of a given job stint is relatively short. Examples might include specialist consultancy or contract-worker markets, such as website designers or actors for movies and plays. A Nash-bargaining model of wage determination would predict that high-access workers in these markets would be employed much more frequently than less well-connected peers, whereas the auction-based model indicates the possibility that high-access workers would optimally choose a strategy giving them high wages when working, while spending more time in unemployment.

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A   Proofs

Proof of Lemma 2. As this is a first-price auction in which the lowest wage demand wins, $\rho(w)$ is necessarily non-increasing in w irrespective of the behavior of other agents. The argument for strict monotonicity proceeds by contradiction. Suppose there are two wage bids $w_1 < w_2$ for which $\rho(w_1) = \rho(w_2)$. Since ρ is non-decreasing, ρ must be constant on (w_1, w_2). However, this implies the bidder bidding w_1 is not bidding optimally, as he could raise his bid to $w_1 + \varepsilon$ for sufficiently small ε, not affect his chances of winning, but strictly increase the wage he is paid. This is therefore inconsistent with an equilibrium, which establishes the desired contradiction.

To establish continuity, again we argue by contradiction. Suppose that ρ is discontinuous at a wage demand w. This implies that there is a positive mass of types submitting a wage demand of w, and therefore a positive probability of a tie, which will be broken at random. However, in this case, a bidder who might be involved in such a tie would find it profitable to bid $w - \varepsilon$ for sufficiently small $\varepsilon > 0$ to avoid the tiebreaker and win the job for sure in such a contingency, which contradicts the assumption that bids are in equilibrium. $\qquad\square$

Proof of Proposition 3. The proof is by contradiction. Suppose that $\nu_1 > \nu_2$ but $w(\nu_1) \leq w(\nu_2)$. Let $w_i = w(\nu_i)$, $i = 1, 2$. Since w_i are optimal for their respective ν_i,

$$P(w_1, \nu_1)w_1 \;\geq\; P(w_2, \nu_1)w_2$$
$$P(w_2, \nu_2)w_2 \;\geq\; P(w_1, \nu_2)w_1.$$

Then, we have

$$P(w_1, \nu_1)w_1 \geq P(w_2, \nu_1)w_2 = \frac{P(w_2, \nu_1)}{P(w_2, \nu_2)}P(w_2, \nu_2)w_2 \geq \frac{P(w_2, \nu_1)}{P(w_2, \nu_2)}P(w_1, \nu_2)w_1,$$

which implies that

$$\frac{P(w_1, \nu_1)}{P(w_1, \nu_2)} \geq \frac{P(w_2, \nu_1)}{P(w_2, \nu_2)}. \tag{13}$$

For any w,

$$\frac{P(w, \nu_1)}{P(w, \nu_2)} = \frac{\alpha(\nu_1)}{\alpha(\nu_2)} \times \frac{1 + \alpha(\nu_2)\rho(w)\delta\kappa_0}{1 + \alpha(\nu_1)\rho(w)\delta\kappa_0}. \tag{14}$$

Inspecting (14), note that $\alpha(\nu_1) > \alpha(\nu_2)$ and $\rho(w)$ is a strictly decreasing function of w. Therefore, the ratio (14) is strictly increasing as a function of w, which contradicts the requirement of (13). $\qquad\square$

Proof of Proposition 4. Consider a worker with type ν who is currently unemployed. He will get a job this period if he hears about a vacancy, and then further is the candidate who has the lowest

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type, among candidates for the same vacancy. By the Poisson property of the matching process, these two events are independent, so we can write the probability of this worker getting a job in the current period as

$$\eta(\nu) = \alpha(\nu) \exp\left[-\frac{C(\nu)}{V}\right]. \tag{15}$$

In steady-state, the flow of workers of each type ν into employment must equal the flow into unemployment,

$$g_U(\nu)\eta(\nu) = \lambda g_E(\nu). \tag{16}$$

Combining (15) and (16) and writing in terms of ϕ, we have

$$\phi(\nu) \equiv \frac{g_E(\nu)}{g_U(\nu)} = \frac{\alpha(\nu)}{\lambda} \exp\left[-\frac{C(\nu)}{V}\right] \tag{17}$$

Differentiating the identity (17) with respect to ν, we obtain

$$\phi'(\nu) = \left[\frac{\alpha'(\nu)}{\lambda} - \frac{\alpha(\nu)}{\lambda} \cdot \frac{C'(\nu)}{V}\right] \exp\left[-\frac{C(\nu)}{V}\right]$$

Apply the definition of $C(\nu)$ and rearrange to find

$$\phi'(\nu) = \left[\frac{\alpha'(\nu)}{\alpha(\nu)} - \frac{\alpha(\nu)g_U(\nu)}{V}\right]\frac{\alpha(\nu)}{\lambda} \exp\left[-\frac{C(\nu)}{V}\right] = \left[\frac{\alpha'(\nu)}{\alpha(\nu)} - \frac{\alpha(\nu)g_U(\nu)}{V}\right]\phi(\nu). \tag{18}$$

Because $g_U(\nu) + g_E(\nu) = g(\nu)$, it follows that we can write

$$g_U(\nu) = \frac{g(\nu)}{1 + \phi(\nu)}. \tag{19}$$

Applying (19) to (18) we conclude

$$\phi'(\nu) = \left[\frac{\alpha'(\nu)}{\alpha(\nu)} - \frac{\alpha(\nu)}{V} \cdot \frac{g(\nu)}{1 + \phi(\nu)}\right]\phi(\nu). \tag{20}$$

The differential equation (5) is constrained by boundary conditions at each end of the interval of types. At ν_L, evaluate (17) and note that by definition $C(\nu_L) = 0$ to obtain

$$\phi(\nu_L) = \frac{\alpha(\nu_L)}{\lambda}.$$

For the largest type ν_H, the boundary condition is specified by the fact that the total flows into and out of unemployment must equal. The total flow into unemployment is $\lambda(J - V)$. To calculate the flow into employment, observe that a vacancy is filled whenever there is at least one candidate. As

the number of candidates per vacancy is distributed Poisson with mean $\frac{C(\nu_H)}{V}$, the probability there is no candidate for a vacancy is $\exp\left[-\frac{C(\nu_H)}{V}\right]$. Therefore,

$$\left\{1 - \exp\left[-\frac{C(\nu_H)}{V}\right]\right\} V = \lambda(J - V)$$

which can be rearranged to

$$\exp\left[-\frac{C(\nu_H)}{V}\right] = 1 - \lambda\left[\frac{J}{V} - 1\right]. \tag{21}$$

Using (21) in (17) evaluated at ν_H we have

$$\phi(\nu_H) = \frac{\alpha(\nu_H)}{\lambda}\left\{1 - \lambda\left[\frac{J}{V} - 1\right]\right\}.$$

$\square$

Proof of Proposition 5. A solution to the problem involves finding both a function ϕ and a scalar V. For any fixed V, let $\hat{\phi}(\nu|V)$ denote the solution of (5) starting from the lower boundary condition (6). Note that condition (6) is independent of V. Assumption 1 implies that the in square brackets on the right side of (5) is continuous and strictly decreasing in V. This implies that $\hat{\phi}(\nu|V)$ is continuous and strictly decreasing in V for all ν, and, in particular, for ν_H. Further, by inspection of (17), $\hat{\phi}(\nu_H|V) \leq \frac{\alpha(\nu_H)}{\lambda}$.

Equation (21) gives the probability there are no candidates for a given vacancy; as a probability, it must lie in $[0, 1]$, which implies that $V \in \left[\frac{\lambda}{1+\lambda}J, J\right]$. The upper boundary condition (7) therefore is continuous and strictly increasing in V, and takes on all values in $\left[0, \frac{\alpha(\nu_H)}{\lambda}\right]$ as V is varied. Equally, $\hat{\phi}(\nu_H|V)$ is continuous and strictly decreasing in V and is bounded from above by $\frac{\alpha(\nu_H)}{\lambda}$. It therefore follows that a joint solution for ϕ and V exists, and is unique. $\square$

Proof of Proposition 6. The first-order necessary condition for a wage w to solve (8) is

$$\frac{\partial P}{\partial w}(w, \nu)w + P(w, \nu) = 0. \tag{22}$$

Using the definition of P from (2) and differentiating,

$$\frac{\partial P}{\partial w}(w, \nu) = \frac{\delta\kappa_0}{1 - \delta} \times \frac{\alpha(\nu)\rho'(w)}{[1 + \alpha(\nu)\rho(w)\delta\kappa_0]^2}. \tag{23}$$

Applying (23) to (22) and rearranging,

$$\rho'(w)w + [1 + \alpha(\nu)\rho(w)\delta\kappa_0]\,\rho(w) = 0. \tag{24}$$

Because the wage demand function is monotonic, we can denote its inverse as $\nu(w)$. Let $\psi(\nu)$ be the probability that there is no candidate for the vacancy with type less than ν. Then, in equilibrium, $\rho(w) = \psi(\nu(w))$, and $\rho'(w) = \psi'(\nu(w)) \cdot \nu'(w)$. Applying these facts to (24) and solving for $w'(\nu)$,

$$w'(\nu) = -\frac{1}{1 + \alpha(\nu)\psi(\nu)\delta\kappa_0}\frac{\psi'(\nu)}{\psi(\nu)}w(\nu). \tag{25}$$

Necessarily by definition it must be that $\psi'(\nu) < 0$, so therefore $w'(\nu) > 0$: workers with better access indeed make higher wage demands. The boundary condition on the differential equation is determined by observing that a candidate of type ν_H only gets the job if she is the only candidate for the vacancy. Therefore, in equilibrium, $w(\nu_H) = 1$, the maximum willingness to pay of the firm.

Recalling (1) and applying the fact that this is an equation that holds in equilibrium, we can also express (25) as

$$w'(\nu) = -\left[\pi_U(w(\nu);\nu)\cdot\frac{1-\delta}{\alpha(\nu)\psi(\nu)\delta\kappa_0}\right]\cdot\frac{\psi'(\nu)}{\psi(\nu)}. \tag{26}$$

Writing the wage demand function in the form (26) permits a comparison with the equilibrium in a standard symmetric independent private-values first-price auction. Let v denote the private value of a bidder. A bidder with a given value v solves

$$\max_b (v - b)P(b),$$

where $P(b)$ is the probability a bid b wins the auction. The first-order condition to maximize this is

$$(v - b)P'(b) - P(b) = 0$$

Let $\Theta(v)$ be the probability that all other bidders have type less than v. Therefore, $P(b) = \Theta(v(b))$ and $P'(b) = \Theta'(v(b))v'(b)$. The first-order condition then implies

$$(v - b(v))\frac{\Theta'(v)}{b'(v)} - \Theta(v) = 0.$$

This rearranges to

$$b'(v) = [v - b(v)]\cdot\frac{\Theta'(v)}{\Theta(v)}. \tag{27}$$

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In both (26) and (27), the terms in square brackets reflect the amount by which the bidder shades his bid relative to his value, or, in other words, the additional rent the bidder earns conditional on winning the auction versus losing.

It is straightforward to obtain expressions for $\psi(\nu)$ in equilibrium based on $\phi(\nu)$. Directly by definition,

$$\psi(\nu) = \frac{\lambda}{\alpha(\nu)}\phi(\nu). \tag{28}$$

Differentiating with respect to ν,

$$\psi'(\nu) = \lambda\frac{\alpha(\nu)\phi'(\nu) - \phi(\nu)\alpha'(\nu)}{\alpha(\nu)^2}.$$

Therefore,

$$
\begin{aligned}
\frac{\psi'(\nu)}{\psi(\nu)} &= \lambda\frac{\alpha(\nu)\phi'(\nu) - \phi(\nu)\alpha'(\nu)}{\alpha(\nu)^2} \times \frac{\alpha(\nu)}{\lambda\phi(\nu)} \\
&= \frac{\phi'(\nu)}{\phi(\nu)} - \frac{\alpha'(\nu)}{\alpha(\nu)} \\
&= -\frac{\alpha(\nu)}{V} \times \frac{g(\nu)}{1 + \phi(\nu)},
\end{aligned}
\tag{29}
$$

where the last line follows from (5).

At this point, we have established only that $w(\nu)$ is an extremum. We now turn to showing that it is in fact a maximizer. The proof proceeds by establishing the pseudoconcavity of the function $\pi_U(w, \nu)$ as defined in (3), which is a sufficient condition for a solution to (22) to be a maximizer. Let $\nu^\star(w)$ denote an inverse bid function determined by the solution to (22). Then, the expected payoff to a worker of type ν who bids w assuming other workers bid according to $\nu^\star(w)$ is

$$\pi_U(w, \nu) = P(w, \nu)w.$$

First we will argue that $\frac{\partial^2 \pi_U}{\partial\nu\partial w} > 0$. Differentiate first by ν to obtain

$$\frac{\partial\pi_U}{\partial\nu}(w, \nu) = \frac{\partial P}{\partial\nu}(w, \nu) \cdot w$$

and then by w to obtain

$$\frac{\partial^2 \pi_U}{\partial\nu\partial w}(w, \nu) = \frac{\partial^2 P}{\partial\nu\partial w}(w, \nu) \cdot w + \frac{\partial P}{\partial\nu}(w, \nu). \tag{30}$$

Since the other workers bid according to $\nu^\star(w)$,

$$P(w, \nu) = \frac{1}{1 - \delta} \times \frac{\alpha(\nu)\psi(\nu^\star(w))\delta\kappa_0}{1 + \alpha(\nu)\psi(\nu^\star(w))\delta\kappa_0}.$$

Differentiating by ν,

$$\frac{\partial P}{\partial \nu}(w, \nu) = \frac{1}{1 - \delta} \times \frac{\psi(\nu^\star(w))\delta\kappa_0}{[1 + \alpha(\nu)\psi(\nu^\star(w))\delta\kappa_0]^2}. \tag{31}$$

Equation (31) shows $\frac{\partial P}{\partial \nu} > 0$. Differentiating further now by w,

$$
\begin{aligned}
\frac{\partial^2 P}{\partial \nu \partial w} &= \frac{\delta\kappa_0}{1 - \delta} \frac{[1 + \alpha(\nu)\psi(\nu^\star)\delta\kappa_0]^2 \psi'(\nu^\star)(\nu^\star)' - 2\psi'(\nu^\star)(\nu^\star)'(1 + \alpha(\nu)\psi(\nu^\star)\delta\kappa_0)}{[1 + \alpha(\nu)\psi'(\nu^\star)\delta\kappa_0]^4} \\
&= \frac{\psi'(\nu^\star)(\nu^\star)'}{1 - \delta} \times \frac{1 - \psi(\nu^\star)\nu^\star}{[1 + \alpha(\nu)\psi'(\nu^\star)\delta\kappa_0]^3}. \tag{32}
\end{aligned}
$$

The expression (30) is positive if and only if, applying (31) and (32),

$$
\begin{aligned}
\frac{\partial^2 \pi_U}{\partial \nu \partial w}(w, \nu) &> 0 \\
\frac{\psi'(\nu^\star)'\delta\kappa_0}{1 - \delta} \cdot \frac{1 - \psi\nu^\star}{[1 + \nu\psi\delta\kappa_0]^3}w + \frac{1}{1 - \delta}\frac{\psi\delta\kappa_0}{[1 + \nu\psi\delta\kappa_0]^2} &> 0 \\
\psi'(\nu^\star)'\frac{1 - \psi\nu}{1 + \nu\psi\delta\kappa_0}w + \psi &> 0 \\
\frac{\psi'}{w'}(1 - \psi\nu)w + \psi(1 + \nu\phi\delta\kappa_0) &> 0
\end{aligned}
$$

Because the bidding function satisfies (25),

$$
\begin{aligned}
-\psi'\frac{\psi}{\psi'}(1 + \nu\psi\delta\kappa_0)(1 - \psi\nu) + \psi(1 + \nu\psi\delta\kappa_0) &> 0 \\
-(1 - \psi\nu) + 1 &> 0 \\
\psi\nu &> 0,
\end{aligned}
$$

As $\psi\nu$ is a probability, it is positive; tracing backwards through this series of if-and-only-if statements allows us to conclude that the expression in (30) is positive.

Now we turn to the pseudoconcavity of $\pi_U(w, \nu)$ in w. Pick ν and w that satisfy (22). We want to show that $\pi_U(w, \nu)$ is nonincreasing for $\hat{w} \in (w, 1]$. Pick such a $\hat{w}$, and let $\hat{\nu} = \nu^\star(\hat{w})$. Note that since $\nu^\star$ is increasing, $\hat{\nu} > \nu$. Since $\hat{\nu}$ and $\hat{w}$ also satisfy (22), $\frac{\partial \pi_U}{\partial w}(\hat{w}, \hat{\nu}) = 0$. Because $\hat{\nu} > \nu$ and $\frac{\partial^2 \pi_U}{\partial \nu \partial w} > 0$, it follows that $\frac{\partial \pi_U}{\partial w}(\hat{w}, \nu) < 0$, as desired.

The argument that $\pi_U(w, \nu)$ is nondecreasing for $\hat{w} \in [w^\star(\nu_L), w)$ is analogous. Taken together, this establishes the pseudoconcavity of $\pi_U(w, \nu)$ in w, and therefore (22) in fact characterizes a maximizing bid. □

Proof of Proposition 7. It is sufficient to show that, if $\phi'(\hat{\nu}) < 0$, then the term in square brackets in (5),

$$\frac{\alpha'(\nu)}{\alpha(\nu)} - \frac{\alpha(\nu)}{V} \cdot \frac{g(\nu)}{1 + \phi(\nu)}, \tag{33}$$

is decreasing at $\hat{\nu}$. The first term is decreasing by assumption. Turning to the second term,

$$\frac{d}{d\nu} \left[\frac{\alpha(\nu)}{V} \cdot \frac{g(\nu)}{1 + \phi(\nu)} \right] = \frac{\alpha'(\nu)}{V} \cdot \frac{g(\nu)}{1 + \phi(\nu)} + \frac{\alpha(\nu)}{V} \cdot \frac{(1 + \phi(\nu))g'(\nu) - \phi'(\nu)g(\nu)}{(1 + \phi(\nu))^2}. \tag{34}$$

Because g is uniform, $g'(\nu) = 0$, and by assumption, $\phi'(\hat{\nu}) < 0$, which imply the expression in (34) is positive. This expression enters negatively into (33), which completes the proof. □