A Model of Momentum and Market States: Theory and Evidence

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Abstract

Momentum profits vary substantially across different market states. Motivated by this phenomenon, I develop a model to connect market states and momentum profits, and test the model's empirical implications. The model applies the mechanism of overconfidence and self-attribution bias into a setting of multiple risky assets with correlated payoffs. A novel insight from the model is that overconfidence can vary asymmetrically between winners and losers, which leads to asymmetric return behaviors between winners and losers. The model generates a set of implications regarding the relation between market states and returns on the winner, loser, and momentum portfolios. These implications are consistent with empirical patterns in the literature and those newly documented in this paper. A calibration exercise shows that the model can match key empirical patterns with reasonable parameters. Overall, this paper unifies momentum, negative momentum profits under certain market states, and long-run reversals.

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1 Introduction

Momentum (Jegadeesh and Titman 1993) is one of the most prominent anomalies in stock markets. A momentum strategy that buys winner stocks with higher recent returns and shorts loser stocks with lower recent returns generates positive average profits in the short run. However, it appears that momentum profits vary substantially over time and across different market states. Cooper, Gutierrez, and Hameed (2004) document that momentum profits are higher following positive market returns (UP markets) than following negative market returns (DOWN markets). Daniel and Moskowitz (2013) show that negative momentum profits often occur under DOWN/UP states, where a DOWN market is followed by an UP market.

In this paper, I further explore the relation between momentum profits and market states. Two relevant market returns for a momentum strategy are the lagged one in the ranking period and the contemporaneous one in the subsequent holding period. Since both lagged and contemporaneous markets can be in an UP or a DOWN state, there are four market states. For example, UP/DOWN corresponds to the state where an UP market is followed by a DOWN market. I show empirically that the average momentum profit is positive under three of the four market states: UP/UP, UP/DOWN, DOWN/DOWN, and is negative under the DOWN/UP state. These patterns survive after risk adjustment including adjustment for time-varying risk (market beta) of the momentum portfolio. More importantly, these patterns present themselves as puzzles to the literature since existing models of momentum have limited success in explaining them.[1]

Thus motivated, I develop a model to explain the patterns of momentum profits across different market states. The model also generates two sets of new testable implications.

[1]As pointed out by Asem and Tian (2010), the underreaction model of momentum by Hong and Stein (1999) and the rational model of momentum by Sagi and Seasholes (2007) predict that the momentum profit is lower under DOWN/DOWN states than under DOWN/UP states. There are also two heuristic stories linking market states with overconfidence, based on the model of Daniel, Hirshleifer, and Subrahmanyam (1998). Cooper, Gutierrez, and Hameed (2004)'s story of UP markets leading to higher overconfidence also predicts that the momentum profit is lower under DOWN/DOWN states than under DOWN/UP states. Asem and Tian (2010)'s story of market reversal leading to declining overconfidence predicts that the momentum profit should be negative under UP/DOWN states. These predictions are at odds with the data.

First, the abnormal return is asymmetric between winners and losers and this asymmetry persists across different market states. Second, long-run reversals are prevalent across different market states. I test these implications empirically and find strong support to the model.

My model applies Daniel, Hirshleifer, and Subrahmanyam (1998)'s mechanism of overconfidence and self-attribution bias into a setting of multiple risky assets of correlated payoffs. The multi-asset setting makes it feasible to derive momentum profits and market states (returns) from the model and examine the relation between them. Self-attribution bias and overconfidence are two closely-related and well-known psychological phenomena. According to Bem (1965), Wolosin, Sherman, and Till (1973), Langer and Roth (1975), Miller and Ross (1975), Fischhoff (1982), among others, people bear self-attribution bias: they tend to attribute success to their own abilities and attribute failure to external reasons. Gervais and Odean (2001) show that investors can become overconfident when learning with self-attribution bias.

As in Daniel, Hirshleifer, and Subrahmanyam (1998) and Gervais and Odean (2001), informed investors in my model can be overconfident about their private information. With self-attribution bias, investors' overconfidence increases when public signals confirm the private information. When public signals disconfirm the private information, investors' overconfidence is dampened, but not completely. A public signal confirms (disconfirms) a private signal when they have the same sign (opposite signs).

In the model, informed investors receive private signals about asset payoffs. Assets with positive private information become winners in the ranking period and assets with negative private information become losers in the ranking period. The payoffs of all assets are correlated through a common factor, and there are public signals about the common factor available in the economy. These public signals play two important roles. First and intuitively, they drive the market movement: an UP market reflects a positive public signal while a DOWN market reflects a negative public signal. Second, they affect investors' overconfidence by confirming or disconfirming private signals. As a result, overconfidence is correlated with market states, which leads to the relation between momentum profits and market states.

The model's mechanism can be understood as follows. Under UP/UP states, since the first positive public signal confirms positive private information and disconfirms negative private information, investors become overconfident about winners and are not overconfident about losers. The second positive public signal further increases overconfidence about winners but does not change confidence about losers. As a result, the abnormal return of winners is positive while that of losers is zero, and the momentum profit is positive and comes mainly from the winner side. Under UP/DOWN states, the second negative public signal reduces overconfidence about winners by disconfirming positive private information, and generates overconfidence about losers by confirming negative private information. As a result, the abnormal returns of both winners and losers are negative and the momentum profit is ambiguous.

Under DOWN/DOWN states, since the first negative public signal confirms negative private information and disconfirms positive private information, investors become overconfident about losers and are not overconfident about winners. The second negative public signal further increases overconfidence about losers but does not change confidence about winners. As a result, the abnormal return of losers is negative while that of winners is zero, and the momentum profit is positive and comes mainly from the loser side. Under DOWN/UP states, the second positive public signal reduces overconfidence about losers by disconfirming negative private information, and generates overconfidence about winners by confirming positive private information. As a result, the abnormal returns of both winners and losers are positive and the momentum profit is ambiguous.

The model also predicts that long-run reversals are prevalent under different market states. The intuition is straightforward: there is earlier overconfidence for either winners or losers or both, under all four market states. Pricing errors from earlier overconfidence are always corrected in the long run, which leads to long-run reversals.

Taken together, the model generates the following predictions about returns on the momentum portfolio: (1) under UP/UP and DOWN/DOWN market states, momentum abnormal returns are positive; under UP/DOWN and DOWN/UP market states, momentum abnormal returns are ambiguous; (2) under all four market states, long-run abnormal returns of a momentum portfolio are negative. The asymmetry between winners and losers translates to a distinctive set of predictions regarding asymmetric return behaviors between winners and losers in the short run: (3a) under UP/UP market states, abnormal returns are positive for winners and zero for losers; (3b) under DOWN/DOWN market states, abnormal returns are negative for losers and zero for winners; (3c) under UP/DOWN market states, abnormal returns are negative for both winners and losers; (3d) under DOWN/UP market states, abnormal returns are positive for both winners and losers.

As argued by Grundy and Martin (2001) and Daniel and Moskowitz (2013), time-varying risk (market beta) also contributes to the relation between market states and momentum profits. Hence, it is necessary to adjust for time-varying risk to test the model's implications. The literature has cautioned about potential underconditioning and overconditioning biases in estimating momentum alpha when researchers use an inappropriate proxy for conditional beta under the conditional CAPM. To mitigate these two biases, I employ the instrumental-variables-based (IV-based) conditional CAPM (CCAPM) framework proposed in Boguth, Carlson, Fisher, and Simutin (2011), and use profits adjusted by this model (referred to as CCAPM-adjusted profits throughout) as the main evidence to test the model.

The model's predictions find strong support in the data. From January 1929 to December 2010, the mean monthly CCAPM-adjusted momentum profit is 1.32%, 1.71%, 1.21%, and −0.97%, under UP/UP, DOWN/DOWN, UP/DOWN, and DOWN/UP states, respectively, consistent with prediction (1).[2] Supporting prediction (2), the mean monthly CCAPM-adjusted long-run return of the momentum portfolio is uniformly negative under all four

[2] As a comparison, the mean monthly raw momentum profit is 1.37%, 5.38%, 1.07%, and -4.84%, under UP/UP, DOWN/DOWN, UP/DOWN, and DOWN/UP states, respectively. Consistent with Daniel and Moskowitz (2013), I also find that time-varying risk contributes to the negative momentum profit under DOWN/UP states significantly.

market states. It is -0.41%, -0.86%, -0.48%, and -0.50% under UP/UP, DOWN/DOWN, UP/DOWN, and DOWN/UP states, respectively.

As for the asymmetry between winners and losers, the results of CCAPM-adjusted winner and loser returns are largely consistent with prediction (3). Under UP/UP states, the mean monthly CCAPM-adjusted return for winners is 1.31%, while that for losers is -0.01%. Under DOWN/DOWN states, the mean monthly CCAPM-adjusted return for winners is 0.41%, while that for losers is -1.31%. Under UP/DOWN states, the mean monthly CCAPM-adjusted return for winners is -0.05%, while that for losers is -1.26%. Under DOWN/UP states, the mean monthly CCAPM-adjusted return for winners is 0.92% while that for losers is 1.89%. A calibration exercise shows that the model can match empirical patterns of abnormal returns of winners and losers across different market states, with reasonable parameters.

As illustrated by the calibration exercise, the model also offers an explanation to the difference in momentum profits between UP/DOWN and DOWN/UP states, i.e. the positive momentum profit under UP/DOWN states and the negative momentum profit under DOWN/UP states. The intuition is as follows. Numerous studies including Daniel, Hirshleifer, and Subrahmanyam (1998, 2001) and Zhang (2006) show that overconfidence is more amplified when information uncertainty is higher. Since stock market volatility is higher in a DOWN market than in an UP market, we expect that overconfidence is more amplified in a DOWN market than in an UP market. Under UP/DOWN states, the amplifying effect of information uncertainty increases confidence over the holding period. This offsets confidence decrease for winners and strengthens confidence increase for losers, and as a result the momentum profit is positive. Under DOWN/UP states, the amplifying effect of information uncertainty decreases confidence over the holding period. This strengthens confidence decrease for losers and offsets confidence increase for winners, and as a result the momentum profit is negative.

This paper contributes to the literature along several dimensions. First, the paper contributes to the literature on market states and momentum and more broadly to the vast literature on momentum. To the best of my knowledge, my model is the first to formally connect market states and momentum profits, and is able to explain a broad set of empirical patterns. The paper is also the first to uncover the asymmetry between winners and losers across different market states, both theoretically and empirically.

This paper also unifies three return patterns of momentum portfolios, i.e. momentum, negative momentum profits under certain market states (referred to as “momentum crashes” by Daniel and Moskowitz 2013), and long-run reversals. It shows that these patterns can arise from a uniform driving force, time-varying overconfidence. Intuitively, increases in confidence in the short run lead to momentum and decreases in confidence in the short run lead to negative momentum profits; the eventual correction of earlier overconfidence gives rise to long-run reversals. To this end, this paper goes along the direction of Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer, and Subrahmanyam (1998), and Hong and Stein (1999), of explaining anomalous patterns in financial markets in a unified way, with a small number of well-documented behavioral features.

This paper also reconciles a seeming inconsistency within the overreaction framework. Cooper, Gutierrez, and Hameed (2004) document that after past DOWN market states, long-run reversals exist despite negative (although insignificant) short-run momentum profits. This pattern has been interpreted as inconsistent with two overreaction models—Daniel, Hirshleifer, and Subrahmanyam (1998) and Hong and Stein (1999)—because if long-run reversals are the outcome of a correction of earlier overreaction, then one might expect to always observe long-run reversals and short-run momentum together. My model shows that this is not necessarily the case. When there is overconfidence (and overreaction) prior to the short-run holding period and this overconfidence gets corrected gradually over the short-run and long-run holding periods, we would observe both negative momentum profits and long-run reversals. Hence, the pattern documented by Cooper, Gutierrez, and Hameed (2004) is not inconsistent with an overreaction/overconfidence model. In fact, my model predicts, and subsequent empirical analysis confirms, that under DU market states, long-run reversals follow significant negative momentum profits.

The remainder of this paper is organized as follows. Section 2 presents the model. Section 3 derives testable hypotheses from the model. Section 4 carries out empirical analysis to test these hypotheses. Section 5 presents a calibration exercise and further discussion. Section 6 concludes.

2 The Model

2.1 Setup

The model applies Daniel, Hirshleifer, and Subrahmanyam (1998)'s mechanism of overconfidence into a setting of multiple risky assets with correlated payoffs. This extension makes it feasible to derive market states and momentum profits from the model, and also captures the fact that asset payoffs depend on both common and firm-specific factors.

The following ingredients are borrowed from Daniel, Hirshleifer, and Subrahmanyam (1998). First, there are four dates: $t = 0, 1, 2, 3$. There is a risk-free asset, and the risk-free rate is zero. Second, there are two groups of investors, the informed denoted by I, and the uninformed denoted by U. The informed are risk neutral and receive private signals; the uninformed are risk averse and do not receive private signals. Third, private signals arrive at $t = 1$ and informed investors determine their confidence about private signals through self-attribution bias. Informed investors' confidence changes from $t = 1$ to $t = 2$, as public signals confirm or disconfirm private signals. Informed investors are rational about public signals that are received by all investors. Conclusive public information arrives at $t = 3$ and resolves uncertainty.

Unlike Daniel, Hirshleifer, and Subrahmanyam (1998), in my model, there are N risky assets indexed by $i = 1, 2, ..., N$. I consider the case where N is sufficiently large. Each risky asset i liquidates at $t = 3$ and pays a dividend θ_i.

The terminal payoff θ_i of each asset i consists of a common factor and a firm-specific component:[3]

$$\theta_i = f + \eta_i, \ \ \forall i = 1, 2, ..., N, \tag{1}$$

where f is the liquidation value of the common factor and is normally distributed: $f \sim \mathcal{N}(0, 2\sigma_f^2)$; η_i is the liquidation value of the firm-specific component and is i.i.d. and normally distributed: $\eta_i \sim \mathcal{N}(0, \sigma_\eta^2)$; f and each η_i are independent.[4] Containing f as a common component, the payoffs of any two assets i and j are correlated. The prior distribution of θ_i is normal: $\theta_i \sim \mathcal{N}(0, \sigma_\theta^2)$, where $\sigma_\theta^2 = \sigma_f^2 + \sigma_\eta^2$. Without loss of generality, the mean of θ_i is set to zero.[5]

Due to risk neutrality of informed investors and a zero risk-free rate, asset prices are derived as conditional expectation of terminal asset payoffs, based on the beliefs of informed investors.

2.2 Information Structure

At $t = 1$, there is a private signal s_i about each terminal payoff θ_i:

$$s_i = f + \eta_i + \epsilon_f + \epsilon_i, \ \ \forall i = 1, 2, ..., N, \tag{2}$$

where $\epsilon_f \sim \mathcal{N}(0, \sigma_{\epsilon f}^2)$ is a common noise and $\epsilon_i \sim \mathcal{N}(0, \sigma_\epsilon^2)$ is a signal-specific noise. ϵ_i is i.i.d. and is independent of f and η_i. In total, there are N signals $s_{i=1,2,...,N}$. Informed investors receive these N private signals. The existence of the common noise, ϵ_f, prevents informed

[3] For simplicity, I assume cash-flow betas of all N assets to be 1. I have verified that in a setting with cash-flow betas different across assets, i.e. $\theta_i = \beta_i f + \eta_i$, $\forall i = 1, 2, ..., N$, where $\beta_i > 0$, similar results would be obtained.

[4] Expressing the variance of f as $2\sigma_f^2$ instead of σ_f^2 is for notational convenience only, which will become clear shortly.

[5] This is also consistent with Daniel, Hirshleifer, and Subrahmanyam (1998), where the terminal value θ of the risky asset is normally distributed with mean zero.

investors from learning f perfectly when N goes to infinity.[6] I define the average of the N private signals as:

$$\bar{s} = \frac{1}{N}\sum_{i=1}^{N} s_i. \tag{3}$$

There are also public signals about the common factor f. These public signals are state variables in the model and play two roles. First, they affect confidence about private signals. Second, they drive returns of every asset and therefore drive market returns. Intuitively, we expect that market returns in different periods are driven by information about different components of f. Thus motivated, I decompose f into $f = f_1 + f_2$, where f_1 and f_2 are i.i.d. and follow $f_1 \sim \mathcal{N}(0, \sigma_f^2)$ and $f_2 \sim \mathcal{N}(0, \sigma_f^2)$. One way to interpret f_1 and f_2 is that they are earnings generated by the common factor in period 1 ($t = 0$ to $t = 1$) and period 2 ($t = 1$ to $t = 2$), respectively, and are distributed a date 3 when the asset is liquidated.

For simplicity and symmetry, I assume that there is a public signal at $t = 1$ about f_1:

$$s_{f,1} = f_1 + \xi_{f,1}, \tag{4}$$

where $\xi_{f,1} \sim \mathcal{N}(0, \sigma_\xi^2)$ is the noise of $s_{f,1}$, and a public signal at $t = 2$ about f_2:

$$s_{f,2} = f_2 + \xi_{f,2}, \tag{5}$$

where $\xi_{f,2} \sim \mathcal{N}(0, \sigma_\xi^2)$ is the noise of $s_{f,2}$.

2.3 Self-Attribution Bias and Asymmetric Overconfidence

As in Daniel, Hirshleifer, and Subrahmanyam (1998), I implement overconfidence as overestimation of the precision of private signals or, equivalently, underestimation of noise variances of private signals. Specifically, while the true variance of ϵ_i is σ_ϵ^2, investors perceive the variance of ϵ_i as $\sigma_{t,i}^2 \leq \sigma_\epsilon^2$ at $t = 1$ and 2. Throughout, confidence about a private signal s_i at $t = 1, 2$ is characterized by $\sigma_{t,i}^2$. A lower $\sigma_{t,i}^2$ corresponds to a higher degree of overconfidence.

[6] See a related discussion in Hughes, Liu, and Liu (2007).

With self-attribution bias, informed investors determine confidence about a private signal based on whether it is confirmed by public signals. When a public signal confirms the private signal, informed investors' confidence increases. When a public signal disconfirms the private signal, informed investors' confidence decreases, but to a smaller extent. For tractability, I implement the following intuitive belief updating rule:

$$\sigma_{t,i}^2 = min((k_+)^{n_+}(k_-)^{n_-}\sigma_\epsilon^2,\ \sigma_\epsilon^2),\quad t = 1, 2, \tag{6}$$

where $k_+ < 1$ is the degree to which confidence is increased by a confirming public signal, $k_- > 1$ is the degree to which confidence is decreased by a disconfirming public signal, n_+ is the number of times that s_i has been confirmed until t, and n_- is the number of times that s_i has been disconfirmed until t. With self-attribution bias, the effect of a confirming public signal does not get offset completely by a disconfirming public signal. Accordingly, we have $k_+k_- < 1$. The minimum operator in Equation (6) leads to $\sigma_{t,i}^2 \leq \sigma_\epsilon^2$, in accordance with my setting of overconfident investors.

From Equation (6), when there have been no confirming public signals, $\sigma_{t,i}^2 = \sigma_\epsilon^2$ and investors remain rational. This is consistent with the specification of self-attribution bias in Gervais and Odean (2001). Note that by setting $k_+ = k_- = 1$ in Equation (6), the channel of self-attribution bias and overconfidence is turned off and we obtain the rational-expectations counterpart of the model.

Since all private signals are also about f_1 and f_2, they can be confirmed or disconfirmed by public signals $s_{f,1}$ and $s_{f,2}$. As in Daniel, Hirshleifer, and Subrahmanyam (1998), a public signal confirms a private signal s_i when they have the same sign and disconfirms s_i when they have different signs. Applying the updating rule in Equation (6) at $t = 1$, we can determine endogenously the confidence at $t = 1$ as:

$$\sigma_{1,i}^2 = \begin{cases} k_+\sigma_\epsilon^2 < \sigma_\epsilon^2 & \text{if } sign(s_{f,1}) = sign(s_i) \\ \sigma_\epsilon^2 & \text{if } sign(s_{f,1}) \neq sign(s_i) \end{cases},\quad \forall i = 1, 2, ..., N. \tag{7}$$

Similarly, applying the updating rule in Equation (6) at $t = 2$, we can determine endogenously the confidence at $t = 2$ as:

$$\sigma_{2,i}^2 = \begin{cases} k_+^2 \sigma_\epsilon^2 < \sigma_\epsilon^2 & \text{if } sign(s_{f,1}) = sign(s_i) \ \ \& \ \ sign(s_{f,2}) = sign(s_i) \\ k_+ k_- \sigma_\epsilon^2 < \sigma_\epsilon^2 & \text{if } sign(s_{f,1}) = sign(s_i) \ \ \& \ \ sign(s_{f,2}) \neq sign(s_i) \\ k_+ k_- \sigma_\epsilon^2 < \sigma_\epsilon^2 & \text{if } sign(s_{f,1}) \neq sign(s_i) \ \ \& \ \ sign(s_{f,2}) = sign(s_i) \\ \sigma_\epsilon^2 & \text{if } sign(s_{f,1}) \neq sign(s_i) \ \ \& \ \ sign(s_{f,2}) \neq sign(s_i) \end{cases} . \tag{8}$$

As shown in Section 2.3.1, Equations (7) and (8) generate the asymmetric overconfidence between winners and losers, which is the key driving force of my model and is absent in Daniel, Hirshleifer, and Subrahmanyam (1998).

2.3.1 Definition of Winners and Losers, and Asymmetric Overconfidence

Winners and losers are defined in this section.[7] In accordance with empirical analysis, the number of winners and losers is the same and is denoted by N_{wl}. For a given $f + \epsilon_f$, the cross-sectional distribution of s_i is normal with mean $f + \epsilon_f$ and variance $\sigma_\eta^2 + \sigma_\epsilon^2$. Figure 1 illustrates this distribution for the case of a positive $f + \epsilon_f$ (Panel A) and the case of a negative $f + \epsilon_f$ (Panel B), respectively. Without loss of generality, I assume that the values of the N signals are ranked as $s_1 < s_2 < s_3 < ... < s_{N-2} < s_{N-1} < s_N$.

Intuitively, winners and losers should have s_i at the two ends of the cross-sectional distribution. I define N_{wl} as $N_{wl} = min[N_p, N_n]$,where N_p is the number of assets with positive s_i and N_n is the number of assets with negative s_i. Therefore, N_{wl} is the minimum of N_p and N_n.

I then define assets $i = 1, 2, ..., N_{wl}$ as losers and assets $i = N - N_{wl} + 1, N - N_{wl} + 2, ..., N$ as winners. Throughout, I use w to denote a winner and l to denote a loser; $i \in w$ indicates that asset i is a winner and $i \in l$ indicates that asset i is a loser. The shaded areas in Figure

[7] As shown in Appendix B.1, in the model winners have higher ranking-period returns than losers, consistent with empirical convention.

1 illustrate the definition of winners and losers. When $f + \epsilon_f$ is positive (Panel A), $N_p > N_n$ and $N_{wl} = N_n$. When $f + \epsilon_f$ is negative (Panel B), $N_p < N_n$ and $N_{wl} = N_p$.

With this definition of winners and losers, we have:

$$s_w > 0, \ s_l < 0, \tag{9}$$

which states that winners have positive signals and losers have negative signals.

Since winners have positive signals and losers have negative signals, confidence is asymmetric between winners and losers, by Equations (7) and (8). For example, when $s_{f,1}$ is positive, the perceived variance at $t = 1$ is $\kappa_+ \sigma_\epsilon^2$ for winners and σ_ϵ^2 for losers. In other words, a positive $s_{f,1}$ confirms positive s_i and makes investors overconfident about winners; it simultaneously disconfirms negative s_i and keeps investors rational about losers.

2.4 Asset Prices and Returns

At $t = 0$, the price of risky asset i, $P_{0,i}$, is simply the prior mean of θ_i:

$$P_{0,i} = \mathbb{E}[\theta_i] = 0, \quad \forall i = 1, 2, ..., N. \tag{10}$$

The price of asset i at $t = 1$, $P_{1,i}$, is the conditional expectation of θ_i at $t = 1$:

$$P_{1,i} = \mathbb{E}_{C,1}[\theta_i | s_1, s_2, ..., s_N, s_{f,1}], \quad \forall i = 1, 2, ..., N, \tag{11}$$

where $\mathbb{E}_{C,1}$ denotes the expectation taken with the perceived variance in Equation (7). The detailed formula of $P_{1,i}$ is in Appendix A.1.

Similarly, the price of asset i at $t = 2$, $P_{2,i}$, is the conditional expectation of θ_i at $t = 2$:

$$P_{2,i} = \mathbb{E}_{C,2}[\theta_i | s_1, s_2, ..., s_N, s_{f,1}, s_{f,2}], \quad \forall i = 1, 2, ..., N, \tag{12}$$

where $\mathbb{E}_{C,2}$ denotes the expectation taken with the perceived variance in Equation (8). The detailed formula of $P_{2,i}$ is in Appendix A.2.

At $t = 3$, there is conclusive public information about θ_i. The price of asset i, $P_{3,i}$, is:

$$P_{3,i} = \theta_i = f + \eta_i, \quad \forall i = 1, 2, ..., N. \tag{13}$$

For tractability, I analyze the case where the variance of the common noise, $\sigma^2_{\epsilon f}$, is sufficiently large. As shown in Appendices A.1 and A.2, the formulas of $P_{1,i}$ and $P_{2,i}$ can then be simplified as:

$$P_{1,i} = \frac{\sigma_f^2}{\sigma_f^2 + \sigma_\xi^2} s_{f,1} + \frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_{1,i}^2}(s_i - \bar{s}), \quad \forall i = 1, 2, ..., N, \tag{14}$$

and

$$P_{2,i} = \frac{\sigma_f^2}{\sigma_f^2 + \sigma_\xi^2}(s_{f,1} + s_{f,2}) + \frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_{2,i}^2}(s_i - \bar{s}), \quad \forall i = 1, 2, ..., N. \tag{15}$$

Equations (14) and (15) are intuitive. Prices contain two terms: the first term arises from price reaction to public signals, and the second term arises from price reaction to private signals. Since I do not model the cross-sectional difference in cash-flow betas, price reaction to public signals is the same across all N assets.

It is convenient to define returns as price changes, as in Daniel, Hirshleifer, and Subrahmanyam (1998, 2001) and Easley and O'Hara (2004). The time period $t = 0$ to $t = 1$ in the model corresponds to the ranking period of momentum strategies in empirical analysis during which winners and losers are identified. The ranking-period return is:

$$\begin{aligned}\Delta P_{1,i} &= P_{1,i} - P_{0,i} \\ &= \frac{\sigma_f^2}{\sigma_f^2 + \sigma_\xi^2} s_{f,1} + \frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_{1,i}^2}(s_i - \bar{s}), \quad \forall i = 1, 2, ..., N.\end{aligned} \tag{16}$$

The time period $t = 1$ to $t = 2$ corresponds to the short-run holding period in empirical analysis. The short-run return is:

$$\begin{aligned}\Delta P_{2,i} &= P_{2,i} - P_{1,i} \\ &= \frac{\sigma_f^2}{\sigma_f^2 + \sigma_\xi^2} s_{f,2} + \frac{\sigma_\eta^2(\sigma_{1,i}^2 - \sigma_{2,i}^2)}{(\sigma_\eta^2 + \sigma_{1,i}^2)(\sigma_\eta^2 + \sigma_{2,i}^2)}(s_i - \bar{s}), \quad \forall i = 1, 2, ..., N. \end{aligned} \tag{17}$$

The time period $t = 2$ to $t = 3$ corresponds to the long-run holding period in empirical analysis. The long-run return is:

$$\begin{aligned}\Delta P_{3,i} &= P_{3,i} - P_{2,i} \\ &= f + \eta_i - \frac{\sigma_f^2}{\sigma_f^2 + \sigma_\xi^2}(s_{f,1} + s_{f,2}) - \frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_{2,i}^2}(s_i - \bar{s}), \quad \forall i = 1, 2, ..., N. \end{aligned} \tag{18}$$

2.5 Market States

The lagged market return MKT_1 is defined as the (equally weighted) market return in the ranking period:

$$MKT_1 = \frac{1}{N}\sum_i \Delta P_{1,i}. \tag{19}$$

The contemporaneous market return MKT_2 is defined as the market return in the short-run holding period:

$$MKT_2 = \frac{1}{N}\sum_i \Delta P_{2,i}. \tag{20}$$

Let $\sigma_{1,w}^2$ and $\sigma_{1,l}^2$ denote $\sigma_{1,i}^2$ in Equation (7) for winners and losers, respectively. Let $\sigma_{2,w}^2$ and $\sigma_{2,l}^2$ denote $\sigma_{2,i}^2$ in Equation (8) for winners and losers, respectively. Let $A = \frac{\sigma_\eta^2}{N_{wl}}\sum_{i\in w}(s_i - \bar{s}) = -\frac{\sigma_\eta^2}{N_{wl}}\sum_{i\in l}(s_i - \bar{s}) > 0.$[8] We then have the following proposition.

Proposition 1. *Lagged and contemporaneous market returns are given by:*

$$MKT_1 = \frac{\sigma_f^2}{\sigma_f^2 + \sigma_\xi^2} s_{f,1} + \frac{\sigma_{1,l}^2 - \sigma_{1,w}^2}{(\sigma_\eta^2 + \sigma_{1,w}^2)(\sigma_\eta^2 + \sigma_{1,l}^2)} \frac{N_{wl}}{N} A. \tag{21}$$

[8] A is positive due to $s_w > \bar{s} > s_l$.

and

$$MKT_2 = \frac{\sigma_f^2}{\sigma_f^2 + \sigma_\xi^2} s_{f,2} + [\frac{\sigma_{1,w}^2 - \sigma_{2,w}^2}{(\sigma_\eta^2 + \sigma_{1,w}^2)(\sigma_\eta^2 + \sigma_{2,w}^2)} - \frac{\sigma_{1,l}^2 - \sigma_{2,l}^2}{(\sigma_\eta^2 + \sigma_{1,l}^2)(\sigma_\eta^2 + \sigma_{2,l}^2)}] \frac{N_{wl}}{N} A. \tag{22}$$

Proof: See Appendix B.2.

Intuitively, market returns contain two terms: the first term arises from price reaction to public signals, and the second term arises from the feedback effect of overconfidence. There are four cases of public signals: $s_{f,1} > 0$ and $s_{f,2} > 0$, $s_{f,1} < 0$ and $s_{f,2} < 0$, $s_{f,1} > 0$ and $s_{f,2} < 0$, and $s_{f,1} < 0$ and $s_{f,2} > 0$, and market states are different across the four cases. It is straightforward to calculate $\sigma_{1,w}^2$, $\sigma_{1,l}^2$, $\sigma_{2,w}^2$, and $\sigma_{2,l}^2$ in these four cases, and the results are shown in Table 1. Substituting these perceived variances into Equations (21) and (22) gives the market states in four cases. Throughout, I refer to a positive (negative) market return as an UP (a DOWN) market state. Recall that two parameters of self-attribution bias are $k_+ < 1$ and $k_- > 1$ with $k_+ k_- < 1$. Panels A to D of Figure 2 illustrate the market price movement for the four cases of public signals below.

2.5.1 Case I: $s_{f,1} > 0$ and $s_{f,2} > 0$

In this case, two market returns are:

$$MKT_1 = \frac{\sigma_f^2}{\sigma_f^2 + \sigma_\xi^2} s_{f,1} + (1 - k_+) \frac{\sigma_\epsilon^2}{(\sigma_\eta^2 + k_+ \sigma_\epsilon^2)(\sigma_\eta^2 + \sigma_\epsilon^2)} \frac{N_{wl}}{N} A > 0, \tag{23}$$

and

$$MKT_2 = \frac{\sigma_f^2}{\sigma_f^2 + \sigma_\xi^2} s_{f,2} + k_+(1 - k_+) \frac{\sigma_\epsilon^2}{(\sigma_\eta^2 + k_+ \sigma_\epsilon^2)(\sigma_\eta^2 + k_+^2 \sigma_\epsilon^2)} \frac{N_{wl}}{N} A > 0. \tag{24}$$

The lagged market return is positive due to the positive $s_{f,1}$ and overconfidence about winners in the ranking period. The contemporaneous market return is positive due to the positive $s_{f,2}$ and continuing overconfidence about winners in the short-run holding period. Market states are UP/UP in this case.

2.5.2 Case II: $s_{f,1} < 0$ and $s_{f,2} < 0$

In this case, two market returns are:

$$MKT_1 = \frac{\sigma_f^2}{\sigma_f^2 + \sigma_\xi^2} s_{f,1} - (1 - k_+) \frac{\sigma_\epsilon^2}{(\sigma_\eta^2 + k_+ \sigma_\epsilon^2)(\sigma_\eta^2 + \sigma_\epsilon^2)} \frac{N_{wl}}{N} A < 0, \tag{25}$$

and

$$MKT_2 = \frac{\sigma_f^2}{\sigma_f^2 + \sigma_\xi^2} s_{f,2} - k_+ (1 - k_+) \frac{\sigma_\epsilon^2}{(\sigma_\eta^2 + k_+ \sigma_\epsilon^2)(\sigma_\eta^2 + k_+^2 \sigma_\epsilon^2)} \frac{N_{wl}}{N} A < 0. \tag{26}$$

The lagged market return is negative due to the negative $s_{f,1}$ and overconfidence about losers in the ranking period. The contemporaneous market return is negative due to the negative $s_{f,2}$ and continuing overconfidence about losers in the short-run holding period. Market states are DOWN/DOWN in this case.

2.5.3 Case III: $s_{f,1} > 0$ and $s_{f,2} < 0$

In this case, two market returns are:

$$MKT_1 = \frac{\sigma_f^2}{\sigma_f^2 + \sigma_\xi^2} s_{f,1} + (1 - k_+) \frac{\sigma_\epsilon^2}{(\sigma_\eta^2 + k_+ \sigma_\epsilon^2)(\sigma_\eta^2 + \sigma_\epsilon^2)} \frac{N_{wl}}{N} A > 0.$$

$$MKT_2 = \frac{\sigma_f^2}{\sigma_f^2 + \sigma_\xi^2} s_{f,2} + [(k_+ - k_+ k_-) \frac{1}{\sigma_\eta^2 + k_+ \sigma_\epsilon^2} - (1 - k_+ k_-) \frac{1}{\sigma_\eta^2 + \sigma_\epsilon^2}] \frac{\sigma_\epsilon^2}{\sigma_\eta^2 + k_+ k_- \sigma_\epsilon^2} \frac{N_{wl}}{N} A < 0. \tag{27}$$

The lagged market return is positive due to the positive $s_{f,1}$ and overconfidence about winners in the ranking period. The contemporaneous market return is negative due to the negative $s_{f,2}$ and decreasing overconfidence about winners and increasing overconfidence about losers in the short-run holding period. Market states are UP/DOWN in this case.

2.5.4 Case IV: $s_{f,1} < 0$ and $s_{f,2} > 0$

In this case, two market returns are:

$$MKT_1 = \frac{\sigma_f^2}{\sigma_f^2 + \sigma_\xi^2} s_{f,1} - (1 - k_+) \frac{\sigma_\epsilon^2}{(\sigma_\eta^2 + k_+ \sigma_\epsilon^2)(\sigma_\eta^2 + \sigma_\epsilon^2)} \frac{N_{wl}}{N} A < 0.$$

$$MKT_2 = \frac{\sigma_f^2}{\sigma_f^2 + \sigma_\xi^2} s_{f,2} + [(1 - k_+ k_-) \frac{1}{\sigma_\eta^2 + \sigma_\epsilon^2} - (k_+ - k_+ k_-) \frac{1}{\sigma_\eta^2 + k_+ \sigma_\epsilon^2}] \frac{\sigma_\epsilon^2}{\sigma_\eta^2 + k_+ k_- \sigma_\epsilon^2} \frac{N_{wl}}{N} A > 0. \tag{28}$$

The lagged market return is negative due to the negative $s_{f,1}$ and overconfidence about losers in the ranking period. The contemporaneous market return is positive due to the positive $s_{f,2}$ and decreasing overconfidence about losers and increasing overconfidence about winners in the short-run holding period. Market states are DOWN/UP in this case.

The following proposition summarizes the four cases above.

Proposition 2. Public Signals and Market States. *There are four cases of public signals:* $s_{f,1} > 0$ *and* $s_{f,2} > 0$, $s_{f,1} < 0$ *and* $s_{f,2} < 0$, $s_{f,1} > 0$ *and* $s_{f,2} < 0$, *and* $s_{f,1} < 0$ *and* $s_{f,2} > 0$. *There are four market states: UP/UP (UU), DOWN/DOWN (DD), UP/DOWN (UD), and DOWN/UP (DU). There is a one-to-one correspondence between the four cases of public signals and the four market states. Market states are UU in Case I, DD in Case II, UD in Case III, and DU in Case IV.*

2.6 Abnormal Returns of the Winner, Loser, and Momentum Portfolios

Throughout, I refer to returns from the channel of self-attribution bias and overconfidence as abnormal returns. I use WIN, LOS, and MOM to denote abnormal returns of the winner, loser, and momentum portfolios, respectively. WIN is given as the average abnormal return of winner stocks:

$$WIN = \frac{1}{N_{wl}} \sum_{i \in w} (\Delta P_{2,i} - \Delta P_{2,i}^R), \tag{29}$$

where $\Delta P_{2,i}^R$ is the short-run return of asset i when investors are rational, i.e. setting $\sigma_{1,i}^2 = \sigma_{2,i}^2 = \sigma_\epsilon^2$ in Equation (17).

Similarly, LOS is given as the average abnormal return of loser stocks:

$$LOS = \frac{1}{N_{wl}} \sum_{i \in l} (\Delta P_{2,i} - \Delta P_{2,i}^R). \tag{30}$$

The momentum portfolio is formed as buying winners and shorting losers, and accordingly MOM is:

$$MOM = WIN - LOS. \tag{31}$$

With $A = \frac{\sigma_\eta^2}{N_{wl}} \sum\limits_{i \in w} (s_i - \bar{s}) = -\frac{\sigma_\eta^2}{N_{wl}} \sum\limits_{i \in l} (s_i - \bar{s}) > 0$ given earlier, the following proposition results.

Proposition 3. *Abnormal returns of the winner, loser, and momentum portfolios are given by:*

$$WIN = \frac{\sigma_{1,w}^2 - \sigma_{2,w}^2}{(\sigma_\eta^2 + \sigma_{1,w}^2)(\sigma_\eta^2 + \sigma_{2,w}^2)} A, \tag{32}$$

$$LOS = \frac{\sigma_{1,l}^2 - \sigma_{2,l}^2}{(\sigma_\eta^2 + \sigma_{1,l}^2)(\sigma_\eta^2 + \sigma_{2,l}^2)} A, \tag{33}$$

and

$$MOM = \frac{\sigma_{1,w}^2 - \sigma_{2,w}^2}{(\sigma_\eta^2 + \sigma_{1,w}^2)(\sigma_\eta^2 + \sigma_{2,w}^2)} A - \frac{\sigma_{1,l}^2 - \sigma_{2,l}^2}{(\sigma_\eta^2 + \sigma_{1,l}^2)(\sigma_\eta^2 + \sigma_{2,l}^2)} A. \tag{34}$$

Proof: See Appendix B.3.

Intuitively, abnormal returns depend on two terms characterizing time-varying confidence for winners and losers: $\sigma_{1,w}^2 - \sigma_{2,w}^2$ and $\sigma_{1,l}^2 - \sigma_{2,l}^2$. I then write the following corollary.

Corollary 1. *When the channel of self-attribution bias and overconfidence is turned off, abnormal returns of the winner, loser, and momentum portfolios are zero.*

Investors' confidence is different across the four cases of public signals. Hence, abnormal returns are different across these four cases, or equivalently, across the four market states according to Proposition 2. Panels A to D in Figure 2 illustrate the cross-sectional difference in price movement between winners and losers, in the four cases of public signals (under four market states). Below, I determine the signs of these abnormal returns and the results are summarized in Table 1.

2.6.1 Case I: $s_{f,1} > 0$ and $s_{f,2} > 0$ (UU market states)

In this case, abnormal returns are:

$$WIN = (1 - k_+)\frac{\sigma_\epsilon^2}{(\sigma_\eta^2 + k_+\sigma_\epsilon^2)(\sigma_\eta^2 + k_+^2\sigma_\epsilon^2)}A > 0, \tag{35}$$

$$LOS = 0, \tag{36}$$

and

$$MOM = (1 - k_+)\frac{\sigma_\epsilon^2}{(\sigma_\eta^2 + k_+\sigma_\epsilon^2)(\sigma_\eta^2 + k_+^2\sigma_\epsilon^2)}A > 0. \tag{37}$$

For winners, continuing overconfidence delivers positive abnormal returns. For losers, there is no change in confidence and therefore there are no abnormal returns. The momentum abnormal returns are positive.

2.6.2 Case II: $s_{f,1} < 0$ and $s_{f,2} < 0$ (DD market states)

In this case, abnormal returns are:

$$WIN = 0, \tag{38}$$

$$LOS = -(1 - k_+)\frac{\sigma_\epsilon^2}{(\sigma_\eta^2 + k_+\sigma_\epsilon^2)(\sigma_\eta^2 + k_+^2\sigma_\epsilon^2)}A < 0, \tag{39}$$

and

$$MOM = (1 - k_+) \frac{\sigma_\epsilon^2}{(\sigma_\eta^2 + k_+\sigma_\epsilon^2)(\sigma_\eta^2 + k_+^2\sigma_\epsilon^2)} A > 0. \tag{40}$$

For winners, there is no change in confidence and therefore there are no abnormal returns. For losers, continuing overconfidence delivers negative abnormal returns. The momentum abnormal returns are positive.

2.6.3 Case III: $s_{f,1} > 0$ and $s_{f,2} < 0$ (UD market states)

In this case, abnormal returns are:

$$WIN = k_+(1 - k_-) \frac{\sigma_\epsilon^2}{(\sigma_\eta^2 + k_+\sigma_\epsilon^2)(\sigma_\eta^2 + k_+k_-\sigma_\epsilon^2)} A < 0, \tag{41}$$

$$LOS = -(1 - k_+k_-) \frac{\sigma_\epsilon^2}{(\sigma_\eta^2 + \sigma_\epsilon^2)(\sigma_\eta^2 + k_+^2\sigma_\epsilon^2)} A < 0, \tag{42}$$

and

$$MOM = k_+(1 - k_-) \frac{\sigma_\epsilon^2}{(\sigma_\eta^2 + k_+\sigma_\epsilon^2)(\sigma_\eta^2 + k_+k_-\sigma_\epsilon^2)} A + (1 - k_+k_-) \frac{\sigma_\epsilon^2}{(\sigma_\eta^2 + \sigma_\epsilon^2)(\sigma_\eta^2 + k_+^2\sigma_\epsilon^2)} A \gtreqless 0. \tag{43}$$

For winners, decrease in confidence leads to negative abnormal returns. For losers, increase in confidence leads to negative abnormal returns. The momentum abnormal returns are ambiguous.

2.6.4 Case IV: $s_{f,1} < 0$ and $s_{f,2} > 0$ (DU market states)

In this case, abnormal returns are:

$$WIN = (1 - k_+k_-) \frac{\sigma_\epsilon^2}{(\sigma_\eta^2 + \sigma_\epsilon^2)(\sigma_\eta^2 + k_+k_-\sigma_\epsilon^2)} A > 0, \tag{44}$$

$$LOS = -k_+(1 - k_-) \frac{\sigma_\epsilon^2}{(\sigma_\eta^2 + k_+\sigma_\epsilon^2)(\sigma_\eta^2 + k_+k_-\sigma_\epsilon^2)} A > 0, \tag{45}$$

and

$$MOM = (1-k_+k_-)\frac{\sigma_\epsilon^2}{(\sigma_\eta^2+\sigma_\epsilon^2)(\sigma_\eta^2+k_+k_-\sigma_\epsilon^2)}A + k_+(1-k_-)\frac{\sigma_\epsilon^2}{(\sigma_\eta^2+k_+\sigma_\epsilon^2)(\sigma_\eta^2+k_+k_-\sigma_\epsilon^2)}A \gtreqless 0. \tag{46}$$

For winners, increase in confidence leads to positive abnormal returns. For losers, decrease in confidence leads to positive abnormal returns. The momentum abnormal returns are ambiguous.

The following two propositions summarize the four cases above.

Proposition 4. Momentum and Market States. *Under UU and DD market states, momentum abnormal returns are positive. Under UD and DU market states, momentum abnormal returns are ambiguous.*

Proposition 5. Asymmetry between Winners and Losers. *Under UU market states, abnormal returns are positive for winners and zero for losers. Under DD market states, abnormal returns are negative for losers and zero for winners. Under UD market states, abnormal returns are negative for both winners and losers. Under DU market states, abnormal returns are positive for both winners and losers.*

2.7 Long-Run Reversals

I use REV (standing for reversals) to denote the long-run abnormal return of the momentum portfolio. It is defined as:

$$REV = \frac{1}{N_{wl}}\sum_{i\in w}(\Delta P_{3,i} - \Delta P_{3,i}^R) - \frac{1}{N_{wl}}\sum_{i\in l}(\Delta P_{3,i} - \Delta P_{3,i}^R), \tag{47}$$

where $\Delta P_{3,i}^R$ is the long-run return of asset i when investors are rational, i.e. $\sigma_{1,i}^2 = \sigma_{2,i}^2 = \sigma_\epsilon^2$ in Equation (18), $\frac{1}{N_{wl}}\sum_{i\in w}(\Delta P_{3,i} - \Delta P_{3,i}^R)$ is the average long-run abnormal return of winner stocks and $\frac{1}{N_{wl}}\sum_{i\in l}(\Delta P_{3,i} - \Delta P_{3,i}^R)$ is the average long-run abnormal return of loser stocks. As shown in Appendix B.4, the formula of REV is:

$$REV = \frac{\sigma_{2,w}^2 - \sigma_\epsilon^2}{(\sigma_\eta^2 + \sigma_{2,w}^2)(\sigma_\eta^2 + \sigma_\epsilon^2)}A + \frac{\sigma_{2,l}^2 - \sigma_\epsilon^2}{(\sigma_\eta^2 + \sigma_{2,l}^2)(\sigma_\eta^2 + \sigma_\epsilon^2)}A. \tag{48}$$

Intuitively, the correction of overconfidence at date 2 in the long run, characterized by $\sigma_{2,w}^2 - \sigma_\epsilon^2$ and $\sigma_{2,l}^2 - \sigma_\epsilon^2$, leads to long-run reversals. From Equation (8), investors are overconfident about either winners or losers, or both of them, at date 2. In other words, either $\sigma_{2,w}^2 < \sigma_\epsilon^2$, or $\sigma_{2,l}^2 < \sigma_\epsilon^2$, or both. Therefore, the following proposition results.

Proposition 6. Long-Run Reversals and Market States. *Under all four market states, long-run abnormal returns of momentum portfolios are negative.*

This proposition is also summarized in Table 1.

2.8 Unifying Return Patterns of Momentum Portfolios

Propositions 4 and 6 unify momentum, negative momentum profits under certain market states, and long-run reversals through the mechanism of self-attribution bias and confidence. A popular view in the literature is that short-run momentum and long-run reversals should be always coupled within an overreaction framework, and we should not observe long-run reversals without short-run momentum. For example, Cooper, Gutierrez, and Hameed (2004) document that long-run reversals can exist without initial short-run momentum. They treat this phenomenon as evidence inconsistent with two overreaction models: Daniel, Hirshleifer, and Subrahmanyam (1998) and Hong and Stein (1999). Propositions 4 and 6 reconcile this inconsistency. The coexistence of negative momentum profits and long-run reversals is feasible within my model, when there is overconfidence prior to the short-run holding period, and this overconfidence gets corrected gradually over the short-run and long-run holding periods.

3 Testable Hypotheses

Propositions 4 to 6 can be directly converted into the following hypotheses for empirical tests.

Hypothesis 1. *Under UU and DD market states, momentum abnormal returns are positive. Under UD and DU market states, momentum abnormal returns are ambiguous.*

Hypothesis 2. *Under all four market states, long-run abnormal returns of momentum portfolios are negative.*

Hypothesis 3. *(a) Under UU market states, abnormal returns are positive for winners and zero for losers. (b) Under DD market states, abnormal returns are negative for losers and zero for winners. (c) Under UD market states, abnormal returns are negative for both winners and losers. (d) Under DU market states, abnormal returns are positive for both winners and losers.*

4 Empirical Analysis

In this section, I conduct empirical analysis to test the three hypotheses above.

4.1 Data and Methods

The data cover all NYSE and AMEX stocks listed on the CRSP monthly file. The sample period is from January 1926 to December 2010. At the end of each month $t-1$, stocks are sorted into deciles based on their returns over months $t-12$ to $t-2$. Following the literature on momentum, I construct a momentum portfolio as long in winners (stocks in the top decile) and short in losers (stocks in the bottom decile).

Following Fama and French (1996) and Grundy and Martin (2001), short-run momentum profits MOM_t are computed as the return of the momentum portfolio in month t, skipping month $t-1$ to mitigate bid-ask bounce effects. Similar to Lee and Swaminathan (2000), Jegadeesh and Titman (2001), and Cooper, Gutierrez, and Hameed (2004), long-run momentum profits REV_t are computed as the cumulative return of the momentum portfolio over the four-year holding period $t+12$ to $t+59$. This computation starts one year after the portfolio formation:

$$REV_t = \sum_{k=12}^{59} r_{t+k}, \tag{49}$$

where r_{t+k} is the return in month $t+k$ of the momentum portfolio formed at $t-1$.

4.1.1 Standard Risk Adjustment

Besides examining raw short-run and long-run momentum profits, I also analyze standard risk-adjusted profits. The CAPM and the Fama-French three-factor model are widely used risk benchmarks in the literature of empirical asset pricing. Nevertheless, it is arguably not useful to use the Fama-French-adjusted profits to test behavioral models, as pointed out by numerous researchers including Daniel, Hirshleifer, and Subrahmanyam (2001) and Cooper, Gutierrez, and Hameed (2004). The reason is that the cross-sectional return difference for stocks sorted according to book-to-market ratios can also arise within a behavioral framework through investor irrationality. In other words, the book-to-market ratio may represent a mispricing measure instead of a risk measure. I still include Fama-French-adjusted profits to ensure thoroughness.

I follow the procedure in Cooper, Gutierrez, and Hameed (2004) to obtain risk-adjusted momentum profits. For each holding-period month k ($k = 0$ for short-run profits and $k = 12$ to 59 for long-run profits), I perform the following time-series regression of raw profits on relevant factors and a constant:

$$r_{t+k} = \alpha_k + \sum_i \beta_{ik} f_{i,t+k} + \epsilon_{t+k}, \tag{50}$$

where r_{t+k} is the raw return in month $t+k$ of the momentum portfolio formed at the end of month $t-1$, and $f_{i,t+k}$ is the value of factor i in month $t+k$. For the CAPM, the single factor is the excess return of the CRSP value-weighted (VW) index over the one-month

T-bill rate. The Fama-French three-factor model involves two additional factors: the return on the small-minus-big (SMB) portfolio and the return on the high-book-to-market-minus-low-book-to-market (HML) portfolio.[9]

For each holding-period month k, the time series of risk-adjusted profits is:

$$r_{t+k}^{adj} = r_{t+k} - \sum_i \hat{\beta}_{ik} f_{i,t+k}, \tag{51}$$

where $\hat{\beta}_{ik}$ are the estimated factor loadings from Equation (50).

The risk-adjusted short-run momentum profit is:

$$MOM_t^{adj} = r_t^{adj}, \tag{52}$$

and the risk-adjusted long-run momentum profit is:

$$REV_t^{adj} = \sum_{k=12}^{59} r_{t+k}^{adj}. \tag{53}$$

By gathering short-run and long-run momentum profits at each month $t-1$, I obtain the time series of raw and risk-adjusted short-run profits.

4.1.2 Adjustment for Time-Varying Risk

A potential driving force for the relation between market states and momentum profits is the time-varying risk (beta) of the momentum portfolio. For example, Grundy and Martin (2001) argue that when the market excess return in the ranking period is positive (negative), the momentum strategy may have a positive (negative) loading on the market factor. Furthermore, this difference in market beta may carry over to the holding period. For example, when the market factor in the ranking period is negative, losers can have a higher beta than winners. If this difference in beta does not vanish in the short-run holding period, then the

[9] The monthly data for these factors are obtained from Kenneth French's website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data/library.html.

momentum strategy would generate negative (positive) profits when the market factor in the short-run holding period is positive (negative). In this case, time-varying risk can contribute to the existence of momentum under DD states and negative momentum profits under DU states.

Therefore, it is important to control for time-varying risk to test the model's implications. Under the conditional CAPM, the true conditional beta is the conditional exposure to the market excess return, given investors' information set. Boguth, Carlson, Fisher, and Simutin (2011) show that estimates of momentum alphas can be biased when researchers employ an inappropriate proxy for the conditional beta. For example, using contemporaneous or future realized betas as proxies for conditional betas may generate overconditioning bias in momentum alphas since information not available to informed investors *ex ante* is used. On the other hand, using lagged realized betas as proxies for conditional betas could lead to underconditioning in momentum alphas since not all information available to informed investors is used.

Boguth, Carlson, Fisher, and Simutin (2011) propose an instrumental-variables-based (IV-based) conditional CAPM (CCAPM) to mitigate these two sources of biases in the evaluation of the conditional performance of momentum strategies. I use this approach to adjust short-run and long-run momentum profits for time-varying risk. Within this framework, the conditional time-series regression is specified as:

$$r_{i,t} = \alpha_i + \gamma_i [1 \quad Z_{i,t-1}]' r_{m,t} + \epsilon_{i,t}, \tag{54}$$

where $r_{i,t}$ is the excess return in month t of portfolio i, $r_{m,t}$ is the excess return on the CRSP VW index in month t, γ_i is a $1 \times (k+1)$ parameter vector, and $Z_{i,t-1}$ is a $1 \times k$ instrument vector.

In Equation (54), the conditional beta estimate is $\beta_{i,t} = \gamma_i[1 \quad Z_{i,t-1}]'$, and is a linear combination of lagged instrumental variables $Z_{i,t-1}$. Following Boguth, Carlson, Fisher, and Simutin (2011), I include eight instrumental variables, and $Z_{i,t-1}$ is given as:

$$Z_{i,t-1} = [DY_{t-1} \; DS_{t-1} \; TS_{t-1} \; TB_{t-1} \; \beta^{6}_{i,t-1} \; \beta^{36}_{i,t-1} \; RU^{6}_{t-1} \; RU^{36}_{t-1}]. \tag{55}$$

The first four instruments are common risk premium predictors: dividend yield (DY), default spread (DS), term spread (TS), and short-term Treasury Bill rate (TB). For each month t, lagged DY, lagged DS, and lagged one-month TB at the end of month $t-1$ and lagged TS at the end of the previous year are collected. DY is computed in the same manner as in Fama and French (1988). It is the sum of dividends accruing to the CRSP VW index over the previous 12 months divided by the price level of the index and is calculated using data from CRSP. DS is the yield spread between Moody's Baa and Aaa corporate bonds and is calculated using data from the Federal Reserve Bank of St. Louis. The one-month TB rate is obtained from CRSP. TS is the yield spread between ten-year and one-year Treasury bonds, and its yearly data is obtained from Robert Shiller's website.[10]

The next two instruments are two lagged market betas of portfolio i. $\beta^{6}_{i,t-1}$ is the lagged six-month beta estimated in the prior six-month window from $t-6$ to $t-1$, and $\beta^{36}_{i,t-1}$ is the lagged 36-month beta estimated in the prior 36-month window from $t-36$ to $t-1$. It is important to include $\beta^{6}_{i,t-1}$ and $\beta^{36}_{i,t-1}$ as instruments since they capture the effect that the beta for winner and loser portfolios may vary over time due to change of their portfolio compositions. They equal the portfolio-weighted average of stock-level lagged betas in the two prior windows, with each stock's weight corresponding to its weight in portfolio i at t. $\beta^{36}_{i,t-1}$ is estimated using monthly returns and $\beta^{6}_{i,t-1}$ is estimated using daily returns. The CRSP VW index is employed as the market factor in these regressions. To mitigate the effect of nonsynchronous trading on the estimation of $\beta^{6}_{i,t-1}$ with daily returns, I include both current and lagged market factors as suggested by Dimson (1979). I use the following

[10] http://www.econ.yale.edu/~shiller/data.htm.

daily regression specification as in Lewellen and Nagel (2006) and Boguth, Carlson, Fisher, and Simutin (2011), for $\tau \in [t-6, t-1]$:

$$r_{i,\tau} = \alpha_i + \beta_{i0} r_{m,\tau} + \beta_{i1} r_{m,\tau-1} + \beta_{i2}[r_{m,\tau-2} + r_{m,\tau-3} + r_{m,\tau-4}]/3 + \epsilon_{i,\tau}, \tag{56}$$

and $\beta^6_{i,t-1}$ is then the sum of the estimated β_{i0}, β_{i1}, and β_{i2}.

As for the last two instruments, RU^6 is the return on the CRSP VW index over $t-6$ to $t-1$, and RU^{36} is the return on the CRSP VW index over $t-36$ to $t-1$. Boguth, Carlson, Fisher, and Simutin (2011) show that these two instruments have predictability for future portfolio betas, and it is helpful to include them.

To compute CCAPM-adjusted momentum profits, I run the following time-series regression analogous to Equation (54) for winner and loser portfolios and for each holding-period month k ($k = 0$ for short-run profits and $k = 12$ to 59 for long-run profits):

$$r_{i,t+k} = \alpha_{i,k} + \gamma_{i,k}[1 \quad Z_{i,t+k-1}]' r_{m,t+k} + \epsilon_{i,t+k}, \quad i = w, l, \tag{57}$$

where $i = w$ denotes the winner portfolio and $i = l$ represents the loser portfolio, $r_{i,t+k}$ is the excess return in month $t+k$ of portfolio i formed at the end of month $t-1$, $r_{m,t+k}$ is the excess return on the CRSP VW index in month $t+k$, and similar to Equation (55), $Z_{i,t+k-1}$ is given as:

$$Z_{i,t+k-1} = [DY_{t+k-1}\ DS_{t+k-1}\ TS_{t+k-1}\ TB_{t+k-1}\ \beta^6_{i,t+k-1}\ \beta^{36}_{i,t+k-1}\ RU^6_{t+k-1}\ RU^{36}_{t+k-1}].$$

For each holding-period month k, the time series of CCAPM-adjusted profits is then:

$$r^{CCAPM}_{i,t+k} = r_{i,t+k} - \hat{\gamma}_{i,k}[1 \quad Z_{t+k-1}]' r_{m,t+k}, \quad i = w, l, \tag{58}$$

where $\hat{\gamma}_{i,k}$ is the estimated $\gamma_{i,k}$ from Equation (57).

The CCAPM-adjusted short-run momentum profit is then:

$$MOM_t^{CCAPM} = r_{w,t}^{CCAPM} - r_{l,t}^{CCAPM}, \tag{59}$$

and the CCAPM-adjusted long-run momentum profit is:

$$REV_t^{CCAPM} = \sum_{k=12}^{59} (r_{w,t+k}^{CCAPM} - r_{l,t+k}^{CCAPM}). \tag{60}$$

4.1.3 Mean Profits Conditional on Market States

To test the three hypotheses, I calculate mean short-run and long-run momentum profits conditional on market states. To directly test the model's implications, I compute these two market returns empirically in a way that closely follows Equations (19) and (20) in the model. At the end of each month $t-1$, the lagged market return MKT_1 is calculated as the average return of all stocks in the sample over the ranking period $t-12$ to $t-2$; the contemporaneous market return MKT_2 is calculated as the average return of all stocks in the sample over the short-run holding period t.

In accordance with the model, I classify a market return as an UP (a DOWN) market state if it is positive (negative). There are four market states: UP/UP (UU), DOWN/DOWN (DD), UP/DOWN (UD), and DOWN/UP (DU). By defining market states for each month $t-1$, I obtain the time series of four market-state dummy variables: UU_t, UD_t, DU_t, and DD_t.

The mean profit conditional on a certain type of market state is calculated by regressing the time series of profits on the time series of the corresponding market-state dummy, with no intercept. For long-run profits, the mean conditional profits are divided by the length of the holding period, 48, to obtain mean monthly profits. The heteroskedasticity robust t-statistics are calculated for all regressions. For long-run profits, since cumulative returns overlap, the robust t-statistics are further corrected for autocorrelation using the Newey-

West method (Newey and West 1987) with the number of lags set to be 47, the number of overlapping months in the holding period window.

Throughout the empirical tests, I use results of CCAPM-adjusted profits as main evidence to test my model's implications. I also discuss results of raw, CAPM, and Fama-French-adjusted profits for completeness and for the connection of the existing literature.

4.2 Momentum and Market States

In this section, I test Hypothesis 1: momentum abnormal returns are positive under UU and DD states, and ambiguous under UD and DU states. This set of results is presented in Panel A of Table 2. The mean monthly momentum profits conditional on four market states are reported, along with the corresponding unconditional mean profits shown in the last row labeled as "All".

Column 2 contains results for the CCAPM-adjusted short-run momentum profits. The unconditional mean profit, 1.00% ($t = 4.79$), is positive and significant. The mean profit is 1.32% ($t = 4.39$) under UU states and 1.71% ($t = 2.73$) under DD states. Therefore, we observe short-run momentum under UU and DD states, consistent with Hypothesis 1. Also consistent with Hypothesis 1, the mean profit under UD states is 1.21% ($t = 5.15$) and that under DU states is -0.97% ($t = -1.26$).

Column 3 presents results for raw short-run momentum profits. The unconditional mean profit, 0.85% ($t = 3.36$), is positive and significant, consistent with the literature documenting short-run momentum. The conditional mean profit shows remarkable variation across market states. It is 1.37% ($t = 4.12$) under UU states and 5.38% ($t = 9.74$) under DD states. The mean profit under UD states, 1.07% ($t = 4.16$), is positive and significant, while that under DU states, -4.84% ($t = -4.67$), is negative and significant. Clearly, we observe negative momentum profits that are both statistically and economically significant under DU states, consistent with Daniel and Moskowitz (2013). As expected, the CCAPM adjustment reduces the magnitude and significance of these negative momentum profits under DU states.

Columns 4 and 5 present results for the CAPM-adjusted and the Fama-French-adjusted short-run momentum profits. The results for CAPM-adjusted short-run momentum profits are shown in Column 3. The unconditional mean profit, 1.13% ($t = 4.71$), is positive and significant. The mean profit is 2.82% ($t = 8.66$) under UU states, 2.97% ($t = 5.42$) under DD states, 0.65% ($t = -2.13$) under UD states, and -2.50% ($t = -2.71$) under DU states. These results are consistent with Hypothesis 1. Results for Fama-French-adjusted profits reported in Column 4 are qualitatively similar to those for CAPM-adjusted profits.

4.3 Long-Run Reversals and Market States

In this section, I test Hypothesis 2 that long-run reversals exist under all four market states. This set of results is presented in Panel B of Table 2. The mean long-run momentum profits conditional on four market states are reported, along with the corresponding unconditional mean shown in the last row labeled as "All".

Column 2 contains results for the CCAPM-adjusted long-run momentum profits. Long-run reversals are prevalent after controlling for the CCAPM. The unconditional mean profit is -0.49% ($t = -3.94$). The conditional mean profit is negative and significant under all four market states. It is -0.41% ($t = -3.74$) under UU states, -0.48% ($t = -4.57$) under UD states, -0.50% ($t = -2.62$) under DU states, and -0.86% ($t = -3.26$) under DD states. These results support Hypothesis 2.

Column 3 presents results for raw long-run momentum profits. The unconditional mean profit, -0.57% ($t = -3.59$), is negative and significant, consistent with the literature of long-run reversals. The conditional mean profit is negative under all four market states. It is -0.38% ($t = -3.26$) under UU states, -0.47% ($t = -4.03$) under UD states, -0.76% ($t = -3.10$) under DU states, and -1.33% ($t = -3.16$) under DD states.

Column 4 presents results for the CAPM-adjusted long-run momentum profits. Long-run reversals remain prevalent after controlling for the CAPM. The unconditional mean profit is -0.48% ($t = -2.93$). The conditional mean profit is negative and significant under all four market states. It is -0.29% ($t = -2.33$) under UU states, -0.38% ($t = -2.95$) under UD states, -0.66% ($t = -2.72$) under DU states, and -1.20% ($t = -2.98$) under DD states.

When long-run momentum profits are adjusted by the Fama-French three-factor model, we expect that they will become less pronounced, as documented by Fama and French (1996). Column 5 reports results for Fama-French-adjusted long-run momentum profits. The unconditional mean profit is -0.21% ($t = -1.90$). Mean profits under UU states and UD states, -0.12% ($t = -1.35$) and -0.11% ($t = -1.10$), respectively, are insignificant. Interestingly, the conditional analysis reveals that the Fama-French adjustment does not entirely eliminate long-run reversals. The mean profit is still economically and statistically significant under DU states, -0.33% ($t = -1.96$), and under DD states, -0.68% ($t = -2.38$).

4.4 The Asymmetry between Winners and Losers

In this section, I test Hypothesis 3 regarding the asymmetry between winners and losers. For this purpose, I focus on results of CCAPM-adjusted profits reported in Table 3. Under UU states, the mean CCAPM-adjusted return for winners is 1.31% ($t = 7.82$) and that for losers is -0.01% ($t = -0.03$). This is consistent with Hypothesis (4a) that abnormal returns are positive for winners and around zero for losers under UU states. This evidence provides support to the model mechanism that continuing overconfidence for winners drives momentum under UU states. Under DD states, the mean CCAPM-adjusted return for winners is 0.41% ($t = 1.69$) and that for losers is -1.31% ($t = -2.12$). The negative abnormal return for losers is consistent with Hypothesis (4b), while the positive abnormal return for winners is inconsistent with Hypothesis (4b), although this inconsistence is weak with a marginal significance at the 10% level. Nevertheless, the positive abnormal return for winners is smaller in magnitude than that of the negative abnormal return for losers. Overall, the evidence under DD states is consistent with the model mechanism that momentum under DD states comes mainly from continuing overconfidence for losers.

Under UD states, the mean CCAPM-adjusted return for winners is -0.05% ($t = -0.28$) and that for losers is -1.26% ($t = -5.14$). This is consistent with Hypothesis (4c) that abnormal returns are negative for both winners and losers. This supports the model mechanism that under UD states, confidence for winners decreases while that for losers increases. Under DU states, the mean CCAPM-adjusted return for winners is 0.92% ($t = 3.73$) and that for losers is 1.89% ($t = 2.33$). This is consistent with Hypothesis (4d) that abnormal returns are positive for both winners and losers. This supports the model mechanism that under DU states, confidence for winners increases while that for losers decreases.

5 Calibration and Discussion

In this section, I calibrate the model to match the empirical patterns in Table 3. By doing so, I also aim to shed light to the following question: can the model explain the difference between momentum abnormal returns under UD and DU states? In the data, the momentum abnormal return is positive, 1.21% ($t = 5.15$), under UD states and negative, -0.97% ($t = -1.26$), under DU states. This difference comes mainly from the side of confidence decrease. Specifically, the negative abnormal return for winners under UD states, -0.05%, is quite small in magnitude, compared to the positive abnormal return for losers, 1.89%, under DU states.

With the basic updating rule of overconfidence in Equation (6), the model does not generate the difference of momentum profits between UD and DU states. The momentum profit from the model is the same between UD and DU states, as evident by comparing Equations (43) and (46). Intuitively, this is due to two reasons. First, confidence decrease is driven by the same parameter k_- and therefore is the same in magnitude, for winners under UD states and losers under DU states. Second, confidence increase is driven by the same parameter k_+ and therefore is the same in magnitude, for losers under UD states and winners under DU states.

If we augment the basic updating rule in Equation (6) with the amplifying effect of information uncertainty (IU) on overconfidence incorporated, the model can match the patterns in Table 3 and explain the difference of momentum abnormal returns under UD and

DU states. The IU-augmented updating rule is motivated by numerous studies including Daniel, Hirshleifer, and Subrahmanyam (1998, 2001) and Zhang (2006). These studies show that investors tend to be more overconfident when information uncertainty is higher. The IU-augmented updating rule does not alter the model results presented in Section 2. It is implemented as follows:

$$\sigma_{t,i}^2 = \begin{cases} q(IU_t)(k_+)^{n_+}(k_-)^{n_-}\sigma_\epsilon^2 & if\ (k_+)^{n_+}(k_-)^{n_-}\sigma_\epsilon^2 < \sigma_\epsilon^2 \\ \sigma_\epsilon^2 & if\ (k_+)^{n_+}(k_-)^{n_-}\sigma_\epsilon^2 \geq \sigma_\epsilon^2 \end{cases} \quad t = 1, 2, \tag{61}$$

where $q(\cdot)$ is a decreasing function, i.e. $q'(\cdot) < 0$, and IU_t is the aggregate information uncertainty at t. Intuitively, when investors are overconfident after observing public signals, their overconfidence is further amplified by $q(IU_t)$. I denote $q(IU_t)$ by q_t throughout. Note that setting $q_t = 1$, i.e. ignoring the effect of information uncertainty on overconfidence, reduces the IU-augmented updating rule to the basic updating rule in (6).

Empirically, one measure of aggregate information uncertainty is stock market volatility. The conventional wisdom suggests that stock market volatility and therefore aggregate information uncertainty is higher in a DOWN market than in an UP market. Therefore, we expect that information uncertainty is higher and the value of the q function is lower in a DOWN market than in an UP market. For simplicity, I take two values of q: q_D and q_U corresponding to DOWN markets and UP markets, respectively. We expect $q_D < q_U$ and I set $q_U = 1$ for normalization. In other words, there is no amplifying effect in an UP market when information uncertainty is lower. Under UU states, $q_1 = q_2 = q_U = 1$; under DD states, $q_1 = q_2 = q_D$; under UD states, $q_1 = q_U = 1$ and $q_2 = q_D$; under DU states, $q_1 = q_D$ and $q_2 = q_U = 1$. Applying these values of q to Equation (61), we obtain IU-augmented perceived variances under four market states, listed in Table 4. Substituting these perceived variances into Equations (32) to (34) gives the formulas for the abnormal returns.

Next, I do the calibration exercise. Again, empirical patterns in Table 3 are the target to match. Empirically, the winner (loser) portfolio is the top (bottom) decile of stocks, ranked according to their ranking-period returns. In the calibration, I set winner (loser) stocks as those in the top (bottom) decile, ranked according to the value of their private signal s_i. In the large N limit, the term A in Equations (32) to (34) is $A = \frac{\sigma_\eta^2}{N_{wl}}\sum_{i \in w}(s_i - \bar{s}) = \sigma_\eta^2\sqrt{\sigma_\eta^2 + \sigma_\epsilon^2}\frac{\int_{p90}^{\infty} x\phi(x)dx}{\int_{p90}^{\infty}\phi(x)dx}$, where $\phi(\cdot)$ and p_{90} are the probability density function and the 90th percentile of the standard normal distribution. Specifically, $A \cong 1.75\sigma_\eta^2\sqrt{\sigma_\eta^2 + \sigma_\epsilon^2}$.

There are five parameters that determine abnormal returns in the model: σ_η^2, σ_ϵ^2, k_+, k_-, and q_D. In the model, the idiosyncratic return variance in the ranking period is $Var(\frac{\sigma_\eta^2}{\sigma_\eta^2+\sigma_{1,i}^2}(s_i - \bar{s})) = \sigma_\eta^2\frac{\sigma_\eta^2+\sigma_\epsilon^2}{\sigma_\eta^2+\sigma_{1,i}^2} \cong \sigma_\eta^2$. Hong and Sraer (2013) estimate that the idiosyncratic variance of monthly stock returns is 0.029. Since the momentum portfolio has a 11-month ranking period, σ_η^2 is calibrated to $\sigma_\eta^2 = 0.029 \times 11 = 0.32$. The other four parameters, σ_ϵ^2, k_+, k_-, and q_D, are calibrated by matching return moments. There are eight moments: winner and loser abnormal returns under four market states. Since in the model, loser abnormal returns under UU states and winner abnormal returns under DD states are zero, I eliminate these two moments. There are then 6 moments left. I choose four of them to match so that we can pin down the four parameters and use the other two to verify the calibrated parameters. The four moments to match are: winner abnormal returns under UU states, 1.31%, winner abnormal returns under UD states, −0.05%, loser abnormal returns under UD states, −1.26%, and loser abnormal returns under DU states, 1.89%.

The calibrated parameters are: $\sigma_\epsilon^2 = 0.33$, $k_+ = 0.96$, $k_- = 1.03$, and $q_D = 0.97$. The theoretical counterpart of Table 3 with the calibrated parameters is shown in Table 5. It is encouraging that other two empirical moments, winner abnormal returns under DU states and loser abnormal returns under DD states, are also matched reasonably well with the calibrated parameters. The calibrated parameters of self-attribution bias and overconfidence are only modest and satisfy the following expected relationship: $k_+ < 1$, $k_- > 1$, $k_+k_- < 1$, and $q_D < 1$. Overall, the fact that the model is able to match key empirical patterns of winner and loser abnormal returns under four market states lends further support to the model.

The calibration exercise provides an explanation to the difference between momentum abnormal returns under UD and DU states. Under UD states, information uncertainty in the short-run holding period is higher than that in the ranking period. Hence, the amplifying effect of information uncertainty increases confidence over the short-run holding period. This dampens confidence decrease for winners and strengthens confidence increase for losers and therefore the momentum effect is stronger. Under DU states, information uncertainty in the short-run holding period is lower than that in the ranking period. Hence, the amplifying effect of information uncertainty decreases confidence over the short-run holding period. This strengthens confidence decrease for losers and dampens confidence increase for winners and therefore the momentum effect is weaker.

6 Concluding Remarks

Momentum profits show substantial variation across different market states. This variation remains after controlling for time-varying risk of the momentum portfolio. This paper offers a model to establish the relation between market states and momentum profits through the channel of self-attribution bias and overconfidence. The model shows that in a setting of multiple risky assets with correlated payoffs, overconfidence can vary asymmetrically between winners and losers, which leads to asymmetric return behaviors between winners and losers. The model is self-contained and is able to explain the patterns of momentum and market states in the literature. Furthermore, the model generates new distinctive implications regarding asymmetric return behaviors between winners and losers, which are supported by further empirical analysis. Overall, the model unifies momentum, negative momentum profits under certain market states, and long-run reversals.

The results presented in this paper have broad implications. First, the paper sheds light on the time-varying performance of momentum portfolios, which may have important implications for momentum trading strategies in practice. Furthermore, this paper contributes to the research agenda of understanding financial market phenomena with well-known behavioral features. It reinforces the potential importance of self-attribution bias and overconfidence in generating asset pricing anomalies in a way that complements Daniel, Hirshleifer, and Subrahmanyam (1998).

A Appendix A

The derivation of Equations (11) and (12) uses the law of iterated expectations and standard Bayesian updating rules of normal variables.

A.1 Derivation of Date 1 Prices

Define $I_1 = \{s_1, s_2, ..., s_N, s_{f,1}\}$ as the information set at $t = 1$. From Equation (11), we have:

$$\begin{aligned} P_{1,i} &= \mathbb{E}_{C,1}[\theta_i | I_1] \\ &= \mathbb{E}_{C,1}[f_1|I_1] + \mathbb{E}_{C,1}[f_2|I_1] + \mathbb{E}_{C,1}[\eta_i|I_1]. \end{aligned} \tag{A.1}$$

Next, we calculate the three conditional expectation terms in Equation (A.1), by applying the law of iterated expectations and standard Bayesian updating rules of normal variables. Plugging these three terms into (A.1) gives the formula of $P_{1,i}$, $\forall i = 1, 2, ..., N$.

(1) Calculation of $\mathbb{E}_{C,1}[f_1|I_1]$. We have:

$$\begin{aligned} &\mathbb{E}_{C,1}[f_1|I_1] \\ =&\mathbb{E}_{C,1}[\mathbb{E}_{C,1}[f_1|f_2 + \epsilon_f, I_1]|I_1] \\ =&\mathbb{E}_{C,1}[\frac{\frac{1}{\sigma_\xi^2} s_{f,1} + \sum\limits_i \frac{1}{\sigma_\eta^2 + \sigma_{1,i}^2}(s_i - f_2 - \epsilon_f)}{\frac{1}{\sigma_f^2} + \frac{1}{\sigma_\xi^2} + \sum\limits_i \frac{1}{\sigma_\eta^2 + \sigma_{1,i}^2}} |I_1] \\ =&\frac{\frac{1}{\sigma_\xi^2} s_{f,1} + \sum\limits_i \frac{1}{\sigma_\eta^2 + \sigma_{1,i}^2} s_i}{\frac{1}{\sigma_f^2} + \frac{1}{\sigma_\xi^2} + \sum\limits_i \frac{1}{\sigma_\eta^2 + \sigma_{1,i}^2}} - \frac{\sum\limits_i \frac{1}{\sigma_\eta^2 + \sigma_{1,i}^2}}{\frac{1}{\sigma_f^2} + \frac{1}{\sigma_\xi^2} + \sum\limits_i \frac{1}{\sigma_\eta^2 + \sigma_{1,i}^2}} \mathbb{E}_{C,1}[f_2 + \epsilon_f|I_1] \end{aligned}$$

$$
\begin{aligned}
&=\frac{\frac{1}{\sigma_\xi^2}s_{f,1}+\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}s_i}{\frac{1}{\sigma_f^2}+\frac{1}{\sigma_\xi^2}+\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}-\frac{\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}{\frac{1}{\sigma_f^2}+\frac{1}{\sigma_\xi^2}+\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}\mathbb{E}_{C,1}[\mathbb{E}_{C,1}[f_2+\epsilon_f|f_1,I_1]|I_1]\\
&=\frac{\frac{1}{\sigma_\xi^2}s_{f,1}+\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}s_i}{\frac{1}{\sigma_f^2}+\frac{1}{\sigma_\xi^2}+\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}-\frac{\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}{\frac{1}{\sigma_f^2}+\frac{1}{\sigma_\xi^2}+\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}\mathbb{E}_{C,1}[\frac{\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}(s_i-f_1)}{\frac{1}{\sigma_f^2+\sigma_{\epsilon f}^2}+\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}|I_1]\\
&=\frac{\frac{1}{\sigma_\xi^2}s_{f,1}+\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}s_i}{\frac{1}{\sigma_f^2}+\frac{1}{\sigma_\xi^2}+\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}-\frac{\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}{\frac{1}{\sigma_f^2}+\frac{1}{\sigma_\xi^2}+\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}(\frac{\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}s_i}{\frac{1}{\sigma_f^2+\sigma_{\epsilon f}^2}+\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}-\frac{\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}{\frac{1}{\sigma_f^2+\sigma_{\epsilon f}^2}+\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}\mathbb{E}_{C,1}[f_1|I_1]),
\end{aligned}
\tag{A.2}
$$

rearrangement of which leads to:

$$
\mathbb{E}_{C,1}[f_1|I_1]=\frac{\frac{1}{\sigma_\xi^2}(\frac{1}{\sigma_f^2+\sigma_{\epsilon f}^2}+\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2})s_{f,1}+\frac{1}{\sigma_f^2+\sigma_{\epsilon f}^2}\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}s_i}{(\frac{1}{\sigma_f^2}+\frac{1}{\sigma_\xi^2})\frac{1}{\sigma_f^2+\sigma_{\epsilon f}^2}+(\frac{1}{\sigma_f^2}+\frac{1}{\sigma_\xi^2}+\frac{1}{\sigma_f^2+\sigma_{\epsilon f}^2})\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}. \tag{A.3}
$$

(2) Calculation of $\mathbb{E}_{C,1}[f_2|I_1]$. We have:

$$
\begin{aligned}
&\mathbb{E}_{C,1}[f_2|I_1]\\
=&\mathbb{E}_{C,1}[\mathbb{E}_{C,1}[f_2|f_1+\epsilon_f,I_1]|I_1]\\
=&\mathbb{E}_{C,1}[\frac{\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}(s_i-f_1-\epsilon_f)}{\frac{1}{\sigma_f^2}+\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}|I_1]\\
=&\frac{\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}s_i}{\frac{1}{\sigma_f^2}+\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}-\frac{\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}{\frac{1}{\sigma_f^2}+\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}\mathbb{E}_{C,1}[f_1+\epsilon_f|I_1]\\
=&\frac{\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}s_i}{\frac{1}{\sigma_f^2}+\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}-\frac{\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}{\frac{1}{\sigma_f^2}+\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}\mathbb{E}_{C,1}[\mathbb{E}_{C,1}[f_1+\epsilon_f|f_1,f_2,I_1]|I_1]
\end{aligned}
$$

$$
\begin{aligned}
&=\frac{\sum_i \frac{1}{\sigma_\eta^2+\sigma_{1,i}^2} s_i}{\frac{1}{\sigma_f^2}+\sum_i \frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}} - \frac{\sum_i \frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}{\frac{1}{\sigma_f^2}+\sum_i \frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}(\mathbb{E}_{C,1}[f_1|I_1]+\mathbb{E}_{C,1}[\mathbb{E}_{C,1}[\epsilon_f|f_1,f_2,I_1]|I_1])\\
&=\frac{\sum_i \frac{1}{\sigma_\eta^2+\sigma_{1,i}^2} s_i}{\frac{1}{\sigma_f^2}+\sum_i \frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}} - \frac{\sum_i \frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}{\frac{1}{\sigma_f^2}+\sum_i \frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}(\mathbb{E}_{C,1}[f_1|I_1]+\mathbb{E}_{C,1}[\frac{\sum_i \frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}(s_i-f_1-f_2)}{\frac{1}{\sigma_{\epsilon f}^2}+\sum_i \frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}|I_1])\\
&=\frac{\sum_i \frac{1}{\sigma_\eta^2+\sigma_{1,i}^2} s_i}{\frac{1}{\sigma_f^2}+\sum_i \frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}} - \frac{\sum_i \frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}{\frac{1}{\sigma_f^2}+\sum_i \frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}(\frac{\sum_i \frac{1}{\sigma_\eta^2+\sigma_{1,i}^2} s_i}{\frac{1}{\sigma_{\epsilon f}^2}+\sum_i \frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}+\frac{\frac{1}{\sigma_{\epsilon f}^2}\mathbb{E}_{C,1}[f_1|I_1]}{\frac{1}{\sigma_{\epsilon f}^2}+\sum_i \frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}-\frac{\sum_i \frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}\mathbb{E}_{C,1}[f_2|I_1]}{\frac{1}{\sigma_{\epsilon f}^2}+\sum_i \frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}),
\end{aligned}
\tag{A.4}
$$

rearrangement of which leads to:

$$
\mathbb{E}_{C,1}[f_2|I_1] = \frac{\frac{1}{\sigma_{\epsilon f}^2}\sum_i \frac{1}{\sigma_\eta^2+\sigma_{1,i}^2} s_i - \frac{1}{\sigma_{\epsilon f}^2}\sum_i \frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}\mathbb{E}_{C,1}[f_1|I_1]}{\frac{1}{\sigma_f^2}\frac{1}{\sigma_{\epsilon f}^2}+(\frac{1}{\sigma_f^2}+\frac{1}{\sigma_{\epsilon f}^2})\sum_i \frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}. \tag{A.5}
$$

(3) Calculation of $\mathbb{E}_{C,1}[\eta_i|I_1]$. We have:

$$
\begin{aligned}
&\mathbb{E}_{C,1}[\eta_i|I_1]\\
=&\mathbb{E}_{C,1}[\mathbb{E}_{C,1}[\eta_i|f_1+f_2+\epsilon_f,I_1]|I_1]\\
=&\mathbb{E}_{C,1}[\frac{\frac{1}{\sigma_{1,i}^2}(s_i-f_1-f_2-\epsilon_f)}{\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_{1,i}^2}}|I_1]\\
=&\frac{\frac{1}{\sigma_{1,i}^2}s_i}{\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_{1,i}^2}} - \frac{\frac{1}{\sigma_{1,i}^2}}{\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_{1,i}^2}}\mathbb{E}_{C,1}[f_1+f_2+\epsilon_f|I_1]\\
=&\frac{\frac{1}{\sigma_{1,i}^2}s_i}{\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_{1,i}^2}} - \frac{\frac{1}{\sigma_{1,i}^2}}{\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_{1,i}^2}}\mathbb{E}_{C,1}[\mathbb{E}_{C,1}[f_1+f_2+\epsilon_f|f_1,I_1]|I_1]\\
=&\frac{\frac{1}{\sigma_{1,i}^2}s_i}{\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_{1,i}^2}} - \frac{\frac{1}{\sigma_{1,i}^2}}{\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_{1,i}^2}}\mathbb{E}_{C,1}[f_1+\frac{\sum_i \frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}(s_i-f_1)}{\frac{1}{\sigma_f^2+\sigma_{\epsilon f}^2}+\sum_i \frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}|I_1]
\end{aligned}
$$

$$=\frac{\frac{1}{\sigma_{1,i}^2}s_i}{\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_{1,i}^2}}-\frac{\frac{1}{\sigma_{1,i}^2}}{\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_{1,i}^2}}\mathbb{E}_{C,1}[\frac{\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}s_i}{\frac{1}{\sigma_f^2+\sigma_{\epsilon f}^2}+\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}+\frac{\frac{1}{\sigma_f^2+\sigma_{\epsilon f}^2}f_1}{\frac{1}{\sigma_f^2+\sigma_{\epsilon f}^2}+\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}|I_1]$$

$$=\frac{\frac{1}{\sigma_{1,i}^2}s_i}{\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_{1,i}^2}}-\frac{\frac{1}{\sigma_{1,i}^2}}{\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_{1,i}^2}}[\frac{\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}s_i}{\frac{1}{\sigma_f^2+\sigma_{\epsilon f}^2}+\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}+\frac{\frac{1}{\sigma_f^2+\sigma_{\epsilon f}^2}\mathbb{E}_{C,1}[f_1|I_1]}{\frac{1}{\sigma_f^2+\sigma_{\epsilon f}^2}+\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}]. \tag{A.6}$$

A.1.1 Derivation of Equation (14)

To simplify the lengthy formula of $P_{2,i}$, I analyze the case where the variance of the common noise, $\sigma_{\epsilon f}^2$, is sufficiently large. Since $\bar{s}\to f+\epsilon_f\sim\mathcal{N}(0,\sigma_f^2+\sigma_{\epsilon f}^2)$, $|\bar{s}|$ is also likely to be large with a large $\sigma_{\epsilon f}^2$. This allows an approximation of $\sum_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}s_i\cong\bar{s}\sum_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}$, substituting which and $\sigma_{\epsilon f}^2\to\infty$ into Equations (A.3), (A.5), and (A.6) gives:

$$\mathbb{E}_{C,1}[f_1|I_1]\cong\frac{\sigma_f^2}{\sigma_f^2+\sigma_\xi^2}s_{f,1}, \tag{A.7}$$

$$\mathbb{E}_{C,1}[f_2|I_1]\cong 0, \tag{A.8}$$

and

$$\mathbb{E}_{C,1}[\eta_i|I_1]=\frac{\sigma_\eta^2}{\sigma_\eta^2+\sigma_{1,i}^2}(s_i-\bar{s}). \tag{A.9}$$

Substituting these three equations into Equation (A.1) gives Equation (14).

A.2 Derivation of Date 2 Prices

Define $I_2=\{s_1,s_2,...,s_N,s_{f,1},s_{f,2}\}$ as the information set at $t=2$. From Equation (12), we have:

$$\begin{aligned}P_{2,i}&=\mathbb{E}_{C,2}[\theta_i|I_2]\\&=\mathbb{E}_{C,2}[f_1+f_2|I_2]+\mathbb{E}_{C,2}[\eta_i|I_2].\end{aligned} \tag{A.10}$$

Next, we calculate the two conditional expectation terms in Equation (A.10), by applying the law of iterated expectations and standard Bayesian updating rules of normal variables. Plugging these two terms into (A.10) gives the formula of $P_{2,i},\ \forall i=1,2,...,N$.

(1) Calculation of $\mathbb{E}_{C,2}[f_1+f_2|I_2]$. First, we have:

$$
\begin{aligned}
&\mathbb{E}_{C,2}[f_1|I_2]\\
=&\mathbb{E}_{C,2}[\mathbb{E}_{C,2}[f_1|f_2+\epsilon_f,I_2]|I_2]\\
=&\mathbb{E}_{C,2}[\frac{\frac{1}{\sigma_\xi^2}s_{f,1}+\sum\limits_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}(s_i-f_2-\epsilon_f)}{\frac{1}{\sigma_f^2}+\frac{1}{\sigma_\xi^2}+\sum\limits_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}|I_2]\\
=&\frac{\frac{1}{\sigma_\xi^2}s_{f,1}+\sum\limits_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}s_i}{\frac{1}{\sigma_f^2}+\frac{1}{\sigma_\xi^2}+\sum\limits_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}-\frac{\sum\limits_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}{\frac{1}{\sigma_f^2}+\frac{1}{\sigma_\xi^2}+\sum\limits_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}\mathbb{E}_{C,2}[f_2+\epsilon_f|I_2]\\
=&\frac{\frac{1}{\sigma_\xi^2}s_{f,1}+\sum\limits_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}s_i}{\frac{1}{\sigma_f^2}+\frac{1}{\sigma_\xi^2}+\sum\limits_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}-\frac{\sum\limits_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}{\frac{1}{\sigma_f^2}+\frac{1}{\sigma_\xi^2}+\sum\limits_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}\mathbb{E}_{C,2}[\mathbb{E}_{C,2}[f_2+\epsilon_f|f_1,f_2,I_2]|I_2]\\
=&\frac{\frac{1}{\sigma_\xi^2}s_{f,1}+\sum\limits_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}s_i}{\frac{1}{\sigma_f^2}+\frac{1}{\sigma_\xi^2}+\sum\limits_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}-\frac{\sum\limits_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}{\frac{1}{\sigma_f^2}+\frac{1}{\sigma_\xi^2}+\sum\limits_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}\mathbb{E}_{C,2}[f_2+\frac{\sum\limits_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}(s_i-f_1-f_2)}{\frac{1}{\sigma_{\epsilon f}^2}+\sum\limits_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}|I_2]\\
=&\frac{\frac{1}{\sigma_\xi^2}s_{f,1}+\sum\limits_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}s_i}{\frac{1}{\sigma_f^2}+\frac{1}{\sigma_\xi^2}+\sum\limits_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}\\
&-\frac{\sum\limits_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}{\frac{1}{\sigma_f^2}+\frac{1}{\sigma_\xi^2}+\sum\limits_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}\mathbb{E}_{C,2}[\frac{\sum\limits_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}s_i}{\frac{1}{\sigma_{\epsilon,f}^2}+\sum\limits_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}-\frac{\sum\limits_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}f_1}{\frac{1}{\sigma_{\epsilon,f}^2}+\sum\limits_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}+\frac{\frac{1}{\sigma_{\epsilon f}^2}f_2}{\frac{1}{\sigma_{\epsilon f}^2}+\sum\limits_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}|I_2]\\
=&\frac{\frac{1}{\sigma_\xi^2}s_{f,1}+\frac{\frac{1}{\sigma_{\epsilon f}^2}}{\frac{1}{\sigma_{\epsilon f}^2}+\sum\limits_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}\sum\limits_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}s_i}{\frac{1}{\sigma_f^2}+\frac{1}{\sigma_\xi^2}+\sum\limits_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}+\frac{\sum\limits_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}{\frac{1}{\sigma_f^2}+\frac{1}{\sigma_\xi^2}+\sum\limits_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}\frac{\sum\limits_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}{\frac{1}{\sigma_{\epsilon f}^2}+\sum\limits_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}\mathbb{E}_{C,2}[f_1|I_2]
\end{aligned}
$$

$$
-\frac{\sum_i \frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}{\frac{1}{\sigma_f^2}+\frac{1}{\sigma_\xi^2}+\sum_i \frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}} \frac{\frac{1}{\sigma_{\epsilon f}^2}}{\frac{1}{\sigma_{\epsilon f}^2}+\sum_i \frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}} \mathbb{E}_{C,2}[f_2|I_2]. \tag{A.11}
$$

By symmetry, we also have:

$$
\begin{aligned}
&\mathbb{E}_{C,2}[f_2|I_2] \\
=&\frac{\frac{1}{\sigma_\xi^2}s_{f,2}+\frac{\frac{1}{\sigma_{\epsilon f}^2}}{\frac{1}{\sigma_{\epsilon f}^2}+\sum_i \frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}\sum_i \frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}s_i}{\frac{1}{\sigma_f^2}+\frac{1}{\sigma_\xi^2}+\sum_i \frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}} + \frac{\sum_i \frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}{\frac{1}{\sigma_f^2}+\frac{1}{\sigma_\xi^2}+\sum_i \frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}} \frac{\sum_i \frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}{\frac{1}{\sigma_{\epsilon f}^2}+\sum_i \frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}\mathbb{E}_{C,2}[f_2|I_2] \\
&-\frac{\sum_i \frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}{\frac{1}{\sigma_f^2}+\frac{1}{\sigma_\xi^2}+\sum_i \frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}} \frac{\frac{1}{\sigma_{\epsilon f}^2}}{\frac{1}{\sigma_{\epsilon f}^2}+\sum_i \frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}} \mathbb{E}_{C,2}[f_1|I_2].
\end{aligned} \tag{A.12}
$$

Combining Equations (A.2) and (A.11) leads to:

$$
\mathbb{E}_{C,2}[f_1+f_2|I_2] = \frac{\frac{1}{\sigma_\xi^2}(\frac{1}{\sigma_{\epsilon f}^2}+\sum_i \frac{1}{\sigma_\eta^2+\sigma_{2,i}^2})(s_{f,1}+s_{f,2})+\frac{2}{\sigma_{\epsilon f}^2}\sum_i \frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}s_i}{(\frac{1}{\sigma_f^2}+\frac{1}{\sigma_\xi^2}+\sum_i \frac{1}{\sigma_\eta^2+\sigma_{2,i}^2})\frac{1}{\sigma_{\epsilon f}^2}+(\frac{1}{\sigma_f^2}+\frac{1}{\sigma_\xi^2})\sum_i \frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}+\frac{1}{\sigma_{\epsilon f}^2}\sum_i \frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}. \tag{A.13}
$$

(2) Calculation of $\mathbb{E}_{C,1}[\eta_i|I_1]$. We have:

$$
\begin{aligned}
&\mathbb{E}_{C,2}[\eta_i|I_2] \\
=&\mathbb{E}_{C,2}[\mathbb{E}_{C,2}[\eta_i|f_1+f_2+\epsilon_f, I_2]|I_2] \\
=&\mathbb{E}_{C,2}[\frac{\frac{1}{\sigma_{2,i}^2}(s_i-f_1-f_2-\epsilon_f)}{\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_{2,i}^2}}|I_2] \\
=&\frac{\frac{1}{\sigma_{2,i}^2}s_i}{\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_{2,i}^2}}-\frac{\frac{1}{\sigma_{2,i}^2}}{\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_{2,i}^2}}\mathbb{E}_{C,2}[f_1+f_2+\epsilon_f|I_2] \\
=&\frac{\frac{1}{\sigma_{2,i}^2}s_i}{\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_{2,i}^2}}-\frac{\frac{1}{\sigma_{2,i}^2}}{\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_{2,i}^2}}\mathbb{E}_{C,2}[\mathbb{E}_{C,2}[f_1+f_2+\epsilon_f|f_1+f_2, I_2]|I_2]
\end{aligned}
$$

$$=\frac{\frac{1}{\sigma_{2,i}^2}s_i}{\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_{2,i}^2}}-\frac{\frac{1}{\sigma_{2,i}^2}}{\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_{2,i}^2}}\mathbb{E}_{C,2}[f_1+f_2+\frac{\sum_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}(s_i-f_1-f_2)}{\frac{1}{\sigma_{\epsilon f}^2}+\sum_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}|I_1]$$

$$=\frac{\frac{1}{\sigma_{2,i}^2}s_i}{\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_{2,i}^2}}-\frac{\frac{1}{\sigma_{2,i}^2}}{\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_{2,i}^2}}\mathbb{E}_{C,2}[\frac{\sum_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}s_i}{\frac{1}{\sigma_{\epsilon f}^2}+\sum_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}+\frac{\frac{1}{\sigma_{\epsilon f}^2}(f_1+f_2)}{\frac{1}{\sigma_{\epsilon f}^2}+\sum_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}|I_1]$$

$$=\frac{\frac{1}{\sigma_{2,i}^2}s_i}{\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_{2,i}^2}}-\frac{\frac{1}{\sigma_{2,i}^2}}{\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_{2,i}^2}}[\frac{\sum_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}s_i}{\frac{1}{\sigma_{\epsilon f}^2}+\sum_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}}+\frac{\frac{1}{\sigma_{\epsilon f}^2}\mathbb{E}_{C,2}[f_1+f_2|I_1]}{\frac{1}{\sigma_{\epsilon f}^2}+\sum_i\frac{1}{\sigma_\eta^2+\sigma_{1,i}^2}}]. \tag{A.14}$$

A.1.2 Derivation of Equation (15)

To simplify the lengthy formula of $P_{2,i}$, I analyze the case where the variance of the common noise, $\sigma_{\epsilon f}^2$, is sufficiently large. Since $\bar{s}\to f+\epsilon_f\sim\mathcal{N}(0,\sigma_f^2+\sigma_{\epsilon f}^2)$, $|\bar{s}|$ is also likely to be large with a large $\sigma_{\epsilon f}^2$. This allows an approximation of $\sum_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}s_i\cong\bar{s}\sum_i\frac{1}{\sigma_\eta^2+\sigma_{2,i}^2}$, substituting which and $\sigma_{\epsilon f}^2\to\infty$ into Equations (A.13) and (A.14) gives:

$$\mathbb{E}_{C,2}[f_1+f_2|I_2]\cong\frac{\sigma_f^2}{\sigma_f^2+\sigma_\xi^2}(f_1+f_2), \tag{A.15}$$

and

$$\mathbb{E}_{C,2}[\eta_i|I_2]=\frac{\sigma_\eta^2}{\sigma_\eta^2+\sigma_{1,i}^2}(s_i-\bar{s}). \tag{A.16}$$

Substituting these two equations into Equation (A.10) gives Equation (15).

B Appendix B

B.1 Ranking-Period Returns

I show that winners have higher ranking-period returns than losers. The difference between the ranking-period return of any winner asset w and that of any loser asset l is:

$$\Delta P_{1,w}-\Delta P_{1,l}=P_{1,w}-P_{1,l}. \tag{B.1}$$

Substituting Equation (14) into Equation (B.1) gives:

$$\Delta P_{1,w} - \Delta P_{1,l} = \frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_{1,w}^2}(s_w - \bar{s}) - \frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_{1,l}^2}(s_l - \bar{s})$$
$$> 0, \tag{B.2}$$

where the last line is due to $s_w > \bar{s} > s_l$. $\square$

B.2 Proof of Proposition 1

For MKT_1, substituting Equation (16) into Equation (19) gives:

$$\begin{aligned} MKT_1 &= \frac{\sigma_f^2}{\sigma_f^2 + \sigma_\xi^2} s_{f,1} + \frac{1}{N}\sum_i \frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_{1,i}^2}(s_i - \bar{s}) \\ &= \frac{\sigma_f^2}{\sigma_f^2 + \sigma_\xi^2} s_{f,1} + \frac{1}{N}\sum_{i\in w} \frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_{1,w}^2}(s_i - \bar{s}) + \frac{1}{N}\sum_{i\in l} \frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_{1,l}^2}(s_i - \bar{s}) \\ &= \frac{\sigma_f^2}{\sigma_f^2 + \sigma_\xi^2} s_{f,1} + \frac{N_{wl}}{N}(\frac{1}{\sigma_\eta^2 + \sigma_{1,w}^2} - \frac{1}{\sigma_\eta^2 + \sigma_{1,l}^2})A \\ &= \frac{\sigma_f^2}{\sigma_f^2 + \sigma_\xi^2} s_{f,1} + \frac{\sigma_{1,l}^2 - \sigma_{1,w}^2}{(\sigma_\eta^2 + \sigma_{1,w}^2)(\sigma_\eta^2 + \sigma_{1,l}^2)}\frac{N_{wl}}{N}A. \end{aligned} \tag{B.3}$$

For MKT_2, substituting Equation (17) into Equation (20) gives:

$$\begin{aligned} MKT_2 &= \frac{\sigma_f^2}{\sigma_f^2 + \sigma_\xi^2} s_{f,2} + \frac{1}{N}\sum_i \frac{\sigma_\eta^2(\sigma_{1,i}^2 - \sigma_{2,i}^2)}{(\sigma_\eta^2 + \sigma_{1,i}^2)(\sigma_\eta^2 + \sigma_{2,i}^2)}(s_i - \bar{s}), \\ &= \frac{\sigma_f^2}{\sigma_f^2 + \sigma_\xi^2} s_{f,1} + \frac{1}{N}\sum_{i\in w} \frac{\sigma_\eta^2(\sigma_{1,w}^2 - \sigma_{2,w}^2)}{(\sigma_\eta^2 + \sigma_{1,w}^2)(\sigma_\eta^2 + \sigma_{2,w}^2)}(s_i - \bar{s}) + \frac{1}{N}\sum_{i\in l} \frac{\sigma_\eta^2(\sigma_{1,l}^2 - \sigma_{2,l}^2)}{(\sigma_\eta^2 + \sigma_{1,l}^2)(\sigma_\eta^2 + \sigma_{2,l}^2)}(s_i - \bar{s}) \\ &= \frac{\sigma_f^2}{\sigma_f^2 + \sigma_\xi^2} s_{f,2} + [\frac{\sigma_{1,w}^2 - \sigma_{2,w}^2}{(\sigma_\eta^2 + \sigma_{1,w}^2)(\sigma_\eta^2 + \sigma_{2,w}^2)} - \frac{\sigma_{1,l}^2 - \sigma_{2,l}^2}{(\sigma_\eta^2 + \sigma_{1,l}^2)(\sigma_\eta^2 + \sigma_{2,l}^2)}]\frac{N_{wl}}{N}A. \end{aligned} \tag{B.4}$$

$\square$

B.3 Proof of Proposition 3

Substituting Equation (17) into Equation (29) gives:

$$\begin{aligned}
WIN &= \frac{\sigma_f^2}{\sigma_f^2+\sigma_\xi^2}s_{f,2} + \frac{1}{N_{wl}}\sum_{i\in w}\frac{\sigma_\eta^2(\sigma_{1,w}^2-\sigma_{2,w}^2)}{(\sigma_\eta^2+\sigma_{1,w}^2)(\sigma_\eta^2+\sigma_{2,w}^2)}(s_i-\bar{s}) - \frac{\sigma_f^2}{\sigma_f^2+\sigma_\xi^2}s_{f,2} \\
&= \frac{\sigma_{1,w}^2-\sigma_{2,w}^2}{(\sigma_\eta^2+\sigma_{1,w}^2)(\sigma_\eta^2+\sigma_{2,w}^2)}A. \qquad \text{(B.5)}
\end{aligned}$$

Substituting Equation (17) into Equation (30) gives:

$$\begin{aligned}
LOS &= \frac{\sigma_f^2}{\sigma_f^2+\sigma_\xi^2}s_{f,2} + \frac{1}{N_{wl}}\sum_{i\in l}\frac{\sigma_\eta^2(\sigma_{1,l}^2-\sigma_{2,l}^2)}{(\sigma_\eta^2+\sigma_{1,l}^2)(\sigma_\eta^2+\sigma_{2,l}^2)}(s_i-\bar{s}) - \frac{\sigma_f^2}{\sigma_f^2+\sigma_\xi^2}s_{f,2} \\
&= -\frac{\sigma_{1,l}^2-\sigma_{2,l}^2}{(\sigma_\eta^2+\sigma_{1,l}^2)(\sigma_\eta^2+\sigma_{2,l}^2)}A. \qquad \text{(B.6)}
\end{aligned}$$

Equation (34) then follows. □

B.3 Derivation of Equation (48)

Substituting Equation (18) into Equation (47) gives:

$$\begin{aligned}
REV &= \frac{1}{N_{wl}}\sum_{i\in w}\left(\frac{\sigma_\eta^2}{\sigma_\eta^2+\sigma_\epsilon^2}-\frac{\sigma_\eta^2}{\sigma_\eta^2+\sigma_{2,i}^2}\right)(s_i-\bar{s}) - \frac{1}{N_{wl}}\sum_{i\in l}\left(\frac{\sigma_\eta^2}{\sigma_\eta^2+\sigma_\epsilon^2}-\frac{\sigma_\eta^2}{\sigma_\eta^2+\sigma_{2,i}^2}\right)(s_i-\bar{s}) \\
&= \frac{\sigma_{2,w}^2-\sigma_\epsilon^2}{(\sigma_\eta^2+\sigma_{2,w}^2)(\sigma_\eta^2+\sigma_\epsilon^2)}A + \frac{\sigma_{2,l}^2-\sigma_\epsilon^2}{(\sigma_\eta^2+\sigma_{2,l}^2)(\sigma_\eta^2+\sigma_\epsilon^2)}A. \qquad \text{(B.7)}
\end{aligned}$$

□

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Table 1: Summary of confidence, market states, and abnormal returns

This table summarizes perceived variances at $t = 1$ and $t = 2$ for private signals of winners and losers, market states MKT, and the signs of winner abnormal returns WIN, loser abnormal returns LOS, momentum abnormal returns MOM, and long-run abnormal returns of the momentum portfolio REV, in the four cases of public signals. Following the model, lagged and contemporaneous market returns are calculated, respectively, as the average returns of all stocks in the sample over the ranking and short-run holding periods. UU (DD) represents that both lagged and contemporaneous market states are UP (DOWN). UD (DU) means that the lagged market state is UP (DOWN) and the contemporaneous market state is DOWN (UP).

		$\sigma^2_{1,w}$	$\sigma^2_{2,w}$	$\sigma^2_{1,l}$	$\sigma^2_{2,l}$	MKT	WIN	LOS	MOM	REV
Case I	$s_{f,1} > 0$ $s_{f,2} > 0$	$k_+\sigma^2_\epsilon$	$k^2_+\sigma^2_\epsilon$	σ^2_ϵ	σ^2_ϵ	UU	> 0	$= 0$	> 0	< 0
Case II	$s_{f,1} < 0$ $s_{f,2} < 0$	σ^2_ϵ	σ^2_ϵ	$k_+\sigma^2_\epsilon$	$k^2_+\sigma^2_\epsilon$	DD	$= 0$	< 0	> 0	< 0
Case III	$s_{f,1} > 0$ $s_{f,2} < 0$	$k_+\sigma^2_\epsilon$	$k_+k_-\sigma^2_\epsilon$	σ^2_ϵ	$k_+k_-\sigma^2_\epsilon$	UD	< 0	< 0	$\gtreqless 0$	< 0
Case IV	$s_{f,1} < 0$ $s_{f,2} > 0$	σ^2_ϵ	$k_+k_-\sigma^2_\epsilon$	$k_+\sigma^2_\epsilon$	$k_+k_-\sigma^2_\epsilon$	DU	> 0	> 0	$\gtreqless 0$	< 0

Table 2: Momentum profits and market states

This table reports mean monthly momentum profits conditional on market states and the unconditional mean monthly momentum profits. Short-run momentum profits are presented in Panel A, and long-run momentum profits are presented in Panel B. The sample period is from January 1929 to December 2010. Robust t-statistics (with Newey-West correction for long-run momentum profits) are reported in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively. Following the model, lagged and contemporaneous market returns are calculated as the average returns of all stocks in the sample over the ranking and short-run holding periods, respectively. UU (DD) represents that both lagged and contemporaneous market returns are positive (negative). UD (DU) means that the lagged market return is positive (negative) and the contemporaneous market return is negative (positive). Momentum profits are reported in percentages. CCAPM α represents momentum profits adjusted by the instrumental-based conditional CAPM model and obtained from Equations (59) and (60). CAPM α and Fama-French α represent momentum profits adjusted by the CAPM and the Fama-French three-factor model, respectively.

Market states	CCAPM α	Raw profits	CAPM α	Fama-French α
Panel A: Short-run momentum profits				
UU	1.32***	1.37***	2.82***	3.25***
	(4.39)	(4.12)	(8.66)	(11.37)
DD	1.71***	5.38***	2.97***	2.81***
	(2.73)	(9.74)	(5.42)	(5.58)
UD	1.21***	1.07***	-0.65**	-0.44
	(5.15)	(4.16)	(-2.13)	(-1.32)
DU	-0.97	-4.84***	-2.50***	-1.88**
	(-1.26)	(-4.67)	(-2.71)	(-2.50)
All	1.00***	0.85***	1.13***	1.46***
	(4.79)	(3.36)	(4.71)	(6.78)
Panel B: Long-run momentum profits				
UU	-0.41***	-0.38***	-0.29**	-0.12
	(-3.74)	(-3.26)	(-2.33)	(-1.35)
DD	-0.86***	-1.33***	-1.20***	-0.68**
	(-3.26)	(-3.16)	(-2.98)	(-2.38)
UD	-0.48***	-0.47***	-0.38***	-0.11
	(-4.57)	(-4.03)	(-2.95)	(-1.10)
DU	-0.50***	-0.76***	-0.66***	-0.33*
	(-2.62)	(-3.10)	(-2.72)	(-1.96)
All	-0.49***	-0.57***	-0.48***	-0.21*
	(-3.94)	(-3.59)	(-2.93)	(-1.90)

Table 3: Asymmetry between winners and losers

This table reports mean monthly CCAPM-adjusted profits of the winner, loser, and momentum portfolios, conditional on market states. The sample period is from January 1929 to December 2010. CCAPM α represents momentum profits adjusted by the instrumental-based conditional CAPM model and obtained from Equations (59) and (60). CCAPM-adjusted profits are reported in percentages. Following the model, lagged and contemporaneous market returns are calculated as the average returns of all stocks in the sample over the ranking and short-run holding periods, respectively. UU (DD) represents that both lagged and contemporaneous market returns are positive (negative). UD (DU) means that the lagged market return is positive (negative) and the contemporaneous market return is negative (positive). Robust t-statistics are reported in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Market States	Winner CCAPM α	Loser CCAPM α	MOM CCAPM α
UU	1.31***	-0.01	1.32***
	(7.82)	(-0.03)	(4.39)
DD	0.41*	-1.31**	1.71***
	(1.69)	(-2.12)	(2.73)
UD	-0.05	-1.26***	1.21***
	(-0.28)	(-5.14)	(5.15)
DU	0.92***	1.89**	-0.97
	(3.73)	(2.33)	(-1.26)

Table 4: Information-uncertainty-amplified overconfidence

This table summarizes information-uncertainty amplified perceived variances at $t = 1$ and $t = 2$ for private signals of winners and losers under four market state, from Equation (61). UU (DD) represents that both lagged and contemporaneous market states are UP (DOWN). UD (DU) means that the lagged market state is UP (DOWN) and the contemporaneous market state is DOWN (UP).

Market States	$\sigma^2_{1,w}$	$\sigma^2_{2,w}$	$\sigma^2_{1,l}$	$\sigma^2_{2,l}$
UU	$k_+\sigma^2_\epsilon$	$k^2_+\sigma^2_\epsilon$	σ^2_ϵ	σ^2_ϵ
DD	σ^2_ϵ	σ^2_ϵ	$q_D k_+\sigma^2_\epsilon$	$q_D k^2_+\sigma^2_\epsilon$
UD	$k_+\sigma^2_\epsilon$	$q_D k_+ k_-\sigma^2_\epsilon$	σ^2_ϵ	$q_D k_+ k_-\sigma^2_\epsilon$
DU	σ^2_ϵ	$k_+ k_-\sigma^2_\epsilon$	$q_D k_+\sigma^2_\epsilon$	$k_+ k_-\sigma^2_\epsilon$

Table 5: Asymmetry between winners and losers: model results

This table is the model counterpart of the empirical results in Table 3. It reports abnormal returns of the winner, loser, and momentum portfolios from the model with the calibrated parameters: $k_+ = 0.96$, $k_- = 1.03$, $q_D = 0.97$, $\sigma^2_\epsilon = 0.33$, and $\sigma^2_\eta = 0.32$. Abnormal returns of the winner, loser, and momentum portfolios are denoted by WIN, LOS, and MOM, respectively. These abnormal returns are calculated by plugging information-uncertainty-augmented perceived variances in Table 4 into the formulas of abnormals returns in Equations (32) to (34). These abnormal returns are in percentages.

Market States	WIN	LOS	MOM
UU	1.31	0	1.31
DD	0	-1.31	1.31
UD	-0.05	-1.26	1.21
DU	0.34	1.89	-1.55

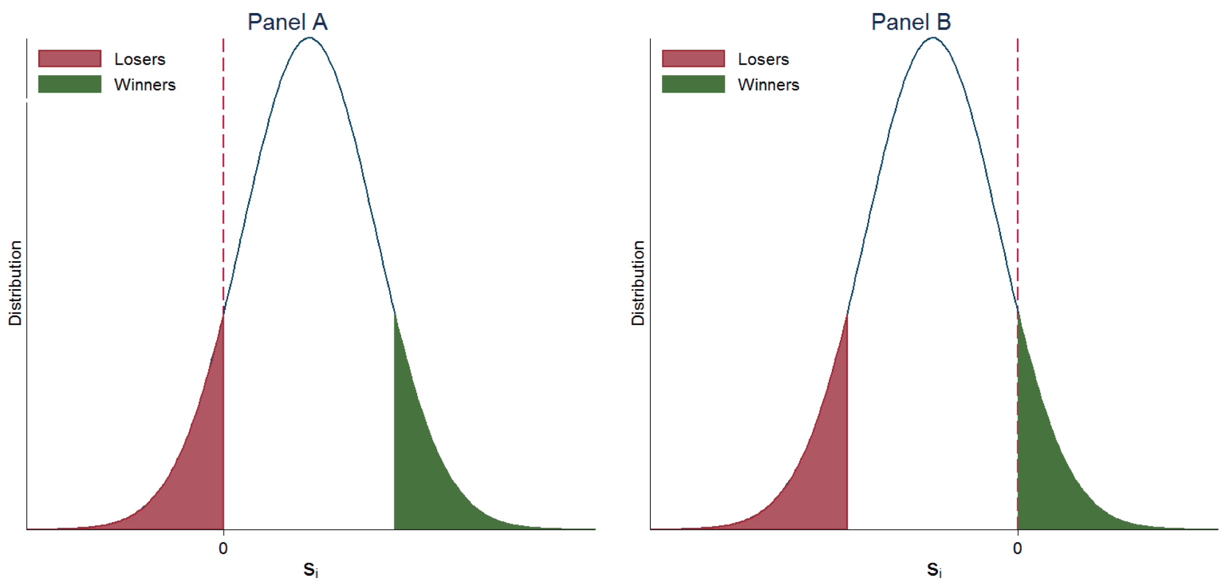

Figure 1: **Definition of winners and losers.** This figure illustrates two cases of cross-sectional distribution of N signals s_i for N risky assets and the corresponding definition of winners and losers. In both panels, the shaded area on the left corresponds to losers, and the shaded area on the right corresponds to winners. Dashed vertical lines indicate $s_i = 0$. Panel A illustrates a case in which $f + \epsilon_f$ is positive and therefore, there are more assets with positive s_i than with negative s_i. Panel B illustrates a case in which $f + \epsilon_f$ is negative and therefore, there are more assets with negative s_i than with positive s_i.

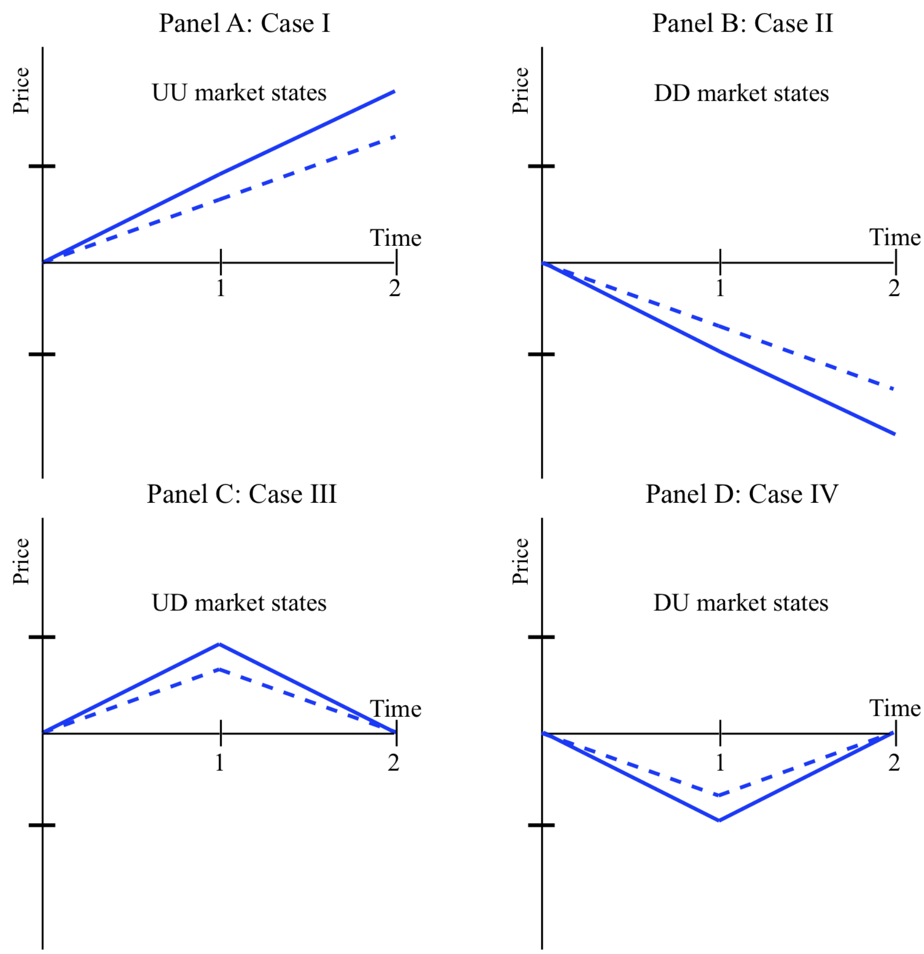

Figure 2: **Market price movement.** This figure illustrates the price movement of the market in the four cases of public signals. Solid lines represent price when investors possess self-attribution bias and overconfidence, and dashed lines represent price with rational investors.

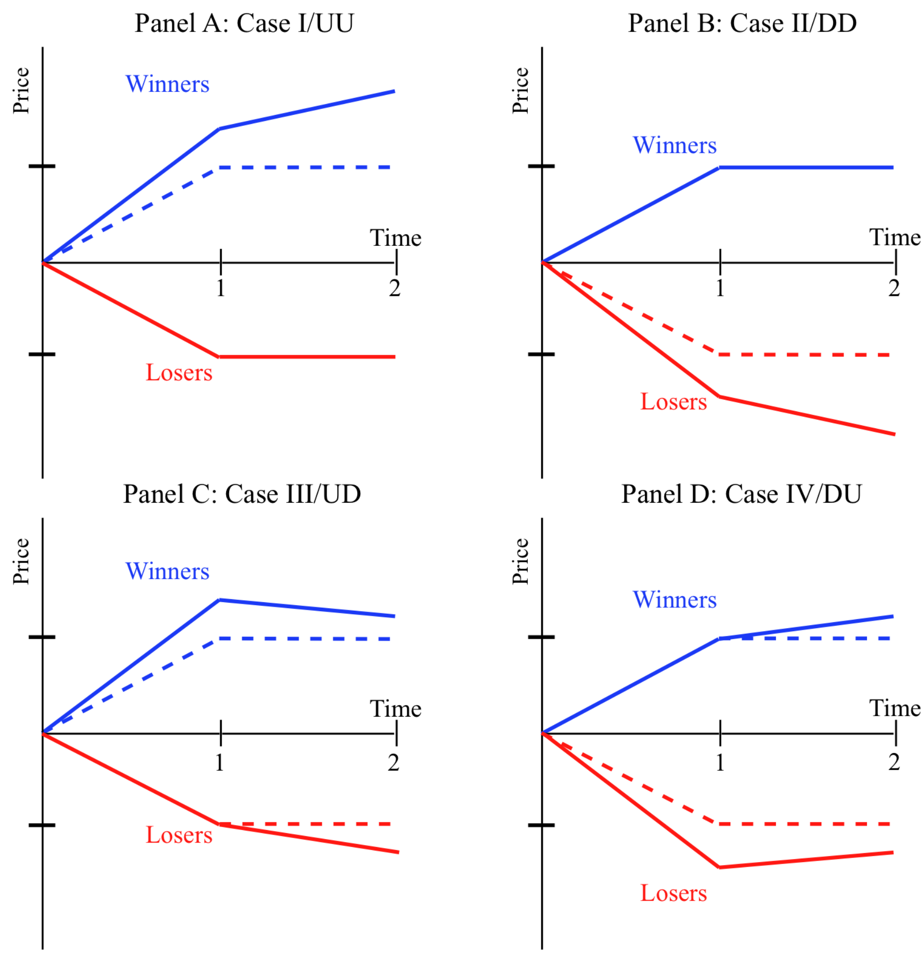

Figure 3: **Cross-sectional difference in price movement.** This figure illustrates the price movement driven by private signals for winners and losers in the four cases of public signals, or equivalently under four market states. Solid lines represent price when investors possess self-attribution bias and overconfidence, and dashed lines represent price with rational investors. Blue lines (above the horizontal time axis) correspond to winners, and red lines (below the horizontal time axis) correspond to losers. For clarity, only average price movement driven by private signals is plotted, since price movement driven by public signals is the same between winners and losers and does not contribute to cross-sectional difference.