Should Buyers use Procurement Service Providers when Suppliers have Private Information about Supply Disruptions?

Zhibin (Ben) Yang * Volodymyr Babich †

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Abstract

We consider a supply chain with one buyer and multiple suppliers, who are subject to disruptions and whose likelihoods of disruption are their private information. In such setting, should the buyer procure directly from the suppliers or engage services of a better-informed procurement service provider (PSP)? Conventional wisdom says that hiring a PSP is always the right choice, because the PSP's knowledge of the supply base improves supplier selection and management. On the other hand, earlier studies prove that using a PSP purely for its superior knowledge about supply costs is always worse for a buyer than contracting with the suppliers directly. Our answer to this research question is more nuanced. Contrary to the findings of the earlier studies, the buyer may benefit from using a PSP. We identify, quantify, and explain all of the benefits and the costs of using a PSP, and describe conditions under which benefits exceed the costs. The positive value of using a PSP is derived with some combinations of suppliers from reduction of information costs due to supplier collusion and improved supply availability. The negative value of using a PSP with some combinations of suppliers comes from the loss of direct control over the supplier's production actions, which leads to reduced supply availability, and increased informational costs; and from the PSP facilitating implicit supplier collusion. Comparative statics analysis indicates that the PSP is valuable only if the buyer diversifies in one procurement model, but not the other one; and that hiring a PSP is not a solution to the problem of unreliable supply base.

*Lundquist College of Business, University of Oregon, Eugene, OR 97403. E-mail: zyang@uoregon.edu

†McDonough School of Business, Georgetown University, Washington, DC 20057. E-mail: vob2@georgetown.edu

1 Introduction

1.1 Research Problem

We investigate whether a buyer, worried about supply risk, should procure directly from the suppliers or engage services of a procurement intermediary firm—a procurement service provider (PSP)—due to the PSP's better information about supply risk. PSP services are popular, and examples of procurement service providers include Li&Fung, Procurian, GEP, IBM, Accenture, and Xerox. There are many benefits that PSPs bring (Wu 2004 offers a classification of procurement intermediation benefits). However, we focus on informational benefits and costs and their interplay with supply risk management.

An immediate intuitive answer to our research question might appear to be "yes—the buyer should use a PSP." Armed with better knowledge about the supply base (e.g., suppliers' costs, capabilities, quality, and reliabilities), a PSP should be able to execute procurement better, selecting the right suppliers and reducing the information costs of procurement. Access to better information is a part of the overall "value proposition" that PSPs advertise to potential clients (buyers).

However, on reflection, another intuitive answer to our research question is "no—the buyer should not use a PSP." It is not obvious that PSPs would share benefits of better information with the buyer. In fact, when using a PSP, the buyer adds another layer to its supply chain, compounding fixed costs in the system and introducing double marginalization. The buyer relinquishes the direct control over procurement and supply-risk management, and cannot guarantee that the PSP will act in the buyer's best interests.

Prior work in economics on intermediation supports the "no" answer. For instance, Mookherjee and Tsumagari (2004) prove that when the PSP has informational advantage over the buyer with respect to the supplier's production costs of substitutable products, the buyer will always be better off contracting with the suppliers directly. This is a surprising result, in light of the popularity of PSP services in practice. It means that the popularity of the procurement outsourcing cannot be explained by informational benefits alone and, thus, the value of the procurement outsourcing must come from transactional or other benefits (e.g., economies of scale, reduced logistics costs, access to a larger supplier pool).

Our analysis shows that the answer to the question on whether a buyer should use a PSP because of the informational benefits is "maybe." Furthermore, we can explain how the benefits of using a PSP are derived and what costs may counterbalance the benefits. Unlike Mookherjee and Tsumagari (2004) and most of other papers on intermediation that emphasize supply costs, we focus on supply risk and asymmetric information about supply risk. Similar to Mookherjee and Tsumagari (2004), we disregard transactional benefits of using a PSP, because the effects of these benefits are predictable. In our model (§2), the suppliers are unreliable, and possess private information about their own reliabilities. The PSP has access to the suppliers' private information. Methodologically, we solve a mechanism design problem in which an agent (the PSP) has two dimensions of private information about supply risk (§3). In general, mechanism design problems with multi-dimensional private information are difficult to solve. However, we are able to derive the optimal procurement contracts for the buyer and the corresponding decisions of the PSP and the suppliers. We contrast these results with the optimal procurement policies in the benchmark model of direct procurement (§4).

In doing so, we derive a number of interesting and practical results. First, contrary to insights from Mookherjee and Tsumagari (2004), there are situations in which the value of using a PSP is positive for the buyer (§5). Second, we decompose the buyer's value of using a PSP by the combination of the suppliers' types, and offer economic insights into forces acting on the value. We explain that the positive value of using a PSP is derived from a reduction of information costs of procurement and improvement of the supply availability (due to a better execution of order diversification by the PSP). We quantify the negative value of using a PSP and show that it comes from the loss of direct control of the supplier's production actions, from the PSP facilitating implicit supplier collusion, and from reduced the supply availability in some cases. Interestingly, supplier collusion under PSP procurement may affect the buyer not only negatively, but also positively. The explanations of various costs and benefits constituting the net value of using a PSP are novel and practically important. Third, by reversing Mookherjee and Tsumagari (2004)'s conclusions, we highlight the importance of supply risk management and the interplay between contracting, asymmetric information, and risk management.

Comparative statics analysis (§6) offers several managerially important observations on the value of using a PSP. We prove that the value of using a PSP is always negative if the disruption cost is too small or too larger, when the buyer will either never diversify or always diversify regardless of the procurement model. Interestingly, we show that the positive PSP value can arise only if the buyer diversifies in one and single-sources in the other procurement model. Numerical analysis indicate that to derive positive PSP benefits, the fraction of reliable suppliers in the economy should be high, and as that fraction increases the value of using a PSP can either increase or decrease. Therefore, unreliable supply base is not a good reason to hire a PSP. Similarly, as the gap between reliabilities increases, the value of using a PSP may decrease, contrary to the a priori intuition. We summarize managerial insights and conclude in §7.

1.2 Literature Review

Our work builds on the extensive research in economics and Operations Management on the use of procurement intermediation. In this literature, Wu (2004) presents bargaining frameworks with complete and incomplete information for modeling supply chain intermediation and review various benefits that intermediation generates. In particular, Wu (2004) classifies benefits into transactional and informational. Belavina and Girotra (2012) show that, beyond transaction and informational benefits, there is also a relational benefit of intermediation. Belavina and Girotra (2012) use the framework of relational contracts to show how a PSP can influence behaviors of both the suppliers and the buyers with an implicit promise of future business. Our approach is that of mechanism design (similar to Mookherjee and Tsumagari 2004 and a number of other papers in information economics), but, unlike most of the prior work, we study the PSP's informational advantage over the buyer with respect to the likelihood of supply disruption. Adida et al. (2012) observe that a PSP can be valuable when supply risk is present. However, they do so for a model without asymmetric information about supply disruptions.

For a benchmark model where the buyer directly contracts with the suppliers, we rely on the results derived by Yang et al. (2012). Other papers that consider asymmetric information about supply risk are Yang et al. (2009); Chaturvedi and Martínez-de Albéniz (2011); Gümüş et al. (2012). However, these papers do not study the use of procurement service providers. These papers, together with our work, belong to the research area of decentralized supply risk management. A review of this research area is available in Aydin et al. (2012).

2 The General Model Setting

In this paper, we shall compare two procurement models: direct procurement and PSP procurement (see Figure 1). As the name implies, under direct procurement, the buyer contracts directly with the suppliers; under PSP procurement, the buyer contracts with a PSP, who then contracts with the suppliers. In both models, the suppliers know their reliabilities, but the buyer only has probabilistic estimates of them. In the latter model, the PSP also knows the supplier's reliabilities. To highlight the effect of supply risk on the buyer's value of using PSP procurement and the interaction between supply risk and intermediation under asymmetric information, we choose the model setting that mirrors that in Mookherjee and Tsumagari (2004), except that in our model asymmetric information is about supply risk, but not supply cost as in Mookherjee and Tsumagari (2004).

The direct- and PSP-procurement models share many modeling assumptions, as follows. We shall introduce the model-specific setting for the two models in §§3 and 4.

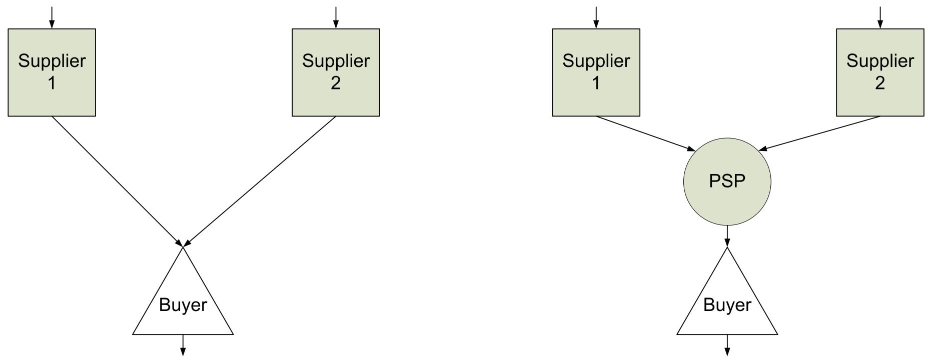


Figure 1: Procurement models: direct procurement (left) and PSP procurement (right)

Both models are of one period, with one buyer and two suppliers. It is possible for only one of the two suppliers in the supply base to receive an order.

A supplier may experience a production disruption, which destroys the entire production batch. The supplier's production cost is incurred as soon as production begins, and cannot be recovered if a disruption happens. Supplier-i's (where $i = 1, 2$) random production yield is ρ_i, where $\rho_i = 1$ with probability θ_i and $\rho_i = 0$ with probability $1 - \theta_i$.

Two types of supplier exist, with the reliability being either "high" or "low," (also denoted as H and L) as measured by the probability of a successful production run. That is, $\theta_i = h$ or $\theta_i = l$ for supplier i, where $0 < l < h < 1$. Because there are two suppliers, the system may have (H,H), (H,L), (L,H), and (L,L) combinations of the suppliers' types. In the economy, the fraction of H-type suppliers is α^H and the fraction of L-type suppliers is α^L, where $\alpha^H + \alpha^L = 1$.

Unit production cost of a supplier depends on its type. Specifically, a high-type supplier-i's production cost is $c_i = c^H$ per unit; and a low-type supplier-i's production cost is $c_i = c^L$ per unit. Because the high type's probability of running successful production is h, its expected cost of yielding one good unit is c^H/h. Similarly, the low type's expected cost of yielding one good unit is c^L/l. Cost c^L may be greater or smaller than c^H, but we assume that $c^L/l > c^H/h$, to model the situation where the less reliable supplier is less cost-efficient in expectation.

To focus on the role of supply risk, we shall assume that the buyer operates in a make-to-order system, and at the time the buyer places orders it knows both the demand D and unit revenue r in the consumer market. Without loss of generality, we normalize the buyer's market demand to be $D = 1$. We can relax the assumption of deterministic r and think of r as the expected revenue or, alternatively, the expected cost of a replacement product on a spot market. Thus, r represents the expected cost of product shortage for the buyer. We restrict the revenue to be greater than the high-reliability supplier-type's expected cost of production.

Assumption 1. $r > c^H/h$.

If this assumption is not satisfied, the buyer will not order in either procurement model.

Finally, all firms have zero reservation values, and would do business as long as this reservation value is met.

3 The PSP-Procurement Model and Solution

3.1 Contracts

In this model, the buyer contracts with the PSP, who then brokers production with the suppliers. This procurement problem is especially challenging, because the suppliers possess private information about their reliability types and the PSP knows them. In contrast, the buyer believes a supplier's reliability is high with probability α^H and is low with probability α^L. The buyer's beliefs regarding the two suppliers' reliability types are identical and independent. The PSP's superior knowledge relative to the buyer may be due to the PSP's history with the suppliers, or geographic proximity, or better knowledge of the suppliers' technologies. Thus, there is information asymmetry between the buyer and the PSP. This is the key feature of this model.

In general, there are an infinite number of mechanisms that might be employed by the principal (the buyer) to govern its interactions with the agent (the PSP) with private information. However, as is usually done in information economics and the contracting literature, to simplify the analysis, we invoke the Revelation Principle and focus only on the class of direct and incentive-compatible mechanisms. Under such mechanisms, the PSP will send the buyer messages that report the suppliers' reliability types, and the buyer structures the contracts in a way such that these reports will be truthful.

The PSP's private information is two-dimensional: one dimension per each supplier. Because each supplier is either H or L type, the PSP can send one of the following four messages: (H,H), (H,L), (L,H), (L,L). Furthermore, in this problem, because the suppliers are *ex ante* symmetric, we can combine (H,L) and (L,H) types/messages. Specifically, the PSP can be (H,H), (H,L) and (L,L) types, in decreasing reliability. Based on its prior beliefs about the suppliers' types, the buyer infers that the PSP is (H,H)-type with probability $(\alpha^H)^2$, (H,L)-type with probability $2(\alpha^H\alpha^L)$, and (L,L)-type with probability $(\alpha^L)^2$. This problem is difficult to solve in that the agent (the PSP) has a discrete type space of more than two types. We shall discuss the technical challenge and our solution procedure in §3.4.

The contract menu between the buyer and the PSP specifies payments and quantities, for each possible message the buyer may receive. Each contract in the menu has three parts: (X, q, p). X is the lump-sum upfront payment from the buyer to the PSP, regardless of the delivery status. q is the quantity that the buyer desires to receive from the PSP. p is the penalty for each unit the PSP falls short from the desired quantity q. Thus, the buyer offers the following menu of contracts to the PSP: $(X, q, p)(H, H)$, $(X, q, p)(H, L)$ and $(X, q, p)(L, L)$.

The lump-sum and the quantity terms are familiar features of procurement contracts. The penalty term p is needed in the contract in the model with supply risk to provide economic incentive for the PSP to deliver goods. It obviates the need to make a strong assumption that production events at the suppliers are contractible (which is demonstrably not true in practice, especially if the buyer and the suppliers are located in different countries, or even different continents). In other words, in the environment where the delivery of goods is uncertain, one needs a variable payment, like the penalty term, to prevent the "take the money and run" behavior of agents.

Having seen the buyer's contract menu, the PSP signs contracts with the suppliers, specifying production quantities and payments. In this model, the PSP is the principal with respect to the suppliers, the contracts between the PSP and the suppliers are general, and the PSP knows the suppliers' types. Therefore, the PSP will always be able to integrate the subsystem consisting of itself and the suppliers and keep the profit of the entire subsystem (e.g., using two-part tariffs). For simplicity, we shall not model contracting between the PSP and the suppliers, but shall study their integrated actions and profits.

The summary of the timing in the PSP-procurement model is: (1) nature reveals the suppliers' types to the suppliers and the PSP; (2) the buyer offers a menu of contracts to the PSP, and the PSP sends the message about its type to the buyer; (3) the PSP receives the payment X from the buyer, and the PSP integrates the supplier subsystem and assigns production to the suppliers; (4) the suppliers begin production, incurring production costs; (5) production uncertainty is realized, and the PSP delivers goods to the buyer, paying penalty p per unit in case of a shortfall from the requirement, q; and finally (6) the buyer sells available goods in the consumer market up to quantity $D = 1$, receiving revenue r.

We shall solve the problem in the reverse chronological order, analyzing first the PSP's production assignment in the next subsection. We shall then use the solution of the PSP's problem in designing the buyer's contract in §3.4.

3.2 The PSP's Optimal Production Assignment

In this subsection, we solve the PSP's production assignment problem, given any contract (X, q, p) from the buyer. Recall that the PSP is able to integrate the PSP-suppliers subsystem and keep the entire subsystem's profit. The PSP's chooses the production assignments to the suppliers, (z_1, z_2), to maximize the expected subsystem profit. Program (1) models the PSP's decision, given the suppliers' reliability types $(t_1, t_2) \in \{(H, H), (H, L), (L, H)\}$:

$$\pi^{t_1,t_2}(X, q, p) = X - \min_{z_1 \geq 0,\, z_2 \geq 0} \left\{ c^{t_1} z_1 + c^{t_2} z_2 + pE\left(q - \rho_1^{t_1} z_1 - \rho_2^{t_2} z_2\right)^+ \right\}. \tag{1}$$

When $(t_1, t_2) = (L, H)$, the PSP's profit $\pi^{LH}(X, q, p) \equiv \pi^{HL}(X, q, p)$, because the PSP is indifferent between type pairs (H,L) and (L,H).

In (1), term $c^{t_i} z_i$ is the production cost of supplier i, and $\rho_i^{t_i} z_i$ is the quantity supplier i delivers, where $\rho_i^{t_i}$ is the production yield of supplier $i = 1, 2$. The expectation is taken over the suppliers' random production yields $(\rho_1^{t_1},\ \rho_2^{t_2})$. We note that the payment X is irrelevant in search for the optimal production assignments.

Lemma 1 presents the PSP's optimal production assignments $(z_1^*, z_2^*)(q, p)$.

Lemma 1. *Under the manufacturer's contract (X, q, p) and $q > 0$, the supplier coalition's optimal production sizes, $(z_1^*, z_2^*)(q, p)$ are:*

(t_1, t_2)	Penalty	$(z_1^*, z_2^*)(q, p)$
(H,H)	$p < \frac{c^H}{h}$	$(0, 0)$
	$\frac{c^H}{h} \leq p < \frac{c^H}{h(1-h)}$	$(q, 0)$
	$p \geq \frac{c^H}{h(1-h)}$	(q, q)
(H,L) and (L,H)	$p < \frac{c^H}{h}$	$(0, 0)$
	$\frac{c^H}{h} \leq p < \frac{c^L}{l(1-h)}$	$(q, 0)$ for (H,L); $(0, q)$ for (L,H)
	$p \geq \frac{c^L}{l(1-h)}$	(q, q)
(L,L)	$p < \frac{c^L}{l}$	$(0, 0)$
	$\frac{c^L}{l} \leq p < \frac{c^L}{l(1-l)}$	$(q, 0)$
	$p \geq \frac{c^L}{l(1-l)}$	(q, q)

We delegate all proofs in this paper to Appendix C.

We now introduce the PSP's profit from production. For this, we first define two intermediate expressions that will be used repeatedly in the rest of the paper. We first define the PSP's incremental *benefit of sole-sourcing* (i.e., sourcing from one supplier relative to none):

$$\psi^{t_1}(q, p) \stackrel{\text{def}}{=} (\theta^{t_1} p - c^{t_1})q, \text{ for } t_1 \in \{H, L\}, \tag{2}$$

where $\theta^H \stackrel{\text{def}}{=} h$ and $\theta^L \stackrel{\text{def}}{=} l$. When the PSP sole-sources from a supplier of type t_1, the PSP-supplier subsystem avoids penalty p if the supplier's production is successful (with probability θ^{t_1}), but incurs production cost of c^{t_1} per unit.

Next, we define the PSP's incremental *benefit of diversifying* (i.e., sourcing from two suppliers relative to one):

$$\psi^{t_1,t_2}(q,p) \stackrel{\text{def}}{=} [\theta^{t_2}(1-\theta^{t_1})p - c^{t_2}]q, \text{ for } (t_1,t_2) \in \{(H,H),(H,L),(L,L)\}. \tag{3}$$

Given that the PSP has assigned production to the supplier of type t_1, the production assignment to the second supplier of type t_2 helps the PSP-suppliers subsystem avoid the penalty with probability $\theta^{t_2}(1-\theta^{t_1})$. This probability represents the likelihood of the joint events that the first supplier's production fails and the second supplier produces successfully.

Using these definitions, we can concisely write the PSP's expected profit to be the sum of incremental benefits relative to the profit of taking no production action, $X - p\,q$.

Lemma 2. *Under contract* (X,q,p)*, the expected profit of the PSP of type* (t_1,t_2) *is*

$$\pi^{t_1,t_2}(X,q,p) = (X - p\,q) + \left[\psi^{t_1}(q,p)\right]^+ + \left[\psi^{t_1,t_2}(q,p)\right]^+, \tag{4}$$

where $(t_1,t_2) \in \{(H,H),(H,L),(L,L)\}$.

3.3 Reliability Advantages of the More Reliable Types of PSP

For a fixed contract (X,q,p), as the PSP's reliability type increases from (L,L), to (H,L) and to (H,H) the PSP's profit increases. The (H,H) type of the PSP earns a premium over the (H,L) type, because the (H,H) type incurs a smaller total expected cost and penalty than the (H,L) type. We refers to the premium earned by (H,H) over (H,L) as the (H,H) type's *reliability advantage* over the (H,L) type. Similarly, the (H,L) type earns a reliability advantage over the (L,L) type. We shall use the following notation for these reliability advantages:

$$\Gamma^{HH}_{HL}(q,p) \stackrel{\text{def}}{=} \pi^{HH}(X,q,p) - \pi^{HL}(X,q,p), \text{ and} \tag{5}$$

$$\Gamma^{HL}_{LL}(q,p) \stackrel{\text{def}}{=} \pi^{HL}(X,q,p) - \pi^{LL}(X,q,p). \tag{6}$$

From Lemma 2,

$$\Gamma^{HH}_{HL}(q,p) = \left[\psi^{HH}(q,p)\right]^+ - \left[\psi^{HL}(q,p)\right]^+, \text{ and} \tag{7a}$$

$$\Gamma^{HL}_{LL}(q,p) = \left[\psi^{H}(q,p)\right]^+ + \left[\psi^{HL}(q,p)\right]^+ - \left[\psi^{L}(q,p)\right]^+ - \left[\psi^{LL}(q,p)\right]^+. \tag{7b}$$

One can apply the expressions for $\psi^t(q,p)$ and $\psi^{t_1,t_2}(q,p)$ in (2) and (3) to obtain the expressions of $\Gamma^{HH}_{HL}(q,p)$ and $\Gamma^{HL}_{LL}(q,p)$. The results are presented in Lemma 6 in Appendix B.

The concept of reliability advantage will play a critical role in the design of the buyer's contracts. Ideally, the buyer would want to design a menu of contracts where each PSP type earns zero profit if the PSP selects the menu option intended for its type. However, a more reliable PSP-type has an incentive to pretend to be a less reliable type (as long as the latter type is invited to do business with the buyer) and to earn a positive profit in the form of reliability advantage. That is, the (H,H) type would want to select a contract intended for the (H,L) or (L,L) types and the (H,L) type would want to pretend to be the (L,L) type. We shall discuss these incentive issues in §3.4.

3.4 The Buyer's Optimal Contract Menu for the PSP

In this section, we analyze the buyer's mechanism design problem, knowing the PSP's optimal production assignments to the suppliers (§3.2) and the more reliable PSP types' incentive of misrepresentation due to their reliability advantages (§3.3). We present the model in (8), outline the solution procedure, and present the optimal contract menu in Proposition 1.

The buyer's optimal contract design problem is presented formally in optimization program (8). There, $(z_1^*, z_2^*)(t_1, t_2) \stackrel{\text{def}}{=} (z_1^*, z_2^*)\big((q,p)(t_1,t_2)\big)$ are the PSP's optimal production assignment under the contract designed for the (t_1, t_2) type of the PSP. $\pi^{t_1,t_2}(s_1, s_2) \stackrel{\text{def}}{=} \pi^{t_1,t_2}\big((X,q,p)(s_1,s_2)\big)$ is the expected profit of PSP of type (t_1, t_2), when it reports type (s_1, s_2) and receives contract $(X, q, p)(s_1, s_2)$.

$$\max_{\substack{(X,q,p)(t_1,t_2),\\ t_1 \text{ and } t_2\in\{H,L\}}} \quad \sum_{t_1,t_2\in\{H,L\}} \alpha^{t_1}\alpha^{t_2}\Big\{ rE\min\Big\{D, \min\big\{q(t_1,t_2), \rho_1^{t_1} z_1^*(t_1,t_2) + \rho_2^{t_2} z_2^*(t_1,t_2)\big\}\Big\}$$
$$- X(t_1,t_2) + p(t_1,t_2)E\Big[q(t_1,t_2) - \rho_1^{t_1} z_1^*(t_1,t_2) - \rho_2^{t_2} z_2^*(t_1,t_2)\Big]^+\Big\}$$

Subject to

$$\text{(IC.HH)} \quad \pi^{HH}(H,H) \geq \pi^{HH}(H,L) \text{ (local, downward)} \tag{8a}$$

$$\pi^{HH}(H,H) \geq \pi^{HH}(L,L) \text{ (global, downward)} \tag{8b}$$

$$\text{(IC.HL)} \quad \pi^{HL}(H,L) \geq \pi^{HL}(H,H) \text{ (local, upward)} \tag{8c}$$

$$\pi^{HL}(H,L) \geq \pi^{HL}(L,L) \text{ (local, downward)} \tag{8d}$$

$$\text{(IC.LL)} \quad \pi^{LL}(L,L) \geq \pi^{LL}(H,H) \text{ (global, upward)} \tag{8e}$$

$$\pi^{LL}(L,L) \geq \pi^{LL}(H,L) \text{ (local, upward)} \tag{8f}$$

$$\text{(IR)} \quad \pi^{HH}(H,H) \geq 0,\ \pi^{HL}(H,L) \geq 0, \text{ and } \pi^{LL}(L,L) \geq 0$$

$$X(t_1,t_2) \geq 0,\ q(t_1,t_2) \geq 0,\ r \geq p(t_1,t_2) \geq 0.$$

The buyer maximizes its expected profit of sourcing via the PSP (before know the PSP's type), subject to the incentive compatibility (IC) and individual rationality (IR) constraints. Because this problem features an agent (the PSP) of three discrete types, the PSP's incentive compatibility is enforced by two categories of constraints: ones that prevent a type from falsely reporting it to be adjacent types and to be non-adjacent types, respectively. In the principal-agent theory, the former are known as *local* IC, and the latter as *global* IC constraints (see Laffont and Martimort, 2002). In our problem, where the agent has three possible types, besides the local IC constraints on all three types, we also impose a global IC constraint on the (H,H) and (L,L) type, respectively, to prevent them from mimicking each other. Furthermore, the IC constraints are directional. An IC constraint that prevent a less reliable type from reporting to be of a more reliable type is an *upward* IC constraint, and is a *downward* IC constraint vice versa. The IR constraints ensure that the PSP's payoff from the game is greater than its reservation profit, which we normalize to be zero. Finally, we impose the "limited liability" constraints on the shortage penalties, which specify that the PSP does not pay a penalty that exceeds the buyer's loss of revenue due to shortage.

We are able to fully characterize the optimal solution when the model parameters satisfy Condition 1 (defined in Appendix A). Condition 1 ensures that the incentive compatibility constraints are non-binding at the optimal solution, and hence we can solve the mechanism design problem following the canonical solution procedure, such as the one in Myerson (1981). In Proposition 1, we present the results under Conditions 1, using these definitions:

$$\psi^t \stackrel{\text{def}}{=} \psi^t(q,p) \text{ at } (q,p) = (1,r), \qquad \text{for } t \in \{H,L\}, \text{ and} \tag{9a}$$

$$\psi^{t_1,t_2} \stackrel{\text{def}}{=} \psi^{t_1,t_2}(q,p) \text{ at } (q,p) = (1,r), \qquad \text{for } (t_1,t_2) \in \{(H,H),(H,L),(L,L)\}. \tag{9b}$$

Proposition 1. *Under the model parameter values that satisfy Condition 1, the buyer's optimal order quantity, $q^*(t_1,t_2)$, and penalty, $p^*(t_1,t_2)$, for the PSP and the PSP's production assignments to the suppliers, $(z_1^*,z_2^*)(t_1,t_2)$, are:*

Model parameters	*Quantity, q^**	*Penalty, p^**	*PSP assignment, (z_1^*,z_2^*)*
(H,H) PSP-type			
$(\alpha^H)^2\psi^{HH} < 0$	1	r	$(1,0)$
$(\alpha^H)^2\psi^{HH} \geq 0$		r	$(1,1)$
(H,L) and (L,H) PSP-types			
$2(\alpha^H\alpha^L)\psi^{HL} - (\alpha^H)^2\phi < 0$	1	$\min\left\{r, \frac{c^H}{h(1-h)}\right\}$	$(1,0)$ *for (H,L)* $(0,1)$ *for (L,H)*
$2(\alpha^H\alpha^L)\psi^{HL} - (\alpha^H)^2\phi \geq 0$		$\frac{c^L}{l(1-h)}$	$(1,1)$
(L,L) PSP-type			
$(\alpha^L)^2\psi^L - [1-(\alpha^L)^2]\phi < 0$ *and* $(\alpha^L)^2(\psi^L+\psi^{LL}) - [1-(\alpha^L)^2]\omega < 0$	0	0	$(0,0)$
$(\alpha^L)^2\psi^L - [1-(\alpha^L)^2]\phi > \left\{(\alpha^L)^2(\psi^L+\psi^{LL}) - [1-(\alpha^L)^2]\omega\right\}^+$	1	$\frac{c^L}{l}$	$(1,0)$
$(\alpha^L)^2(\psi^L+\psi^{LL}) - [1-(\alpha^L)^2]\omega \geq \left\{(\alpha^L)^2\psi^L - [1-(\alpha^L)^2]\phi\right\}^+$		$\tilde{p}$	$(1,1)$

where

$$\tilde{p} \stackrel{def}{=} \begin{cases} \frac{c^L}{l(1-l)} & \text{if } (1-l)^2-(1-h)\geq 0 \\ \min\left\{r, \frac{c^L}{l(1-h)}\right\} & \text{if } (1-l)^2-(1-h)<0 \text{ and } r\geq \frac{c^L}{l(1-l)} \\ n.a. & \text{otherwise} \end{cases}, \tag{10a}$$

$$\phi \stackrel{def}{=} \Gamma^{HH}_{HL}\left(1, \frac{c^L}{l(1-h)}\right) = \Gamma^{HL}_{LL}\left(1, \frac{c^L}{l}\right) = h\left(\frac{c^L}{l}-\frac{c^H}{h}\right), \text{ and} \tag{10b}$$

$$\omega \stackrel{def}{=} \Gamma^{HL}_{LL}\left(1, \tilde{p}\right). \tag{10c}$$

The optimal payment $X^*(t_1, t_2)$ *can be derived from equations* (18) *provided in Remark 1 in Appendix A. The buyer's expected profit under the optimal contract menu is*

$$(\alpha^H)^2\psi^H + (\alpha^H)^2\left(\psi^{HH}\right)^+ \tag{11a}$$

$$+\,2(\alpha^H\alpha^L)\psi^H + \left[2(\alpha^H\alpha^L)\psi^{HL} - \left(\alpha^H\right)^2\phi\right]^+ \tag{11b}$$

$$\left.\begin{aligned} &+\left\{(\alpha^L)^2\psi^L - [1-(\alpha^L)^2]\phi\right\}^+ \\ &+\left(\left\{(\alpha^L)^2(\psi^L+\psi^{LL}) - [1-(\alpha^L)^2]\omega\right\} - \left\{(\alpha^L)^2\psi^L - [1-(\alpha^L)^2]\phi\right\}^+\right)^+. \end{aligned}\right\} \tag{11c}$$

It is useful to discuss, in general, the buyer's tradeoffs when designing the menu of contracts with the PSP. In the face of the PSP's incentives to misrepresent its type to benefit from the reliability advantage (see §3.3), the buyer has several choices. The buyer can simply resign to the fact that the PSP of a more reliable type will earn a positive profit. In this case, the buyer pays the more reliable PSP-types enough to make their lying unnecessary. For instance, the buyer can offer the same contract to the (H,H) and (H,L) types of the PSP, effectively paying the (H,H) type its *reliability advantage* over the (H,L) type. Alternatively, the buyer may fight the more reliable types' incentives, by altering the contracts with the less reliable types, so that lying becomes less lucrative. For example, if the buyer stops doing business with the PSP of type (H,L), this would eliminate the incentive of the (H,H) type to pretend to be (H,L) type. However, by altering the contract with the less reliable PSP-types, the buyer may be deviating from the first-best procurement actions, and the entire system loses profits. The economic insight into the contract design problem in program (8) is that the buyer's choice is a tradeoff between having to pay for information (we shall refer to this as the *information cost*) and deviating from the first-best procurement actions and thus losing profits for the system in the process.

We now introduce the definitions of information costs ϕ and ω in (10b) and (10c), because they will be used repeatedly in the rest of the paper. $\phi \stackrel{\text{def}}{=} \Gamma^{HH}_{HL}\left(1, \frac{c^L}{l(1-h)}\right)$ in (10b) represents the buyer's information cost of using diversification with the (H,L) (or (L,H)) PSP-type. This information cost arises from the (H,H) PSP-type's reliability advantage over the (H,L) (or (L,H)) type, $\Gamma^{HH}_{HL}(q,p)$, when the buyer orders $q = 1$ and sets the optimal penalty $p = \frac{c^L}{l(1-h)}$ for the latter type to diversify. Due to this information cost, the buyer's expected incremental profit from using diversification with the (H,L) and (L,H) PSP-types, $2(\alpha^H \alpha^L)\psi^{HL}$, is reduced by $\left(\alpha^H\right)^2 \phi$ (see the term in $[\,]^+$ of (11b)).

The term ϕ in (10b) also coincides with the buyer's information cost of using sole-sourcing with the (L,L) PSP-type, that is, $\phi \overset{\text{def}}{=} \Gamma^{HL}_{LL}\left(1, \frac{c^L}{l}\right)$. This information cost arises from the (H,L) and (L,H) PSP-types' reliability advantage over the (L,L) type, $\Gamma^{HL}_{LL}(q,p)$, when the buyer orders $q = 1$ from the (L,L) PSP-type and sets the optimal penalty $p = c^L/l$ to induce it to sole-source. As a result of this information cost, the buyer's expected profit from using sole-sourcing with the (L,L) type, $(\alpha^L)^2\psi^L$, is reduced by $[1-(\alpha^L)^2]\phi$ (see the terms in the first $\{\,\}^+$ in (11c)).

Lastly in (10c), $\omega \overset{\text{def}}{=} \Gamma^{HL}_{LL}(1,\tilde{p})$ represents the buyer's information cost of using diversification with the (L,L) PSP-type under the optimal penalty $p = \tilde{p}$. This information cost also arises from the (H,L) and (L,H) PSP-types' reliability advantage over the (L,L) type. Because of this information cost, the buyer's expected profit from using diversification with the (L,L) PSP-type, $(\alpha^L)^2(\psi^L + \psi^{LL})$, is reduced by $[1-(\alpha^L)^2]\omega$ (see the terms in the first $\{\,\}$ in $(\,)^+$ of (11c)).

In §5, we shall analyze the economic forces for the PSP value under different levels of the revenue r. To prepare for this analysis, from the conditions of the model parameters (the leftmost column) in the table of Proposition 1, we define r^{HL}, r^L and r^{LL}.

$$r^{HL} \overset{\text{def}}{=} r \text{ such that } 2(\alpha^H\alpha^L)\psi^{HL} - \left(\alpha^H\right)^2\phi = 0, \tag{12a}$$

$$r^{L} \overset{\text{def}}{=} r \text{ such that } (\alpha^L)^2\psi^L - [1-(\alpha^L)^2]\phi = 0, \tag{12b}$$

$$r^{LL} \overset{\text{def}}{=} r \text{ such that } (\alpha^L)^2(\psi^L + \psi^{LL}) - [1-(\alpha^L)^2]\omega = \left\{(\alpha^L)^2\psi^L - [1-(\alpha^L)^2]\phi\right\}^+. \tag{12c}$$

r^{HL} defines the threshold of the revenue for the buyer to use diversification with the (H,L) and (L,H) PSP-types. r^L and r^{LL} are the thresholds of the revenue for the buyer's choice of procurement action with the (L,L) PSP-types. There are two cases, depending on whether $r^L < r^{LL}$ or not. If $r^L \le r^{LL}$, then the buyer takes no procurement action for $r < r^L$, uses sole-sourcing for $r^L \le r < r^{LL}$, and uses diversification for $r > r^{LL}$. If $r^L > r^{LL}$, then sole-sourcing will not be used. Specifically, the buyer takes no procurement action for $r < r^{LL}$, and uses diversification for $r \ge r^{LL}$.

4 The Direct-Procurement Model

In the direct-procurement model, the buyer orders directly from the suppliers. This is the benchmark model, needed to quantify the value of using the PSP. A similar direct-procurement model was studied in Yang et al. (2012), but with three main differences: in this paper, (1) the setup cost of working with a supplier is $K = 0$, (2) the buyer's market demand is normalized to be $D = 1$, and (3) instead of the variable payment term, we use the penalty term, p. We shall take advantage of the solution derived in Yang et al. (2012), and, without duplicating their analysis, present the results needed in this paper.

In contrast to PSP procurement (§3), in the direct-procurement model the principal (the buyer) faces two agents (the suppliers). However, each supplier has one-dimensional private information about its reliability type. The buyer offers a menu of four pairs of contracts: $(X_1, q_1, p_1)(t_1, t_2)$ and $(X_2, q_2, p_2)(t_1, t_2)$ for $(t_1, t_2) = (H, H)$, (H, L), (L, H) and (L, L).

Having received a contract, (X, q, p), from the buyer, supplier-i independently decides its production size, denoted as z_i, to maximize its expected profit. The supplier's production decision is represented by the optimization program $\max_{z_i \geq 0} \Big\{ X - c^{t_i}\, z_i - p\, E(q - \rho^{t_i}\, z_i)^+ \Big\}$, where t_i is supplier-i's true reliability type. The supplier's optimal production decision and expected profit are summarized in Lemma 3.

Lemma 3. *Under contract (X, q, p), the optimal production size of supplier-i, denoted as $z_i^{t_i}(q, p)$, is $z_i^{t_i}(q, p) = q$ if $p \geq c^{t_i}/\theta^{t_i}$, and $z_i^{t_i}(q, p) = 0$ othwise. The supplier's optimal expected profit is $\pi_i^{t_i}(X, q, p) = X - pq + \big[\psi^{t_i}(q, p)\big]^+$.*

In the supplier optimal profit $\pi_i^{t_i}(X, q, p)$, term $\psi^t(q, p)$ is the supplier's incremental benefit of running production, relative to the profit of taking no action, $X - q$. $\psi^t(q, p)$ coincides with the PSP incremental benefit of sole-sourcing from a supplier of type-t, which is defined in (2). The implication is that, in the face of the same contract for sole-sourcing, the integrated subsystem of the PSP and a single supplier enjoys the same profit as the supplier does under direct procurement.

Under the same contract, the high-reliability supplier type enjoys a premium compared to the low-reliability type, for being more reliable. For a given contract (X, q, p), we define the high type's premium over the low type as the high type's *reliability advantage*, and denote it as $\Gamma^{\frac{H}{L}}(q, p) \stackrel{\text{def}}{=} \pi_i^H(X, q, p) - \pi_i^L(X, q, p)$. Using the expression for $\pi_i^{t_i}(X, q, p)$ in Lemma 3, we derive the expression for $\Gamma^{\frac{H}{L}}(q, p)$ to be $\Gamma^{\frac{H}{L}}(q, p) = \big[\psi^H(q, p)\big]^+ - \big[\psi^L(q, p)\big]^+$.

With such reliability advantage, the high type has an incentive to pretend to be the low type to earn a positive profit. In the face of the suppliers' misrepresention incentives, the buyer designs its contract menu of direct procurement. The buyer's decision is represented by Program (13), in which the incentive-compatibility (IC) constraints ensures that the suppliers will report their true reliability-types.

$$\max_{\substack{(X_i,q_i,p_i)(t_1,t_2)\\ i=1,2;\, t_1,t_2\in\{H,L\}}} \Bigg[\sum_{t_1,t_2\in\{H,L\}} \alpha^{t_1}\alpha^{t_2}\Bigg(r\, E \min\Big\{D,\ \rho_1^{t_1}\ z_1^{t_1}[(q_1,p_1)(t_1,t_2)] + \rho_2^{t_2}\ z_2^{t_2}[(q_2,p_2)(t_1,t_2)]\Big\}$$

$$- X_1(t_1,t_2) + p_1(t_1,t_2)\, E\Big[q_1(t_1,t_2) - \rho_1^{t_1}\ z_1^{t_1}[(q_1,p_1)(t_1,t_2)]\Big]^+ \tag{13a}$$

$$- X_2(t_1,t_2) + p_2(t_1,t_2)\, E\Big[q_2(t_1,t_2) - \rho_2^{t_2}\ z_2^{t_2}[(q_2,p_2)(t_1,t_2)]\Big]^+\Bigg)\Bigg],$$

Subject to For $i = 1, 2$

(IC) $\Pi_i^H(H) \geq \Pi_i^H(L),\ \Pi_i^L(L) \geq \Pi_i^L(H)$ (13b)

(IR) $\Pi_i^H(H) \geq 0,\ \Pi_i^L(L) \geq 0$ (13c)

$X_i(t_1,t_2) \geq 0,\ q_i(t_1,t_2) \geq 0,\ r \geq p_i(t_1,t_2) \geq 0,$ for $t_1, t_2 \in \{H, L\}$,

where $\Pi_i^{t_i}(s_i) \overset{\text{def}}{=} \alpha^H \pi_i^{t_i}\left((X_i,q_i,p_i)(s_i,H)\right) + \alpha^L \pi_i^{t_i}\left((X_i,q_i,p_i)(s_i,L)\right)$ is supplier-i's expected profit of reporting to be of type $s_i \in \{H, L\}$, before knowing the other supplier's type.

To induce the high supplier type to report its true reliability type, the buyer has two choices. First, the buyer can stop ordering from the low type, making it non-profitable for the high type to mimic the low type. Second, the buyer can still order from the low type, but pays the high type an informational rent that equals its reliability advantage, $\Gamma^{\frac{H}{L}}(q,p)$, making misrepresentation unnecessary. Using the second approach, the buyer incurs a cost due to the informational rent because it orders from the low type—the information cost. When ordering from a low type supplier with $q = 1$, to minimize the information cost, it is optimal for the buyer to set the penalty to be the minimum value that turns on the supplier's production, $p = \frac{c^L}{l}$. We find that under the optimal penalty, the buyer's information cost coincides with ϕ defined in (10b) in the PSP-procurement model, that is,

$$\Gamma^{\frac{H}{L}}\left(1, \tfrac{c^L}{l}\right) = \phi. \tag{14}$$

We present the buyer's optimal procurement actions and its optimal expected profit in Proposition 2, where terms ψ^t and ψ^{t_1,t_2} are defined in (9).

Proposition 2. *In the direct-procurement model, the buyer's optimal procurement quantities,* $(q_1^*, q_2^*)(t_1,t_2)$, *are:*

Model parameters	*Penalties,* (p_1, p_2)	*Quantities,* (q_1, q_2)
(H,H) supplier-type pair		
$(\alpha^H)^2\psi^{HH} \le 0$	$(c^H/h, 0)$	$(1, 0)$
$(\alpha^H)^2\psi^{HH} > 0$	$(c^H/h, c^H/h)$	$(1, 1)$
(H,L) supplier-type pair		
$(\alpha^H\alpha^L)\psi^{HL} - (\alpha^H)^2\phi \le 0$	$(c^H/h, 0)$	$(1, 0)$
$(\alpha^H\alpha^L)\psi^{HL} - (\alpha^H)^2\phi > 0$	$(c^H/h, c^L/l)$	$(1, 1)$
(L,H) supplier-type pair		
$(\alpha^L\alpha^H)\psi^{HL} - (\alpha^H)^2\phi \le 0$	$(0, c^H/h)$	$(0, 1)$
$(\alpha^L\alpha^H)\psi^{HL} - (\alpha^H)^2\phi > 0$	$(c^L/l, c^H/h)$	$(1, 1)$
(L,L) supplier-type pair		
$(\alpha^L)^2\psi^L - (\alpha^H\alpha^L)\phi \le 0$	*N/A*	$(0, 0)$
$(\alpha^L)^2\psi^L - (\alpha^H\alpha^L)\phi > 0$ $\ge (\alpha^L)^2\psi^{LL} - (\alpha^L\alpha^H)\phi$	$(c^L/l, 0)$	$(1, 0)$
$(\alpha^L)^2\psi^{LL} - (\alpha^L\alpha^H)\phi > 0$	$(c^L/l, c^L/l)$	$(1, 1)$

The buyer's optimal expected profit is

$$\left(\alpha^H\right)^2 \psi^H + \left(\alpha^H\right)^2 (\psi^{HH})^+ \tag{15a}$$

$$+\, 2\left(\alpha^H\alpha^L\right)\psi^H + \left[2(\alpha^H\alpha^L)\psi^{HL} - 2(\alpha^H)^2\phi\right]^+ \tag{15b}$$

$$+ \left[\left(\alpha^L\right)^2\psi^L - \left(\alpha^H\alpha^L\right)\phi\right]^+ + \left[\left(\alpha^L\right)^2\psi^{LL} - \left(\alpha^L\alpha^H\right)\phi\right]^+ . \tag{15c}$$

Because ψ^{t_1} and ψ^{t_1,t_2} increase in the revenue r, for each type pair (t_1, t_2), there exist revenue thresholds, at which the buyer is indifferent between not ordering and sole-sourcing, and between sole-sourcing and diversification, respectively. We define these thresholds as follows:

$$\tilde{r}^{HL} \stackrel{\text{def}}{=} r \text{ such that } (\alpha^H\alpha^L)\psi^{HL} - (\alpha^H)^2\phi = 0, \tag{16a}$$

$$\tilde{r}^{L} \stackrel{\text{def}}{=} r \text{ such that } (\alpha^L)^2\psi^{L} - (\alpha^H\alpha^L)\phi = 0, \text{ and} \tag{16b}$$

$$\tilde{r}^{LL} \stackrel{\text{def}}{=} r \text{ such that } (\alpha^L)^2\psi^{LL} - (\alpha^H\alpha^L)\phi = 0. \tag{16c}$$

To prepare for the analysis of the value of using the PSP in §5, we compare the buyer's information costs for using diversification with the (L,L) pair under the two procurement models. Under direct procurement, the buyer orders from both low-type suppliers, incurring information cost ϕ with each supplier. Under PSP procurement, the buyer incurs information cost ω (defined in (10c)) with the PSP. We find that

Lemma 4. *$\omega > \phi$.*

The key driver of the result in Lemma 4 is the buyer's loss of control over the suppliers' production actions caused by the adoption of the PSP as the intermediary. The effect of loss-of-control manifests itself in a greater penalty needed for diversification under PSP procurement (i.e., $p = \tilde{p}$ from Proposition 1 for PSP procurement and $p = c^L/l$ from Proposition 2 for direct procurement; $\tilde{p} > c^L/l$). Why is $p = \tilde{p}$ under PSP procurement larger that c^L/l? If the PSP receives the penalty $p = c^L/l$, it has at most the same incentive to produce as a single supplier does. To induce the PSP to diversify, the buyer must increase the penalty. The higher penalty $p = \tilde{p}$ under PSP procurement leads to a larger reliability advantage of the (H,L) pair over the (L,L) pair, and, thus, $\omega > \phi$.

We are now ready to analyze the value of using PSP procurement.

5 The PSP Value

The main research question of this paper is whether the buyer should use a PSP due to the PSP's better information about supply risk. To answer this question, we first define the value of using the PSP for the buyer as the difference between the buyer's expected profits under PSP procurement (11) and direct procurement (15). Can this value be positive? Mookherjee and Tsumagari (2004) results indicate that no, we should expect the PSP value for the buyer to be negative.

Interestingly, our answer is that the PSP value may be positive, and Figure 2 illustrates this. To draw Figure 2, we used the following parameter values: $\alpha^H = 0.9$, $h = 0.7$, $l = 0.6$, and $c^H = c^L = 10$.

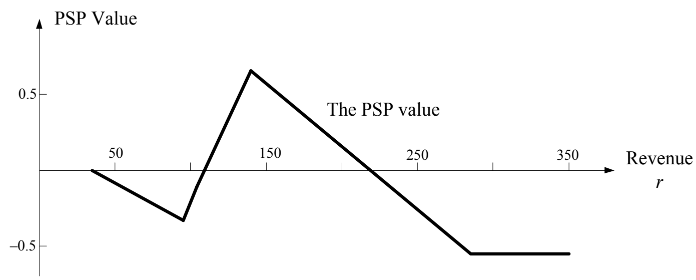


Figure 2: The PSP value as function of r.
Legend: The parameter values are $\alpha^H = 0.9$, $h = 0.7$, $l = 0.6$, and $c^H = c^L = 10$.

The first takeaway from the section is—the PSP value may be positive. For the rest of this section we shall try to understand where the positive PSP value comes from and, at the same time, what explains the negative value (in Mookherjee and Tsumagari, 2004, and in our model).

Derived from (11) and (15), the expression for the PSP value is given in Proposition 3.

Proposition 3. *The buyer's value of using the PSP is*

$$\left.\left[2\left(\alpha^H\alpha^L\right)\psi^{HL} - (\alpha^H)^2\phi\right]^+ - \left[2\left(\alpha^H\alpha^L\right)\psi^{HL} - 2(\alpha^H)^2\phi\right]^+ \right\} \quad (17a)$$

$$\left.\begin{aligned} &+\left\{(\alpha^L)^2\psi^L - [1-(\alpha^L)^2]\phi\right\}^+ \\ &+\left(\left\{(\alpha^L)^2(\psi^L+\psi^{LL}) - [1-(\alpha^L)^2]\omega\right\} - \left\{(\alpha^L)^2\psi^L - [1-(\alpha^L)^2]\phi\right\}^+\right)^+ \\ &-\left[\left(\alpha^L\right)^2\psi^L - \left(\alpha^H\alpha^L\right)\phi\right]^+ - \left[\left(\alpha^L\right)^2\psi^{LL} - \left(\alpha^L\alpha^H\right)\phi\right]^+. \end{aligned}\right\} \quad (17b)$$

In Proposition 3, term (17a) is the PSP value attributable to the contracts with the (H,L) and (L,H) PSP-types (and the corresponding supplier types under direct procurement). Term (17b) is the PSP value attributable to the contract with the (L,L) PSP-type. There is no contribution to the PSP value from the contract with the (H,H) PSP-type.

We plot (17a) and (17b), creating two solid lines in Figure 3. The sum of the two solid lines in Figure 3 is the PSP value in Figure 2. From Figure 3, it appears that (17a) contributes the

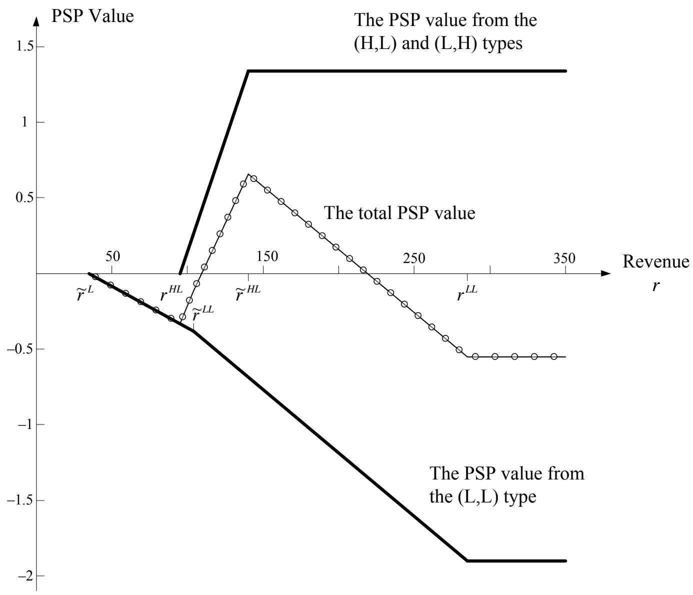


Figure 3: Decomposition of the PSP value by the contracts with (H,L) and (L,H) types and the contract with (L,L) type.
Legend: The parameter values are $\alpha^H = 0.9$, $h = 0.7$, $l = 0.6$, and $c^H = c^L = 10$. $r^L > r^{LL} > \tilde{r}^{HL}$.

benefits of using the PSP while (17b) contributes the costs. This is true in general, according to Proposition 4, and this is the second takeaway from this section.

Proposition 4. *The contribution to the PSP value from the contracts with the (H,L) and (L,H) types (i.e., expression* (17a)*) is positive. The contribution to the PSP value from the contract with the (L,L) type (i.e., expression* (17b)*) is negative.*

5.1 The PSP Value from the (H,L) and (L,H) contracts

In Table 1, the PSP value from the (H,L) and (L,H) contracts (17a) is dissected by different levels of the revenue r and by cause. Table 1 also presents the procurement actions and the buyer's expected profits under the optimal contracts in the two procurement models. For this table, recall (16) for the definition of $\tilde{r}^{HL}$, and (12a) for the definition of r^{HL}, which are marked in Figure 3.

From Table 1, we identify two causes for the PSP value: (i) supplier collusion enabled by the PSP and (ii) change of control of the procurement actions, from the buyer to the PSP. Supplier

Revenue, $r \in$	$(c^H/h, r^{HL})$	$[r^{HL}, \tilde{r}^{HL})$	$[\tilde{r}^{HL}, \infty)$
PSP procurement			
$(z_1^*, z_2^*)(H,L)$	$(1,0)$	$(1,1)$	
$(z_1^*, z_2^*)(L,H)$	$(0,1)$		
Buyer's profit	$2(\alpha^H\alpha^L)\psi^H$	$2(\alpha^H\alpha^L)(\psi^H + \psi^{HL}) - (\alpha^H)^2\phi$	
Direct procurement			
$(q_1, q_2)(H,L)$	$(1,0)$		$(1,1)$
$(q_1, q_2)(L,H)$	$(0,1)$		
Buyer's profit	$2(\alpha^H\alpha^L)\psi^H$		$2(\alpha^H\alpha^L)(\psi^H + \psi^{HL}) - 2(\alpha^H)^2\phi$
The PSP value = PSP profit − Direct profit			
Change of control			
Supply availability	0	$2(\alpha^H\alpha^L)\psi^{HL}$	0
Information cost	0	$-(\alpha^H)^2\phi$	0
Supplier collusion	0	0	$(\alpha^H)^2\phi$
Total	0	$2(\alpha^H\alpha^L)\psi^{HL} - (\alpha^H)^2\phi$	$(\alpha^H)^2\phi$

Table 1: The optimal procurement actions, the buyer' profits, and the buyer's value from PSP, by cause, attributable to the (H,L) and (L,H) types vs. revenue r.

collusion distorts the suppliers' incentives of misrepresentation, and, thus, changes the buyer's information costs for truthful reports. Change of control has two effects. First, it may distort the procurement actions for the suppliers, and, thus, affects the supply availability for the buyer. Second, regardless of whether or not the procurement actions are distorted, change of control affects the buyer's information costs for truthful reports. We shall discuss these causes next.

When $r \in (c^H/h, r^{HL})$, under both procurement models only one of the suppliers (i.e., the high-type supplier) receives an order. Thus, there is no difference in the supply availability between the two procurement models. It turns out that there is no information costs in both models. The PSP value is zero. Intuitively, the buyer is indifferent between dealing with a high-type supplier directly and dealing with an integrated subsystem of the PSP and a high-type supplier (although the PSP has access to two suppliers, the buyer knows that the PSP will optimally order from the supplier that the buyer would have used itself). This case (i.e., $r \in (c^H/h, r^{HL})$) is similar to the Mookherjee and Tsumagari (2004) result, where in the absence of supply risk there is no need for the buyer or the PSP to diversify. As we shall see next, the PSP value is derived by the buyer when diversification does happen, which highlights the importance of supply risk and supply risk management in procurement.

At the other extreme, when $r \in [\tilde{r}^{HL}, \infty)$, under both procurement models, both suppliers receive an order. Hence, there is no difference in the supply availability, again. In this case, PSP procurement dominates direct procurement by $(\alpha^H)^2\phi$. This term represents the buyer's saving in information cost, which the buyer needs to pay the (H,H) PSP-type and the (H,H) supplier-type pair for truthful reporting.

Interestingly, this is a positive effect of implicit collusion between suppliers enabled by the PSP. Specifically, under direct procurement the suppliers are independent. In the (H,H) pair, both suppliers need to be paid to stop pretending to be a low-type one. Under direct procurement, this results in the expected information costs of $2(\alpha^H)^2\phi$ for the buyer. Under PSP procurement, the suppliers are coordinated by the PSP. The coordinated subsystem prefers to misreport itself as either the (H,L) or (L,H) type, but not both. In equilibrium, the dominated form of misreporting will not arise. Therefore, the buyer needs to pay only one information cost $(\alpha^H)^2\phi$. The saving in information cost, $(\alpha^H)^2\phi$, is a benefit of using the PSP for the buyer.

Finally, when $r \in [r^{HL}, \tilde{r}^{HL})$, we observe a combination of two effects from the change of control. First, interestingly, procurement actions under the two procurement models are different. Under direct procurement, the buyer orders from one supplier (the high-type) only. In contrast, under PSP procurement, the PSP diversifies and orders from both suppliers. This action of the PSP reduces the buyer's supply risk. As a result, the buyer has a higher supply availability, and accrues a benefit of $2(\alpha^H\alpha^L)\psi^{HL}$. Second, however, ordering from two suppliers instead of one comes at a cost to the buyer of having to pay an additional information cost $(\alpha^H)^2\phi$, due to collusion. The net effect, $2(\alpha^H\alpha^L)\psi^{HL} - (\alpha^H)^2\phi$, is in favor of using the PSP.

The preceding observations in the case of $r \in [r^{HL}, \tilde{r}^{HL})$ shed lights on the interaction between PSP procurement and the effect of asymmetric information. As Yang et al. (2012) discuss, the buyer has to pay extra information cost every time it diversifies under direct procurement. This imposes an extra cost on using diversification, and, thus, forces the buyer to use less diversification with the (H,L) supplier pair than in the fully-integrated system of symmetric information. In contrast, the PSP, privileged with the perfect knowledge about the suppliers, does not pay this extra cost when diversifying. Thus, we observe that the PSP's better information results in risk management actions that are closer to the ones of the fully integrated system, benefiting the buyer.

To summarize, the third and forth takeaways from this section are—the PSP benefits are derived from the reduction in information costs due to the positive effects of implicit supplier collusion (when diversification happens under PSP procurement) and from the reduction in supply risk (when the PSP diversifies, whereas the buyer under direct procurement would not). The fifth key takeaway in this section is the fact that we can decompose the PSP value into the change-of-control cost and implicit supplier-collusion cost. To the best of our knowledge, this is the first paper to present such decomposition and to articulate the difference between these two costs.

5.2 The PSP Value from the (L,L) Contract

Under the (L,L) PSP-type, supplier collusion and change of control continue to be the causes for the PSP value (negative in this case). For exposition purpose, we shall use the model parameter values that lead to Figures 2 and 3, that is, $r^L > r^{LL} > \tilde{r}^{HL}$. From Lemma 7, the ordering $\tilde{r}^{HL} > \tilde{r}^{LL}$ implies $r^{LL} > \tilde{r}^{LL}$. Hence, in §5.2.1, we shall analyze the PSP value under assumption $r^L > r^{LL} > \tilde{r}^{LL}$. Later, in §5.2.2, we shall extend the analysis to the case $r^L < r^{LL} < \tilde{r}^{LL}$, to provide additional perspectives on the effects of supply collusion and change of control. For the definitions of r^L, r^{LL} and r^{HL}, please see (12b), (12c) and (12a). For the definitions of $\tilde{r}^L$, $\tilde{r}^{LL}$ and $\tilde{r}^{HL}$, please see (16b), (16c) and (16a).

5.2.1 The PSP Value from the (L,L) contract under $r^L > r^{LL} > \tilde{r}^{LL}$

In Table 2, we summarize the expressions for the PSP value from the (L,L) contract, and dissect them by the causes we identified and by the level of revenue r. We also summarize the buyer's optimal profits and procurement actions under the two procurement models.

Similar to Table 1, we identify two causes of the PSP value from the (L,L) PSP-type: (i) supplier collusion enabled by the PSP and (ii) change of control of the procurement actions. However, compared to the effects in Table 1, in Table 2 supplier collusion and change of control have negative effects on the PSP value. We shall discuss these causes starting with the rightmost column of Table 2 and proceeding to the left.

Revenue, $r \in$	$[\tilde{r}^L, \tilde{r}^{LL})$	$[\tilde{r}^{LL}, r^{LL})$	$[r^{LL}, \infty)$
PSP procurement			
$(z_1^*, z_2^*)(L,L)$	$(0,0)$		$(1,1)$
Profit	0		$(\alpha^L)^2(\psi^L + \psi^{LL})$ $-[1-(\alpha^L)^2]\omega$
Direct procurement			
$(q_1, q_2)(L,L)$	$(1,0)$	$(1,1)$	
Profit	$(\alpha^L)^2\psi^L - (\alpha^H\alpha^L)\phi$	$(\alpha^L)^2(\psi^L + \psi^{LL}) - 2(\alpha^H\alpha^L)\phi$	
PSP value = PSP profit − Direct profit			
Change of control			
Supply availability	$-(\alpha^L)^2\psi^L$	$-(\alpha^L)^2(\psi^L + \psi^{LL})$	0
Information cost	$(\alpha^H\alpha^L)\phi$	$2(\alpha^H\alpha^L)\phi$	$-2(\alpha^H\alpha^L)(\omega - \phi)$
Supplier collusion	0		$-(\alpha^H)^2\omega$
Total	$-\big[(\alpha^L)^2\psi^L - (\alpha^H\alpha^L)\phi\big]$	$-(\alpha^L)^2(\psi^L + \psi^{LL})$ $+2(\alpha^H\alpha^L)\phi$	$-2(\alpha^H\alpha^L)(\omega - \phi)$ $-(\alpha^H)^2\omega$

Table 2: The optimal procurement actions, the buyer's profits, and the buyer's value from PSP, by cause, attributable to the (L,L) type vs. the revenue r. Assumption on parameter values: $r^L > r^{LL} > \tilde{r}^{LL}$.

For revenue $r \in \big[r^{LL}, \infty\big)$, the buyer uses diversification with the (L,L) type under both procurement models. Therefore, the buyer's supply availability is the same under both models. This allows us to focus on the information cost due to the two causes for the PSP value: change of control and supplier collusion.

Let's begin with the information cost from change of control, which is due to the possibility of the (H,L) or (L,H) type misreporting to be (L,L). Under direct procurement, the buyer's information cost is ϕ (see equation (14)). Under PSP procurement, the corresponding cost is ω (see its definition in (10c)). According to Lemma 4, $\omega > \phi$. Therefore, by using PSP procurement, the buyer loses $2(\alpha^H\alpha^L)(\omega - \phi) > 0$ in expectation. From the intuition we developed following Lemma 4, we observe that by switching to PSP procurement, the buyer ends up paying more to the intermediary (the PSP) to implement the same production assignments as it had under direct procurement.

Next, we discuss the information cost from supplier collusion, which is due to the possibility of the (H,H) type misreporting to be the (L,L) type. Under direct procurement, in equilibrium, because the suppliers are independent, the suppliers in the (H,H) pair cannot simultaneously report to be low-type (i.e., to be the (L,L) pair). If one of them pretends to be low-type, the other one is better off reporting truthfully to be high-type. In contrast, under PSP procurement, because the PSP coordinates the reports from the suppliers, the (H,H) PSP-type can report to be (L,L)-type. To prevent this, the buyer must pay, in expectation, an amount $(\alpha^H)^2\omega$, due to information cost.

Next, consider the case $\boldsymbol{r \in [\tilde{r}^{LL}, r^{LL})}$ in Table 2. Compared to the case $r \in [r^{LL}, \infty)$, supplier collusion has no effect, because no procurement action is taken under PSP procurement for $r \in [\tilde{r}^{LL}, r^{LL})$. The PSP value from change of control is different as well, compared to the case $r \in [r^{LL}, \infty)$. First, because the procurement actions differ between the two procurement models, there is a new effect—a decrease in the supply availability for the buyer. Specifically, the buyer uses diversification under direct procurement, but the PSP does not order. The expected loss of profit due to decreased supply availability is $-(\alpha^L)^2(\psi^L + \psi^{LL})$. Second, because the PSP does not order, the buyer avoids the information costs for using diversification, $2(\alpha^H\alpha^L)\phi$. The net effect from change of control, $-(\alpha^L)^2(\psi^L + \psi^{LL}) + 2(\alpha^H\alpha^L)\phi$, is negative.

Finally, consider $\boldsymbol{r \in [\tilde{r}^{L}, \tilde{r}^{LL})}$ in Table 2. Similar to the case $\boldsymbol{r \in [\tilde{r}^{LL}, r^{LL})}$, supplier collusion has no contribution to the PSP value. The only contribution to the PSP value comes from change of control. Under direct procurement, the buyer sole-sources. Under PSP procurement, the buyer does not order at all, because ordering under PSP procurement enables supplier collusion and thus increases the information cost for sole-sourcing. Therefore, change of control decreases the buyer's supply availability, but at the benefit of saving an information cost.

5.2.2 The PSP Value from the (L,L) contract under $r^L < r^{LL} < \tilde{r}^{LL}$

In §5.2.1, we assumed $r^L > r^{LL} > \tilde{r}^{LL}$. However, the problem we are solving is complex. There could be different parameter values that lead to different observations. To reveal additional effects of supply collusion and change of control, in this section we assume $r^L < r^{LL} < \tilde{r}^{LL}$. Following the structure of Table 2, we present the procurements actions and the buyer's profits from the two models and the PSP value in Table 3.

Revenue, $r \in$	$[\tilde{r}^L, r^L)$	$[r^L, r^{LL})$	$[r^{LL}, \tilde{r}^{LL})$	$[\tilde{r}^{LL}, \infty)$
PSP procurement				
$(z_1^*, z_2^*)(L,L)$	$(0,0)$	$(1,0)$	$(1,1)$	
Profit	0	$(\alpha^L)^2\psi^L$ $-[1-(\alpha^L)^2]\phi$	$(\alpha^L)^2(\psi^L+\psi^{LL}) - [1-(\alpha^L)^2]\omega$	
Direct procurement				
$(q_1, q_2)(L,L)$	$(1,0)$			$(1,1)$
Profit	$(\alpha^L)^2\psi^L - (\alpha^H\alpha^L)\phi$			$(\alpha^L)^2(\psi^L+\psi^{LL})$ $-2(\alpha^H\alpha^L)\phi$
PSP value = PSP profit − Direct profit				
Change of control				
Supply availability	$-(\alpha^L)^2\psi^L$	0	$(\alpha^L)^2\psi^{LL}$	0
Information cost	$(\alpha^H\alpha^L)\phi$	0	$-\big[(\alpha^L\alpha^H)\omega$ $+(\alpha^H\alpha^L)(\omega-\phi)\big]$	$-2(\alpha^H\alpha^L)(\omega-\phi)$
Supplier collusion	0	$-\big[(\alpha^L\alpha^H)+(\alpha^H)^2\big]\phi$	$-(\alpha^H)^2\omega$	
Total	$-\big[(\alpha^L)^2\psi^L$ $-(\alpha^H\alpha^L)\phi\big]$	$-\big[(\alpha^L\alpha^H)+(\alpha^H)^2\big]\phi$	$(\alpha^L)^2\psi^{LL} - (\alpha^L\alpha^H)\omega$ $-(\alpha^H\alpha^L)(\omega-\phi)$ $-(\alpha^H)^2\omega$	$-2(\alpha^H\alpha^L)(\omega-\phi)$ $-(\alpha^H)^2\omega$

Table 3: The optimal procurement actions, the buyer's profits, and the buyer's value from PSP, by cause, attributable to the (L,L) type vs. revenue r. Assumption on parameter values: $r^L < r^{LL} < \tilde{r}^{LL}$.

Under small and large revenues in Table 3 (the rightmost and the leftmost columns), the effects of supply collusion and change of control are identical to the respective cases in Table 2. Therefore, we focus on the cases of medium revenues in Table 3: $r \in [r^L, r^{LL})$ and $r \in [r^{LL}, \tilde{r}^{LL})$, which did not appear in Table 2.

First, consider the case $r \in [r^{LL}, \tilde{r}^{LL})$. Similar to the case of large revenues (e.g., $r \geq r^{LL}$ in Table 2), supplier collusion increases the buyer expected information cost by $-(\alpha^H)^2\omega$. However, compared to the results in Table 2, change of control has a new effect on the supply availability—an increase in the supply availability for the buyer. Specifically, diversification is employed under PSP procurement, but not under direct procurement. The expected benefit from increased supply availability is $(\alpha^L)^2\psi^{LL}$. However, diversification under PSP procurement increases the buyer's information costs. When sole-sourcing from the low-type supplier-1 under direct procurement, the buyer pays the (H,L) pair expected informational rent $(\alpha^H\alpha^L)\phi$. Under PSP procurement, the buyer's incurs the information cost ω for using diversification with the (L,L) PSP type. Compared to direct procurement, the (H,L) type earns an extra informational rent $(\alpha^H\alpha^L)(\omega-\phi)$, and the (L,H) type earns an expected informational rent $(\alpha^L\alpha^H)\omega$. The net effect from change of control, $(\alpha^L)^2\psi^{LL}-(\alpha^L\alpha^H)\omega-(\alpha^H\alpha^L)(\omega-\phi)$, is negative.

The reason for why there is more diversification under PSP procurement is not obvious. On one hand, the PSP is privy to the suppliers' information, and the PSP can order from the second supplier without paying information cost that the buyer would have to pay in the same situation. On the other hand, we are looking at the optimal contracts designed by the buyer, and, for the buyer, diversification through the PSP comes with costs due to change of control and supplier collusion. The following is an explanation of this result.

The buyer's benefit of ordering from the second supplier, in addition to ordering from the first one, is $(\alpha^L)^2\psi^{LL}$, and it is the same in both the direct-procurement and PSP-procurement models. However, the expected information costs of ordering from the second supplier vary. Under direct procurement, this cost of the second supplier is $(\alpha^L\alpha^H)\phi$. It is due to the incremental information cost the buyer must pay to prevent the second supplier, if it is high-type, from pretending to be low-type (because there are two suppliers, there is also an information cost attributed to the first supplier, but we are only concerned with the incremental effects of diversification). Under PSP procurement, the incremental information cost of ordering from the second supplier is $[1-(\alpha^L)^2](\omega-\phi)$. It is due to the possibility of the PSP of types (H,H), (H,L), and (L,H) pretending to do be (L,L) type. If $(\alpha^L\alpha^H)\phi>[1-(\alpha^L)^2](\omega-\phi)$, then the diversification is more likely to be employed under PSP procurement.

The interactions between supply risk, diversification, collusion, and change of control are novel and unique to this paper. These interactions lead to the new and important managerial insights, reversing the conclusions of previous studies (e.g., Mookherjee and Tsumagari 2004).

For the case $\boldsymbol{r\in\left[r^L,r^{LL}\right)}$, the buyer and the PSP sole-source from the low-type supplier-1 under the two procurement models. Thus, there is no difference in the supply availability between the two models. It turns out that change of control has no effect on the buyer's information costs either. As we discussed in §5.1, intuitively, the buyer is indifferent between dealing with one supplier directly, and dealing with an integrated subsystem of the PSP and one supplier (although the PSP has access to two suppliers, the buyer knows that the PSP will optimally order from the supplier the buyer would have used itself). Therefore, the only PSP value comes from supplier collusion.

Compared to the collusion effects in case $r>r^{LL}$, under which the PSP diversifies (in both

Tables 2 and 3), the collusion in case $r \in [r^L, r^{LL})$ in Tables 3 has a new effect. Under direct procurement, the high-type supplier-1 has an incentive to pretend to be low-type. The buyer pays the expected information cost of $(\alpha^H \alpha^L)\phi$ to prevent that. Under PSP procurement, not only the (H,L) PSP-type, but also the (L,H) and (H,H) PSP-types can pretend to be (L,L). In expectation, the buyer pays extra expected information costs of $[(\alpha^L \alpha^H) + (\alpha^H)^2]\phi$ under PSP procurement. Note that the information cost of using sole-sourcing with the (L,L) PSP-type is ϕ (not ω), the same as the information cost when ordering from one supplier directly. As we mentioned above, ordering from the PSP that sole-sources is equivalent to just ordering from one supplier.

In summary, the PSP value from the (L,L) type is due to supplier collusion and change of control. Collusion increases the buyer's information costs for using the (L,L) supplier-pair any time the PSP takes a procurement action. The contribution of collusion to the PSP value depends on whether the PSP diversifies or sole-sources. Change of control contributes to the PSP value by altering the supply availability for the buyer and its information costs for using the (L,L) type. Interestingly, change of control may increase the supply availability. However, the net contribution of change-of-control is always negative.

6 Comparative Statics Analysis of the PSP Value

In this section, we shall analyze how the PSP value is affected by the revenue, the supply base's reliability, and the reliability gap between the two supplier types.

6.1 The Effect of the Revenue

We have shown numerically (e.g., see Figure 2) that the total PSP value can be positive or negative. As Mookherjee and Tsumagari (2004) predicts, and our result confirms, the PSP value is negative in the absence of diversification, corresponding to small revenues in our model. Next, we shall discuss the PSP value under large and moderate revenues.

Large revenues. The numerical result in Figure 2 suggests that the PSP value must be negative when the revenue is large. The reason for this result is not straightforward. On one hand, the PSP value from the (H,L) and (L,H) is positive. On the other hand, the PSP value from the (L,L) type is negative. One can prove that the net value is negative, as summarized in Proposition 5.

Proposition 5. *When the revenue* $r \geq \max\left\{\tilde{r}^{HL}, \min\{r^L, r^{LL}\}\right\}$, *the total PSP value is negative.*

To understand the intuition, recall from §5 that at large revenues there are positive and negative effects of using PSP procurement. The increases in the information costs due to the negative effects of change-of-control and supplier-collusion are larger than the reduction in the information costs due to the positive effect of supplier collusion. Therefore, the total PSP value is negative.

Moderate revenues. We now characterize the condition for the total PSP value to be strictly positive at moderate revenues $r \in \left[r^{HL}, \max\left\{\tilde{r}^{HL}, \min\{r^L, r^{LL}\}\right\}\right)$. Numerical studies suggest that when α^H is large, and thus $\tilde{r}^{HL} \ll \min\{r^L, r^{LL}\}$, the total PSP value is positive. See Figure 3 for an example.

The total PSP value is positive, if under PSP procurement the buyer uses diversification with the (H,L) and (L,H) PSP-types, but stops ordering from the (L,L) PSP-type. Using diversification with the (H,L) and (L,H) PSP-types, the buyer enjoys one of the following two benefits over direct procurement: a reduction in the information costs of using diversification, or an increase in the supply availability of the (H,L) and (L,H) pairs. (See discussion in §5.1.) On the other hand, by not ordering from the (L,L) PSP-type, the buyer avoids a cost of using the PSP, i.e., the cost of supplier collusion, but does not earn any revenues either. So, there is a negative PSP value associated with (L,L) type. If α^H is large, the probability of drawing the low type for both suppliers simultaneously is negligible, and so is the buyer's cost from the (L,L) type. It is much more likely that the buyer will collect a benefit from using diversification with the (H,L) and (L,H) types.

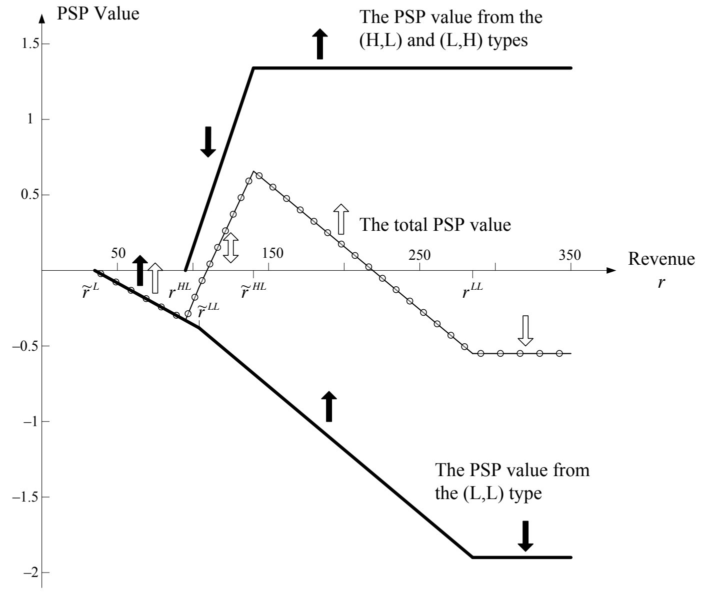
This highlights the importance of risk management strategies in procurement and their interactions with information. Only if the PSP-procurement system uses diversification but the direct-procurement system does not with some supplier types, the PSP is valuable for the buyer.

6.2 The Effect of the Supply Base's Reliability

A salient feature of our model is supply disruption risk and the supply base's reliability. An interesting question is whether an improvement in the supply base's reliability is a substitute for, or a complement to using PSP procurement. To explore this question, we measure the supply base's reliability using α^H, the probability of drawing a high-reliability type, and analyze the marginal change of the PSP value as α^H increases for a fixed revenue r (but not how the breakpoints in r change). We assume $r^L > r^{LL} > \tilde{r}^{HL}$, under which the PSP values are illustrated in Figures 2 and 3, and their expressions are available from Tables 1 and 2. In Figure 4, we illustrate the directions of change of the PSP values.

One may think that, as the supply base becomes more reliable, it is less important for the buyer to take advantage of the PSP's knowledge of the suppliers' risk. Interestingly, Figure 4 shows that the total PSP value may increase in α^H at moderate and small revenues (i.e. $r < r^{LL}$).

The managerial takeaway is that the increasing reliability of the supply base may encourage



The solid black arrows indicate the directions of change of the PSP values by the PSP-type. The arrows filled white indicate the direction of change of the total PSP value. The arrow pointing upward by a line segment indicates that the PSP value (by type) is increasing in α^H and h for the range of r that apply to the line segment. The arrow pointing downward indicates that the PSP value is decreasing in α^H and h. The bi-directional arrow indicate that the PSP value may be increase or decreasing in α^H and h.

Figure 4: The directions of change in α^H and h of the PSP value. (The directions of change in h coincide with those in α^H.)
Legend: The parameter values are such that $r^L > r^{LL} > \tilde{r}^{HL}$.

PSP procurement, even if the PSP value is negative. In this paper, we focus on the informational and risk-management costs and benefits of using PSP procurement. In practice, the manager might also consider other costs and benefits (e.g., transactional). The decreasing informational and risk-management costs (or increasing benefits) of using the PSP enhance other positive benefits, and may make PSP procurement a better choice than direct procurement.

The key driver of this behavior is the reduction of the supply availability of the (L,L) type due to change of control, which occurs only at moderate and low revenues. Recall from the discussion in §5.2.1 that switching to PSP procurement causes the buyer to forgo ordering from the (L,L) pair and the opportunity to earn a profit. The likelihood of incurring the profit loss decreases, as the supply base's reliability increases. In contrast, at large revenues the PSP value decreases in α^H. This highlights the need to understand the driving economic forces that determine the PSP value at different revenue levels, which we explored in §5.

6.3 The Effect of the Reliability Gap between the Two Types

We have quantified the PSP value for the buyer when the reliabilities of the two supplier-types are fixed. A natural question to follow up is whether the PSP becomes more, or less, valuable, when the reliability gap between the high and low type widens. To investigate this question, we increase h (or decrease l), while fixing l (or fixing h), and monitor the marginal change of the PSP value. (We do not analyze how the breakpoints in r change.) We assume $r^L > r^{LL} > \tilde{r}^{HL}$, under which the expressions for the PSP values are available from Tables 1 and 2. The directions of change in h are illustrated in Figure 4, and the directions of change in l are opposite to those in h. The findings are summarized in Proposition 6.

Proposition 6. *Suppose $r^L > r^{LL} > \tilde{r}^{HL}$. As h increases (or, as l decreases) by a small value $\varepsilon > 0$, the total PSP value* (17) *decreases for $r \geq r^{LL}$, increases for $\tilde{r}^{HL} \leq r < r^{LL}$, and may increase or decrease for $r^{HL} \leq r < \tilde{r}^{HL}$.*

Intuitively, as the reliability gap widens, getting the right supplier matters more. Hence, one may think that the PSP's information about the the suppliers' reliabilities should become more valuable. Interestingly, Proposition 6 shows that the PSP value may decrease in h for large revenues (i.e, $r \geq r^{LL}$) and moderate revenues (i.e., $r^{HL} \leq r < \tilde{r}^{HL}$).

The managerial takeaway from Proposition 6 is that the reliability gap cannot be used as a sole indicator for choosing PSP procurement over direct procurement.

We identify two causes for why the PSP value may decreases. First, recall from §6.1 that at large revenues, by switching to PSP procurement, the buyer's profit decreases due to the extra information costs for eliciting the more reliable types' truthful reports. The greater the reliability gap, the more the buyer has to pay in the information costs. Second, at moderate revenues, by switching to PSP procurement, the buyer may enjoy an improvement in supply availability of the (H,L) and (L,H) pairs, from using diversification with these pairs versus sole-sourcing. As the high type becomes more reliable, the relative benefit of using the second supplier (low-type) decreases. This intuition sheds light on the interaction between risk, information and intermediation, which is a unique feature of our model.

7 Managerial Takeaways

What can we tell managers of a buyer firm, who are contemplating whether to hire a procurement service provider? Contrary to naïve intuition about informational benefits that PSPs bring, the PSP value can be negative. But also contrary to the earlier results in the economics literature, the PSP value can be positive as well.

We provide intuitive explanations how various economic forces affect the PSP value for the buyer by breaking the PSP value into positive and negative components (benefits and costs). This paper demonstrates supplier collusion may have both negative and positive effect on PSP value. The benefits of collusion are surprising, a priori. They are accrued because the system with a PSP may have fewer credible ways of deceiving the buyer about the true reliabilities of the suppliers. This paper also shows that changing direct control to PSP control over procurement may have both positive and negative consequences for supply availability. On one hand, with some combinations of suppliers, diversification is cheaper for the PSP to implement, and, hence, the buyer enjoys increased supply availability. On the other hand, with other combinations of suppliers, using diversification (or even ordering) is more expensive via PSP, and supply availability is reduced.

Our findings indicate that hiring a PSP is not the solution to the problem of unreliable supply base. As the fraction of more reliable suppliers in the economy decreases, the value of PSP procurement may decrease or increase. In fact, numerical analysis indicates that in order for the PSP value to be positive the fraction of the reliable suppliers in the economy must be quite high.

Similarly, the increasing reliability gap between high and low reliability suppliers is not a reason to hire a PSP, as the value of PSP may actually decrease in this gap.

This paper focuses on informational and risk management aspects of using a PSP. Besides these aspects, there are other benefits and costs the buyers can experience with PSPs (transactional, relational). Combining some of those other benefits (e.g., reduction of search costs, reduction in logistics costs) with the insights we developed in this paper should be straightforward. On the other hand, some benefits and costs (e.g., order aggregation, relational benefits) might interact with risk management and informational considerations in a non-trivial way and can be interesting subjects for future studies.

The results in this paper are derived using a static game. In a dynamic game, the interactions between information and supply risk would be complicated by the inter-temporal effects of the players' decisions. These interactions are interesting, but are beyond of our study. In future studies, the insights from our model can be used as a benchmark for exploring the informational benefits of using the PSP in dynamic settings.

In this paper, the buyer and the PSP use diversification for supply risk management. In practice, there are other risk management measures, such as backup production option (e.g., see Yang et al., 2009) and process improvement. These risk management measures allow for new forms of interactions between information, supply risk and intermediation. These can be interesting subjects for future studies.

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Term	Definition	Reference
$\theta = h,\, l$	the probability of successful production run by the high- and low-reliability supplier-types	§2
$\alpha^H,\, \alpha^L$	the probability of drawing the high and low supplier-types; $\alpha^H + \alpha^L = 1$	§2
$c^H,\, c^L$	the unit cost of running production by the high and low supplier-types	§2
D	the buyer's market demand; we assume $D = 1$	§2
r	revenue per unit of market demand satisfied	§2
X, q, p	payment, required quantity and penalty per unit of shortfall	§3.1
$(X, q, p)(t_1, t_2)$	the contract intended for the (t_1, t_2) PSP-type in the contract menu	§3.1
$\pi^{t_1,t_2}(X, q, p)$	optimal expected profit of the (t_1, t_2) PSP-type, under contract (X, q, p)	(1)
$(z_1^*, z_2^*)(q, p)$	the PSP's production assignments to the suppliers under (q, p)	§3.2
$\psi^t(q, p)$	$\stackrel{\text{def}}{=} (\theta^t p - c^t)q$, the PSP's benefit of sole-sourcing from a supplier of type t, under given (q, p)	(2)
$\psi^{t_1,t_2}(q, p)$	$\stackrel{\text{def}}{=} [\theta^{t_2}(1 - \theta^{t_1})p - c^{t_2}]q$, the PSP's benefit of diversification with a pair of suppliers of types (t_1, t_2), under given (q, p)	(3)
$\Gamma^{\frac{HH}{HL}}(q, p)$	$\stackrel{\text{def}}{=} \pi^{HH}(X, q, p) - \pi^{HL}(X, q, p)$, the reliability advantage of the (H,H) PSP-type over the (H,L) type, under given (q, p)	(5)
$\Gamma^{\frac{HL}{LL}}(q, p)$	$\stackrel{\text{def}}{=} \pi^{HL}(X, q, p) - \pi^{LL}(X, q, p)$, the reliability advantage of the (H,L) PSP-type over the (L,L) type, under given (q, p)	(5)
$\pi^{t_1,t_2}(s_1, s_2)$	$\stackrel{\text{def}}{=} \pi^{t_1,t_2}\big((X, q, p)(s_1, s_2)\big)$, the optimal profit of the (t_1, t_2) PSP-type under the contract intended for the (s_1, s_2) type	§3.4
$(z_1^*, z_2^*)(t_1, t_2)$	$(z_1^*, z_2^*)\big((q, p)(t_1, t_2)\big)$	§3.4
$\Gamma^{\frac{HH}{HL}}(t_1, t_2)$	$\stackrel{\text{def}}{=} \Gamma^{\frac{HH}{HL}}\big((q, p)(t_1, t_2)\big)$	(33)
$\Gamma^{\frac{HL}{LL}}(t_1, t_2)$	$\stackrel{\text{def}}{=} \Gamma^{\frac{HL}{LL}}\big((q, p)(t_1, t_2)\big)$	(33)
ψ^t	$\stackrel{\text{def}}{=} \psi^t(q, p)$ at $(q, p) = (1, r)$	(9)
ψ^{t_1,t_2}	$\stackrel{\text{def}}{=} \psi^{t_1,t_2}(q, p)$ at $(q, p) = (1, r)$	(9)
$\tilde{p}$	the optimal penalty $p^*(L, L)$ that induces the (L,L) PSP-type to diversify	(10a)
ϕ	$\stackrel{\text{def}}{=} \Gamma^{\frac{HL}{LL}}\left(1, \frac{c^L}{l}\right)$, information cost, the (L,L) PSP-type sole-sources	(10b)
	$\stackrel{\text{def}}{=} \Gamma^{\frac{HH}{HL}}\left(1, \frac{c^L}{l(1-h)}\right)$, information cost, the (H,L) PSP-type diversifies	(10b)
	$\stackrel{\text{def}}{=} \Gamma^{\frac{H}{L}}\left(1, \frac{c^L}{l}\right)$, information cost of a low-type supplier, direct procurement	(14)
ω	$\stackrel{\text{def}}{=} \Gamma^{\frac{HL}{LL}}(1, \tilde{p})$, information cost, the (L,L) PSP-type diversifies	(10c)
r^{HL}	the value of revenue, above which it is optimal for the buyer to induce the (H,L) PSP-type to diversify	(12a)
r^L	the value of revenue, above which it is optimal for the buyer to induce the (L,L) PSP-type to sole-source	(12b)
r^{LL}	the value of revenue, above which it is optimal for the buyer to induce the (L,L) PSP-type to diversify	(12c)
$\tilde{r}^{HL}$	the value of revenue, above which it is optimal for the buyer to diversify with the (H,L) supplier pair in the direct procurement model	(16a)
$\tilde{r}^L$	the value of revenue, above which it is optimal for the buyer to sole-sourcing from a low-type supplier in the direct procurement model	(16b)
$\tilde{r}^{LL}$	the value of revenue, above which it is optimal for the buyer to diversify with the (L,L) supplier pair in the direct procurement model	(16c)

Table 4: List of definitions

A Appendix - Supporting Materials for Proposition 1

Condition 1. *The model parameters satisfy the following conditions:*

1.a *If* $2(\alpha^H\alpha^L)\psi^{HL} - \left(\alpha^H\right)^2\phi \geq 0$*, then no additional restrictions are imposed on the model parameters.*

1.b *If* $2(\alpha^H\alpha^L)\psi^{HL} - \left(\alpha^H\right)^2\phi < 0$*, then* $\frac{c^H}{h(1-h)}$ *must satisfy an additional restriction that is specified in the following table:*

Model parameters satisfy	***Restriction on*** $\frac{c^H}{h(1-h)}$
$(\alpha^L)^2\psi^L - [1-(\alpha^L)^2]\phi < 0$ *and* $(\alpha^L)^2(\psi^L+\psi^{LL}) - [1-(\alpha^L)^2]\omega < 0$	*no additional restriction*
$(\alpha^L)^2\psi^L - [1-(\alpha^L)^2]\phi > \left\{(\alpha^L)^2(\psi^L+\psi^{LL}) - [1-(\alpha^L)^2]\omega\right\}^+$	$\frac{c^H}{h(1-h)} \geq \frac{c^L}{l}$
$(\alpha^L)^2(\psi^L+\psi^{LL}) - [1-(\alpha^L)^2]\omega \geq \left\{(\alpha^L)^2\psi^L - [1-(\alpha^L)^2]\phi\right\}^+$ *and* $(1-l)^2-(1-h) \geq 0$	$\frac{c^H}{h(1-h)} \geq \frac{c^L}{l(1-l)}$

where ϕ *and* ω *are defined in* (10b) *and* (10c)*.*

Remark 1. *Under Condition 1, the optimal transfer payment,* $X^*(t_1,t_2)$*, can be derived from the following equations:*

$$\begin{aligned} X^*(L,L) &= p^*(L,L)q^*(L,L) - \left[\psi^L\Big((q^*,p^*)(L,L)\Big)\right]^+ - \left[\psi^{LL}\Big((q^*,p^*)(L,L)\Big)\right]^+, \\ X^*(H,L) &= p^*(H,L)q^*(H,L) - \left[\psi^H\Big((q^*,p^*)(H,L)\Big)\right]^+ \\ &\quad - \left[\psi^{HL}\Big((q^*,p^*)(H,L)\Big)\right]^+ + \Gamma^{\frac{HL}{LL}}(L,L), \text{ and} \\ X^*(H,H) &= p^*(H,H)q^*(H,H) - \left[\psi^H\Big((q^*,p^*)(H,H)\Big)\right]^+ \\ &\quad - \left[\psi^{HH}\Big((q^*,p^*)(H,H)\Big)\right]^+ + \Gamma^{\frac{HH}{HL}}(H,L) + \Gamma^{\frac{HL}{LL}}(L,L), \end{aligned} \tag{18}$$

where $\Gamma^{\frac{HL}{LL}}(L,L) \stackrel{def}{=} \Gamma^{\frac{HL}{LL}}\Big((q^*,p^*)(L,L)\Big)$ *and* $\Gamma^{\frac{HH}{HL}}(H,L) \stackrel{def}{=} \Gamma^{\frac{HH}{HL}}\Big((q^*,p^*)(H,L)\Big)$*.*

B Appendix - Technical Results

Lemma 5. *Given that* $c^L/l > c^H/h$ *and* $h > l$*, the following inequalities are true:*

$$\frac{c^L}{l(1-h)} > \frac{c^H}{h(1-h)} > \frac{c^H}{h} \text{ and } \frac{c^L}{l(1-h)} > \frac{c^L}{l(1-l)} > \frac{c^L}{l} > \frac{c^H}{h}. \tag{19}$$

Proof. The results follow from the inequalities $c^L/l > c^H/h$ and $h > l$. ■

Lemma 6. *Under order quantity, q, and shortfall penalty, p, the (H,H) PSP-type's reliability advantage over the (H,L) type is:*

$$\Gamma^{HH}_{HL}(q,p) = \begin{cases} 0 & p < \frac{c^H}{h(1-h)} \\ [h(1-h)p - c^H]q & \frac{c^H}{h(1-h)} \le p < \frac{c^L}{l(1-h)} \\ [(h-l)(1-h)p - (c^H - c^L)]q & p \ge \frac{c^L}{l(1-h)} \end{cases}. \tag{20}$$

The (H,L) PSP-type's reliability advantage over the (L,L) type is:

$$\Gamma^{HL}_{LL}(q,p) = \begin{cases} 0 & p < \frac{c^H}{h} \\ (hp - c^H)q & \frac{c^H}{h} \le p < \frac{c^L}{l} \\ [(h-l)p - (c^H - c^L)]q & \frac{c^L}{l} \le p < \frac{c^L}{l(1-l)} \\ \left\{[(1-l)^2 - (1-h)]p - c^H + 2c^L\right\}q & \frac{c^L}{l(1-l)} \le p < \frac{c^L}{l(1-h)} \\ [(h-l)(1-l)p - (c^H - c^L)]q & p \ge \frac{c^L}{l(1-h)} \end{cases}. \tag{21}$$

Both $\Gamma^{HL}_{LL}(q,p)$ and $\Gamma^{HH}_{HL}(q,p)$ are nonnegative and continuous in q and p, and are linearly increasing in q. $\Gamma^{HH}_{HL}(q,p)$ is monotonic increasing in the penalty p. $\Gamma^{HL}_{LL}(q,p)$ is monotonic increasing in p if and only if $(1-l)^2 - (1-h) \ge 0$. Under $(1-l)^2 - (1-h) < 0$, $\Gamma^{HL}_{LL}(q,p)$ decreases in $p \in \left[\frac{c^L}{l(1-l)}, \frac{c^L}{l(1-h)}\right)$.

Proof. To derive the expressions in (20) and (21), one can apply the expressions for $\psi^H(q,p)$, $\psi^L(q,p)$ (defined in (2)), $\psi^{H,H}(q,p)$, $\psi^{H,L}(q,p)$ and $\psi^{L,L}(q,p)$ (defined in (3)) to (7a) and (7b). Details are omitted. ■

Lemma 7. *The following inequalities are true:*

$$\tilde{r}^{HL} > \tilde{r}^{LL} > \tilde{r}^L; \tag{22a}$$

$$r^L > \tilde{r}^L,\ \tilde{r}^{HL} > r^{HL};\ \textit{and}\ r^{HL} > \tilde{r}^L\ \textit{under}\ h \ge \tfrac{1}{2}. \tag{22b}$$

Proof. The inequalities in (22a) follow from the definitions of $\tilde{r}^{HL}$, $\tilde{r}^{LL}$ and $\tilde{r}^L$ (see (16a), (16c) and (16b)). In (22b), inequality $r^L > \tilde{r}^L$ follows from the definitions of r^L (see (12b)) and $\tilde{r}^L$, and inequality $\tilde{r}^{HL} > r^{HL}$ follows from the definitions of $\tilde{r}^{HL}$ and r^{HL} (see (12a)).

Now, we prove $r^{HL} > \tilde{r}^L$ under $h \ge 1/2$. We shall show that if $(\alpha^L)^2\psi^L - (\alpha^L\alpha^H)\phi \le 0$ (i.e., $r \le \tilde{r}^L$), then $2(\alpha^H\alpha^L)\psi^{HL} - \left(\alpha^H\right)^2\phi < 0$ (i.e., $r < r^{HL}$). Recall the definitions $\psi^L = lr - c^L$ and $\psi^{HL} = l(1-h)r - c^L$. One can verify that $\psi^L > 2\psi^{HL}$ under $h \ge 1/2$. Therefore, $2(\alpha^H\alpha^L)\psi^{HL} - \left(\alpha^H\right)^2\phi < (\alpha^H/\alpha^L)\left[(\alpha^L)^2\psi^L - (\alpha^L\alpha^H)\phi\right] \le 0$. The result follows. ■

C Appendix - Proofs (to appear in the online appendix)

Proof of Lemma 1. The PSP's profit (1) can be written as

$$\pi^{t_1,t_2}(X,q,p) = (X - pq) + \max_{z_1\geq 0,\, z_2\geq 0} \left\{ pE\left[\min\left(q, \rho_1^{t_1} z_1 + \rho_2^{t_2} z_2\right)\right] - c^{t_1} z_1 - c^{t_2} z_2 \right\} \tag{23}$$

We define the terms in the maximization operation in (23) as $\xi^{t_1,t_2}(z_1, z_2, q, p)$, and expand the expectation in it, obtaining

$$\begin{aligned}\xi^{t_1,t_2}(z_1,z_2,q,p) = p\Big[&\theta^{t_1}\theta^{t_2}\min(q, z_1+z_2) + \theta^{t_1}(1-\theta^{t_2})\min(q,z_1) \\ &+ (1-\theta^{t_1})\theta^{t_2}\min(q,z_2)\Big] - c^{t_1}z_1 - c^{t_2}z_2\end{aligned} \tag{24}$$

To find the optimal (z_1, z_2), we solve $\max_{z_1\geq 0,\, z_2\geq 0} \xi^{t_1,t_2}(z_1, z_2, q, p)$. $\xi^{t_1,t_2}(z_1, z_2, q, p)$ is piecewise linear in (z_1, z_2), with four corner solutions: $(z_1, z_2) = (0, 0)$, $(q, 0)$, $(0, q)$ and (q, q). At these corners, the objective values are:

$$\xi^{t_1,t_2}(0,0,q,p) = 0 \tag{25a}$$

$$\xi^{t_1,t_2}(q,0,q,p) = \left(\theta^{t_1}p - c^{t_1}\right)q \tag{25b}$$

$$\xi^{t_1,t_2}(0,q,q,p) = \left(\theta^{t_2}p - c^{t_2}\right)q \tag{25c}$$

$$\xi^{t_1,t_2}(q,q,q,p) = \left[\left(\theta^{t_1} + \theta^{t_2} - \theta^{t_1}\theta^{t_2}\right)p - c^{t_1} - c^{t_2}\right]q \tag{25d}$$

To find the optimal (z_1, z_2), it suffices to compare the values of $\xi^{t_1,t_2}(z_1, z_2, q, p)$ at the four corner solutions. We use $(t_1, t_2) = (H, L)$ to illustrate the comparison. The values of $\xi^{HL}(z_1, z_2, q, p)$ at the four corner solutions are

$$\xi^{HL}(0,0,q,p) = 0 \tag{26a}$$

$$\xi^{HL}(q,0,q,p) = \left(hp - c^H\right)q \tag{26b}$$

$$\xi^{HL}(0,q,q,p) = \left(lp - c^L\right)q \tag{26c}$$

$$\xi^{HL}(q,q,q,p) = \left[(hp - c^H) + (l(1-h)p - c^L)\right]q \tag{26d}$$

First, consider $p < c^H/h$. At $(z_1, z_2) = (0, 0)$, the objective $\xi^{HL}(0, 0, q, p) = 0$. It follows from $p < c^H/h$ that at $(z_1, z_2) = (q, 0)$ the objective $\xi^{HL}(q, 0, q, p) < 0$. At $(z_1, z_2) = (0, q)$, $\xi^{HL}(0, q, q, p) = l(p - c^L/l)q$. From the assumption $c^L/l > c^H/h$, we have $\xi^{HL}(0, q, q, p) < l(p - c^H/h)q < 0$. Now, compare $\xi^{HL}(q, q, q, p)$ to 0. Because $(l(1 - h)p - c^L)q < \xi^{HL}(0, q, q, p) < 0$ and $hp - c^H < 0$, it follows $\xi^{HL}(q, q, q, p) < 0$. Therefore, $(z_1, z_2) = (0, 0)$ is the optimal solution.

Second, we consider $p \geq c^H/h$. It follows immediately that $(z_1, z_2) = (q, 0)$ dominates $(0, 0)$. Furthermore, we show that $(z_1, z_2) = (q, 0)$ dominates $(0, q)$ as well. The difference between their objective values is:

$$\xi^{HL}(q,0,q,p) - \xi^{HL}(0,q,q,p) = [h(p - c^H/h) - l(p - c^L/l)]q. \tag{27}$$

From the assumption $c^L/l > c^H/h$, it follows $\xi^{HL}(q,0,q,p) - \xi^{HL}(0,q,q,p) > (h-l)(p - c^H/h)q$. The right-hand-side of this inequality is positive, because $h > l$ and $p \geq c^H/h$.

Therefore, under $p \geq c^H/h$, the optimal solution is between $(z_1, z_2) = (q, 0)$ and (q, q). The difference between their objective values is:

$$\xi^{HL}(q,q,q,p) - \xi^{HL}(q,0,q,p) = [l(1-h)p - c^L]q. \tag{28}$$

Under $p \geq c^L/[l(1-h)]$, $\xi^{HL}(q,q,q,p) - \xi^{HL}(q,0,q,p) \geq 0$. The optimal solution is $(z_1, z_2) = (q, q)$. Conversely, under $c^H/h \leq p < c^L/[l(1-h)]$, $(z_1, z_2) = (q, 0)$ is optimal (note, from Lemma 5, $c^L/[l(1-h)] > c^H/h$).

The proofs for the cases of $(t_1, t_2) = (H, H)$, (L, H) and (L, L) are identical to $(t_1, t_2) = (H, L)$, and thus are omitted. ■

Proof of Lemma 2. We use $(t_1, t_2) = (H, L)$ to illustrate the proof. From (23) and (26) in the proof of Lemma 1, the PSP's optimal profit is:

$$\pi^{t_1,t_2}(q,p) = (X - pq) + \begin{cases} 0 & p < c^H/h \\ (hp - c^H)q & c^H/h \leq p < c^L[l(1-h)] \\ \left[(hp - c^H) + (l(1-h)p - c^L)\right] q & p \geq c^L/[l(1-h)] \end{cases} \tag{29}$$

First, we show this is equivalent to

$$\pi^{t_1,t_2}(q,p) = (X - pq) + \left[(hp - c^H)q\right]^+ + \left\{[(l(1-h)p - c^L)]q\right\}^+ \tag{30}$$

Specifically, we shall show for each of the three cases of p that the value in (30) equal to (29). Under $p < c^H/h$, from (30), the PSP profit is $\pi^{t_1,t_2}(q,p) = (X - pq)$. Under $c^H/h \leq p < c^L[l(1-h)]$, we have $(hp - c^H)q \geq 0$ but $[(l(1-h)p - c^L)]q < 0$. From (30), $\pi^{t_1,t_2}(q,p) = (X - pq) + (hp - c^H)q$. Under $p \geq c^L/[l(1-h)]$, we have $(hp - c^H)q > 0$ and $[(l(1-h)p - c^L)]q \geq 0$. From (30), $\pi^{t_1,t_2}(q,p) = (X - pq) + (hp - c^H)q + [(l(1-h)p - c^L)]q$.

Next, we apply the definitions of $\psi^H(q,p) \stackrel{\text{def}}{=} (hp - c^H)q$ (see (2)) and $\psi^{HL}(q,p) \stackrel{\text{def}}{=} [l(1-h)p - c^L]q$ (see (3)) to (30). The results follows.

The proofs for the cases of $(t_1, t_2) = (H, H)$ and (L, L) are identical to $(t_1, t_2) = (H, L)$, and thus are omitted. ■

Proof of Lemma 4. From the definitions (10b) and (10c), $\omega - \phi = \Gamma^{\frac{HL}{LL}}(1,\tilde{p}) - \Gamma^{\frac{HL}{LL}}\left(1, \frac{c^L}{l}\right)$. We consider two cases: $(1-l)^2 - (1-h) \geq 0$ and the opposite case.

Under $(1-l)^2 - (1-h) \geq 0$, from (10a) $\tilde{p} = c^L[l(1-l)] > c^L/l$. From Lemma 6, $\Gamma^{\frac{HL}{LL}}(q,p)$ increases in p for all $p \geq 0$. It follows that $\Gamma^{\frac{HL}{LL}}(1,\tilde{p}) - \Gamma^{\frac{HL}{LL}}\left(1, \frac{c^L}{l}\right) > 0$.

Under $(1-l)^2 - (1-h) < 0$, from Proposition 1, $\tilde{p} = \min\{r, c^L/[l(1-h)]\}$. Because $\tilde{p} > c^L/[l(1-l)]$ must be true for inducing the (L,L) PSP-type to diversify at the optimal solution, it follows $\tilde{p} \in \left[c^L/[l(1-l)], c^L/[l(1-h)]\right]$. From Lemma 6, one can verify that $\Gamma^{\frac{HL}{LL}}\left(q, c^L/[l(1-l)]\right) > \Gamma^{\frac{HL}{LL}}\left(q, c^L/l\right)$ and $\Gamma^{\frac{HL}{LL}}(q, c^L/[l(1-h)]) > \Gamma^{\frac{HL}{LL}}\left(q, c^L/l\right)$. The result follows from the fact that $\Gamma^{\frac{HL}{LL}}(q,p)$ is linear in p for $p \in \left[c^L/[l(1-l)], c^L/[l(1-h)]\right]$. ■

Proof of Proposition 1. To begin, we express the buyer's objective in program (8) as the difference between the total channel profit and the PSP's profit. To do that, using the expression for the PSP's expected profit (1), we derive the following equality for the (t_1,t_2) PSP-type:

$$\begin{aligned} X(t_1,t_2) - p(t_1,t_2)E\Big[q(t_1,t_2) - \rho_1^{t_1} z_1^*(t_1,t_2) - \rho_2^{t_2} z_2^*(t_1,t_2)\Big]^+ \\ = c^{t_1} z_1^*(t_1,t_2) + c^{t_2} z_2^*(t_1,t_2) + \pi^{t_1,t_2}(t_1,t_2) \end{aligned} \tag{31}$$

The left-hand-side of this equality is the buyer's total cost of using the PSP of the (t_1,t_2) type. The right-hand-side is the sum of the PSP's production costs and its profit. We apply this equality to the buyer's objective in (8), converting it into the following:

$$\begin{aligned} \max_{\substack{(X,q,p)(t_1,t_2),\\ t_1 \text{ and } t_2 \in \{H,L\}}} \sum_{t_1,t_2 \in \{H,L\}} (\alpha^{t_1}\alpha^{t_2}) \Big[rE\min\Big\{D, \min\big\{q(t_1,t_2), \rho_1^{t_1} z_1^*(t_1,t_2) + \rho_2^{t_2} z_2^*(t_1,t_2)\big\}\Big\} \\ - c^{t_1} z_1^*(t_1,t_2) - c^{t_2} z_2^*(t_1,t_2) - \pi^{t_1,t_2}(t_1,t_2)\Big] \end{aligned} \tag{32}$$

Hereafter, without loss of optimality, we replace the buyer's objective in (8) with (32).

Furthermore, we note that the (H,L) and (L,H) PSP-types are identical for the PSP and the buyer. The buyer designs the contracts for these two types to be identical, and collects the same expected profits from these contracts. Therefore, in this proof we drop the notion that the PSP may be of type (L,H), but assume that the PSP may be of type (H,L) with probability $2(\alpha^H\alpha^L)$.

From this point, the proof takes four major steps to complete. **First**, we apply the definitions of *reliability advantage* to the incentive compatibility (IC) constraints. To facilitate presentation, we use the following notation:

$$\begin{aligned} \pi^{t_1,t_2}(s_1,s_2) &\stackrel{\text{def}}{=} \pi^{t_1,t_2}\big((q,p)(s_1,s_2)\big) \\ \Gamma^{\frac{HH}{HL}}(s_1,s_2) &\stackrel{\text{def}}{=} \Gamma^{\frac{HH}{HL}}\big((q,p)(s_1,s_2)\big) \\ \Gamma^{\frac{HL}{LL}}(s_1,s_2) &\stackrel{\text{def}}{=} \Gamma^{\frac{HH}{HL}}\big((q,p)(s_1,s_2)\big) \end{aligned} \tag{33}$$

From the definition of the reliability advantage of the (H,H) PSP-type over the (H,L) type under the contract for the (H,L) type, $\pi^{HH}(H,L) = \pi^{HL}(H,L) + \Gamma^{\frac{HH}{HL}}(H,L)$. We apply this equality to IC constraint (8a), obtaining $\pi^{HH}(H,H) - \pi^{HL}(H,L) \geq \Gamma^{\frac{HH}{HL}}(H,L)$. Similarly, $\pi^{HL}(H,H) = \pi^{HH}(H,H) - \Gamma^{\frac{HH}{HL}}(H,H)$. Applying this equality to IC constraint (8c), we express it as $\Gamma^{\frac{HH}{HL}}(H,H) \geq \pi^{HH}(H,H) - \pi^{HL}(H,L)$. This pair of local IC constraints can be combined into the following form:

$$\Gamma^{\frac{HH}{HL}}(H,H) \geq \pi^{HH}(H,H) - \pi^{HL}(H,L) \geq \Gamma^{\frac{HH}{HL}}(H,L). \tag{34}$$

Similarly, from the definition of the reliability advantage of the (H,L) over the (L,L) type $\Gamma^{\frac{HH}{HL}}(s_1, s_2)$, the pair of local IC constraints between the (H,L) and (L,L) type becomes

$$\Gamma^{\frac{HL}{LL}}(H,L) \geq \pi^{HL}(H,L) - \pi^{LL}(L,L) \geq \Gamma^{\frac{HL}{LL}}(L,L). \tag{35}$$

Now, consider the pair of global IC constraints between the (H,H) type and the (L,L) type. Under the contract designed for the (s_1, s_2) PSP-type, the reliability advantage of the (H,H) type over the (L,L) type is

$$\begin{aligned} &\pi^{HH}(s_1,s_2) - \pi^{LL}(s_1,s_2) = \left[\pi^{HH}(s_1,s_2) - \pi^{HL}(s_1,s_2)\right] + \left[\pi^{HL}(s_1,s_2) - \pi^{LL}(s_1,s_2)\right] \\ &= \Gamma^{\frac{HH}{HL}}(s_1,s_2) + \Gamma^{\frac{HL}{LL}}(s_1,s_2) \end{aligned} \tag{36}$$

Using this equality under the contracts for the (H,H) and (L,L) types, respectively, we express the pair of global IC constraints as

$$\Gamma^{\frac{HH}{HL}}(H,H) + \Gamma^{\frac{HL}{LL}}(H,H) \geq \pi^{HH}(H,H) - \pi^{LL}(L,L) \geq \Gamma^{\frac{HH}{HL}}(L,L) + \Gamma^{\frac{HL}{LL}}(L,L) \tag{37}$$

Second, we shall use IC constraints in the forms of (34), (35) and (37) and the IR constraints to derive the forms of all PSP-types' profits at the optimal solution. We shall then apply the results to program (8). The result is presented in program (43).

We begin this step with a key observation: the buyer's objective function in the form of (32) decreases in the PSP's profits, $\pi^{t_1,t_2}(t_1,t_2)$. At the optimal solution, the PSP's profits must be minimized. With this observation in mind, we identify the lower bounds on the PSP-types' profits from the IR and IC constraints.

From the IR constraints on the (L,L) types, at the optimality we must have

$$\pi^{LL}(L,L) = 0. \tag{38}$$

The rightmost inequality in the IC constraints (35) imposes a lower bound on the profit of the (H,L) PSP-types, $\pi^{HL}(H,L)$. Using this lower bound and the result of $\pi^{LL}(L,L) = 0$, we conclude that at the optimal solution

$$\pi^{HL}(H,L) = \Gamma^{\frac{HL}{LL}}(L,L) \geq 0. \tag{39}$$

The rightmost inequalities in (34) and (37) provide two possible lower bounds on the profit of the (H,H) PSP-types, $\pi^{HH}(H,H)$. Applying $\pi^{HL}(H,L) = \Gamma^{HL}_{LL}(L,L)$ and $\pi^{LL}(L,L) = 0$ to these inequalities, we conclude that at the optimality we must have

$$\pi^{HH}(H,H) = \max\left\{\Gamma^{HH}_{HL}(H,L), \Gamma^{HH}_{HL}(L,L)\right\} + \Gamma^{HL}_{LL}(L,L) \geq 0. \tag{40}$$

We apply (38), (39) and (40), to the leftmost inequalities in the IC constraints (34), (35) and (37). Then, at optimality the IC constraints take the forms of

$$\Gamma^{HH}_{HL}(H,H) \geq \max\{\Gamma^{HH}_{HL}(H,L), \Gamma^{HH}_{HL}(L,L)\} \tag{41a}$$

$$\Gamma^{HL}_{LL}(H,L) \geq \Gamma^{HL}_{LL}(L,L) \tag{41b}$$

$$\Gamma^{HH}_{HL}(H,H) + \Gamma^{HL}_{LL}(H,H) \geq \max\{\Gamma^{HH}_{HL}(H,L), \Gamma^{HH}_{HL}(L,L)\} + \Gamma^{HL}_{LL}(L,L) \tag{41c}$$

We apply (38), (39) and (40) to the objective (32), obtaining

$$\begin{aligned}
&(\alpha^H)^2\Big[rE\min\Big\{D, \min\left\{q(t_1,t_2), \rho^H z_1^*(H,H) + \rho^H z_2^*(H,H)\right\}\Big\} - c^H z_1^*(H,H) - c^H z_2^*(H,H) \\
&\qquad - \max\Big\{\Gamma^{HH}_{HL}(H,L), \Gamma^{HH}_{HL}(L,L)\Big\} - \Gamma^{HL}_{LL}(L,L)\Big] && (42a)\\
&+ 2(\alpha^H\alpha^L)\Big[rE\min\Big\{D, \min\left\{q(t_1,t_2), \rho^H z_1^*(H,L) + \rho^H z_2^*(H,L)\right\}\Big\} \\
&\qquad - c^H z_1^*(H,L) - c^L z_2^*(H,L) - \Gamma^{HL}_{LL}(L,L)\Big] && (42b)\\
&+ (\alpha^L)^2\Big[rE\min\Big\{D, \min\left\{q(t_1,t_2), \rho^L z_1^*(L,L) + \rho^L z_2^*(L,L)\right\}\Big\} - c^L z_1^*(L,L) - c^L z_2^*(L,L)\Big]. && (42c)
\end{aligned}$$

For any $(q,p)(t_1,t_2)$, we can make the IR constraints true, by choosing appropriate values for the payments $X(t_1,t_2)$. The non-negativity constraints on $X(t_1,t_2)$ are true from (38), (39), (40), and the reliability advantages $\Gamma^{HH}_{HL}(q,p)$ and $\Gamma^{HL}_{LL}(q,p)$ being non-negative. Then program (8) is equivalent to

$$\begin{aligned}
&\max_{\substack{(q,p)(H,H),\\(q,p)(H,L),(q,p)(L,L)}} && \text{objective (42)} \\
&\text{subject to} && \text{IC constraints (41a), (41b) and (41c)} \\
& && q(t_1,t_2) \geq 0,\ r \geq p(t_1,t_2) \geq 0,\ \text{for } (t_1,t_2) \in \{(H,H),(H,L),(L,L)\}.
\end{aligned} \tag{43}$$

The solution to program (43) depends on whether $\Gamma^{HH}_{HL}(H,L) \geq \Gamma^{HH}_{HL}(L,L)$ or not. In Lemma 10, we shall show that the optimal solution satisfies this inequality. Therefore, in the rest of the proof, without loss of optimality we assume

$$\Gamma^{HH}_{HL}(H,L) \geq \Gamma^{HH}_{HL}(L,L). \tag{44}$$

In the third step, we apply the additional constraint (44) to program (43) and group the terms in the objective by the PSP's type, leading to program (45):

$$\max_{\substack{(q,p)(H,H),\\(q,p)(H,L),(q,p)(L,L)}} \quad (\alpha^H)^2\Big[rE\min\Big\{D,\min\{q(H,H),\rho^H z_1^*(H,H)+\rho^H z_2^*(H,H)\}\Big\}$$

$$-c^H z_1^*(H,H)-c^H z_2^*(H,H)\Big] \tag{45a}$$

$$+2(\alpha^H\alpha^L)\Big[rE\min\Big\{D,\min\{q(H,L),\rho^H z_1^*(H,L)+\rho^L z_2^*(H,L)\}\Big\}$$

$$-c^H z_1^*(H,L)-c^L z_2^*(H,L)\Big]-(\alpha^H)^2\Gamma^{\frac{HH}{HL}}(H,L) \tag{45b}$$

$$+(\alpha^L)^2\Big[rE\min\Big\{D,\min\{q(L,L),\rho^L z_1^*(L,L)+\rho^L z_2^*(L,L)\}\Big\}$$

$$-c^L z_1^*(L,L)-c^L z_2^*(L,L)\Big]-[(\alpha^H)^2+2(\alpha^H\alpha^L)]\Gamma^{\frac{HL}{LL}}(L,L). \tag{45c}$$

subject to IC constraints (44), (41b), and

$$\Gamma^{\frac{HH}{HL}}(H,H)\geq\Gamma^{\frac{HH}{HL}}(H,L), \tag{45d}$$

$$\Gamma^{\frac{HH}{HL}}(H,H)+\Gamma^{\frac{HL}{LL}}(H,H)\geq\Gamma^{\frac{HH}{HL}}(H,L)+\Gamma^{\frac{HL}{LL}}(L,L), \tag{45e}$$

$$q(t_1,t_2)\geq 0,\ r\geq p(t_1,t_2)\geq 0,\ \text{for } (t_1,t_2)\in\{(H,H),(H,L),(L,L)\}.$$

In the fourth and last step, we solve program (45). We first relax all IC constraints, (44), (45d), (41b) and (45e), obtaining

$$\max_{\substack{(q,p)(H,H),\\(q,p)(H,L),(q,p)(L,L)}} \quad \text{objective (45a) + (45b) + (45c)}$$
$$\text{subject to} \quad q(t_1,t_2)\geq 0,\ r\geq p(t_1,t_2)\geq 0,\ \text{for } (t_1,t_2)\in\{(H,H),(H,L),(L,L)\} \tag{46}$$

The set of $(q,p)(t_1,t_2)$ optimal to relaxation (46) is presented in Lemma 8. If a solution in this set satisfies all IC constraints (44), (45d), (41b) and (45e), then the solution is also optimal to program (45), and to the original program (8).

From the solutions in Lemma 8, we select a subset of solutions by imposing the following restrictions: $p^*(H,H)\geq p^*(H,L)\geq p^*(L,L)$. Lemma 9 establishes that under Condition 1 the subset of solutions obtained by applying these restrictions is non-empty, and all solutions in this subset satisfy the IC constraints (44), (45d), (41b) and (45e). Therefore, these solutions are optimal to program (8). In presenting the results in this proposition, without loss of optimality, we choose $p^*(H,H)$ and $p^*(H,L)$ to be fixed at their upper bounds.

Finally, we show how the optimal transfer payments, $X^*(t_1,t_2)$, can be computed from $(q^*,p^*)(t_1,t_2)$.

From Lemma 2, we have

$$
\begin{aligned}
X^*(t_1,t_2) = p^*(t_1,t_2)q^*(t_1,t_2) &- \left[\psi^{t_1}\Big((q^*,p^*)(t_1,t_2)\Big)\right]^+ \\
&- \left[\psi^{t_1,t_2}\Big((q^*,p^*)(t_1,t_2)\Big)\right]^+ + \pi^{t_1,t_2}\Big((X^*,q^*,p^*)(t_1,t_2)\Big),
\end{aligned}
\tag{47}
$$

where the expressions for $\pi^{t_1,t_2}\big((X^*,q^*,p^*)(t_1,t_2)\big)$ are given in (38), (39) and (40), respectively, depending on types. Furthermore, under the additional constraint (44), equation (40) becomes

$$
\pi^{HH}(H,H) = \Gamma^{\frac{HH}{HL}}(H,L) + \Gamma^{\frac{HL}{LL}}(L,L). \tag{48}
$$

We apply (38), (39) and (48) to equation (47), obtaining the expressions for $X^*(t_1,t_2)$. The results are presented in equations (18) in Appendix A. We can substitute the optimal $(q^*,p^*)(t_1,t_2)$ (presented in this proposition) in (18) to compute $X^*(t_1,t_2)$. ■

Lemma 8. *The optimal solutions to relaxation program* (46) *are:*

Name	*Model parameters satisfy*	$q^*(t_1,t_2)$	$p^*(t_1,t_2)$	(z_1^*,z_2^*)
		$(t_1,t_2) =$ *(H,H)*		
(HH.1)	$(\alpha^H)^2\psi^{HH} < 0$	1	*any* $p \in \left[\frac{c^H}{h}, r\right]$	$(1,0)$
(HH.2)	$(\alpha^H)^2\psi^{HH} \geq 0$		*any* $p \in \left[\frac{c^H}{h(1-h)}, r\right]$	$(1,1)$
		$(t_1,t_2) =$ *(H,L)*		
(HL.1)	$2(\alpha^H\alpha^L)\psi^{HL} - \left(\alpha^H\right)^2\phi < 0$	1	*any* $p \in \left[\frac{c^H}{h}, \min\{r, \frac{c^H}{h(1-h)}\}\right]$	$(1,0)$
(HL.2)	$2(\alpha^H\alpha^L)\psi^{HL} - \left(\alpha^H\right)^2\phi \geq 0$		$\frac{c^L}{l(1-h)}$	$(1,1)$
		$(t_1,t_2) =$ *(L,L)*		
(LL.1)	$(\alpha^L)^2\psi^L - [1-(\alpha^L)^2]\phi < 0$ *and* $(\alpha^L)^2(\psi^L+\psi^{LL}) - [1-(\alpha^L)^2]\omega < 0$	0	0	$(0,0)$
(LL.2)	$(\alpha^L)^2\psi^L - [1-(\alpha^L)^2]\phi >$ $\left\{(\alpha^L)^2(\psi^L+\psi^{LL}) - [1-(\alpha^L)^2]\omega\right\}^+$	1	$\frac{c^L}{l}$	$(1,0)$
(LL.3)	$(\alpha^L)^2(\psi^L+\psi^{LL}) - [1-(\alpha^L)^2]\omega \geq$ $\left\{(\alpha^L)^2\psi^L - [1-(\alpha^L)^2]\phi\right\}^+$		$\tilde{p}$	$(1,1)$

where

$$
\tilde{p} \overset{def}{=} \begin{cases} \frac{c^L}{l(1-l)} & \textit{if } (1-l)^2 - (1-h) \geq 0 \\ \min\left\{r, \frac{c^L}{l(1-h)}\right\} & \textit{if } (1-l)^2 - (1-h) < 0 \textit{ and } r \geq \frac{c^L}{l(1-l)} \\ n.a. & \textit{otherwise} \end{cases},
$$

$\phi = h\left(\frac{c^L}{l} - \frac{c^H}{h}\right)$, *and* $\omega = \Gamma^{\frac{HL}{LL}}(1,\tilde{p})$.

Proof. The optimization program (46) can be treated as the sum of three independent optimization programs:

$$
\left\{\max_{r\geq p(H,H)\geq 0, q(H,H)\geq 0} \text{ objective (45a)}\right\} \tag{49a}
$$

$$
+\left\{\max_{r\geq p(H,L)\geq 0, q(H,L)\geq 0} \text{ objective (45b)}\right\} \tag{49b}
$$

$$+\left\{\max_{r\geq p(L,L)\geq 0, q(L,L)\geq 0} \text{ objective (45c)}\right\} \tag{49c}$$

We shall solve each of these programs, starting with (49c).

The solution procedure for (49c)**. First**, we shall show that without loss of optimality we can focus on solutions such that $q(L,L) = D = 1$.

From Lemma 1, for any given $q(L,L)$, the (L,L) PSP-type's optimal production assignments $(z_1^*, z_2^*)(L,L)$ are $(0,0)$, $\big(q(L,L), 0\big)$ or $\big(q(L,L), q(L,L)\big)$. Specifically, if the buyer sets the penalty $p(L,L) \geq c^L/[l(1-l)]$, the PSP's production assignments are $(z_1^*, z_2^*)(L,L) = \big(q(L,L), q(L,L)\big)$. Objective (45c) becomes

$$(\alpha^L)^2\Big(rE\min\Big\{D, \min\big\{1, \rho_1^L + \rho_2^L\big\}\, q(L,L)\Big\} - 2c^L q(L,L)\Big) - [(\alpha^H)^2 + 2(\alpha^H\alpha^L)]\Gamma_{LL}^{HL}(L,L), \tag{50}$$

where $\min\big\{1, \rho_1^L + \rho_2^L\big\}\, q(L,L)$ is the random quantity delivered by the (L,L) PSP-type. The buyer receives $q(L,L)$ with probability $1-(1-l)^2$ (when one, or both, of ρ_1^L and ρ_2^L equals 1), and receives zero with probability $(1-l)^2$ (when $\rho_1^L = \rho_2^L = 0$). The buyer's expected quantity sold to the market is $\big[1-(1-l)^2\big]\min\big\{D, q(L,L)\big\}$. Then, objective (45c) becomes

$$(\alpha^L)^2\Big(r\big[1-(1-l)^2\big]\min\big\{D, q(L,L)\big\} - 2c^L q(L,L)\Big) - \big[(\alpha^H)^2 + 2(\alpha^H\alpha^L)\big]\Gamma_{LL}^{HL}(L,L). \tag{51}$$

From Lemma 6, for a given $p(L,L)$, $\Gamma_{LL}^{HL}(L,L) \overset{\text{def}}{=} \Gamma_{LL}^{HL}\big((q,p)(L,L)\big)$ is linear in $q(L,L)$. Therefore, objective (51) is piece-wise linear and concave in $q(L,L)$ with two corners being $q(L,L) = 0$ and $q(L,L) = D = 1$.

Furthermore, we can discard $q(L,L) = 0$, because this solution is attainable by setting the requirement to be $q(L,L) = 1$ and making the penalty $p(L,L)$ sufficiently low, so that the PSP makes no production (i.e., $(z_1^*, z_2^*)(L,L) = (0,0)$) and information cost (if any) is zero. Thus, $q(L,L) = 1$ at optimality.

Next, we apply $q(L,L) = 1$ to objective (45c), and search for the optimal penalty $p(L,L)$. The buyer chooses the penalty to induce one of the following production assignments: $(z_1^*, z_2^*)(L,L) = (0,0), (1,0)$ and $(1,1)$ (see Lemma 1 for the penalty values that induces these assignments). In the following analysis, we identify the locally optimal penalties that induces these assignments, and then identify the global optimal penalty.

To induce a void production assignment, $(z_1^*, z_2^*)(L,L) = (0,0)$, from Lemma 1, the buyer selects any $p(L,L) < c^L/l$. The buyer collects zero profit from the market, but incurs a positive information cost, $[(\alpha^H)^2 + 2(\alpha^H\alpha^L)]\Gamma_{LL}^{HL}(1, p(L,L))$ for any $p(L,L) > c^H/h$. Therefore, the buyer sets $p(L,L)$ to be any values less than c^H/h, so that the information cost equals zero, and so does objective (45c).

To induce assignment $(z_1^*, z_2^*)(L,L) = (1,0)$, the buyer selects the penalty $p(L,L)$ from the interval $\left[c^L/l, c^L/[l(1-l)]\right)$. Because $\Gamma^{HL}_{LL}\big(1, p(L,L)\big)$ is increasing in $p(L,L)$ in this interval (see Lemma 6), to minimize the information cost, we fix $p(L,L)$ at its lower bound c^L/l. At this penalty, $\Gamma^{HL}_{LL}(1, c^L/l) = \phi$ (defined in (10b)). The buyer satisfies the market demand $D = 1$ with probability l, and collects expected profit of $lr - c^L$ from the market. Recall that we defined $\psi^L \stackrel{\text{def}}{=} \psi^L(1,r) = lr - c^L$ (see (9)). Thus, objective (45c) equals

$$(\alpha^L)^2\psi^L - [(\alpha^H)^2 + 2(\alpha^H\alpha^L)]\phi. \tag{52}$$

To induce $(z_1^*, z_2^*)(L,L) = (1,1)$, the buyer sets the penalty $p(L,L)$ to be in the interval $\left[c^L/[l(1-l)], \infty\right)$. To choose a penalty from this interval while satisfying $p(L,L) \le r$. We must have $r \ge c^L/[l(1-l)]$. The optimal value of $p(L,L)$ depends on whether $(1-l)^2 - (1-h) \ge 0$ or not. If $(1-l)^2 - (1-h) \ge 0$, the information cost term $\Gamma^{HL}_{LL}\big(1, p(L,L)\big)$ is increasing in $p(L,L) \in \left[c^L/[l(1-l)], \infty\right)$ (see Lemma 6). Therefore, we fix $p(L,L) = c^L/[l(1-l)]$ to minimize the information cost. The limited liability constraint, $p(L,L) \le r$, follows from the fact that the revenue r must exceed $c^L/[l(1-l)]$ for the buyer to collect a positive benefit by using diversification with the (L,L) type. If $(1-l)^2 - (1-h) < 0$, then $\Gamma^{HL}_{LL}\big(1, p(L,L)\big)$ is decreasing in $p \in \left[c^L/[l(1-l)], c^L/[l(1-h)]\right)$, and is increasing in $p \in [c^L/[l(1-h)], \infty)$ (see Lemma 6). The optimal penalty $p(L,L)$ is the smaller value of $c^L/[l(1-h)]$ and r. We define the optimal penalty for the (L,L) PSP-type to diversify to be $\tilde{p}$, formalized in (10a).

We now compute the value of (45c). Under the optimal penalty $p(L,L) = \tilde{p}$, the minimum information cost of diversifying with the (L,L) type is $\omega \stackrel{\text{def}}{=} \Gamma^{HL}_{LL}(1,\tilde{p})$, presented in (10c). Under production assignment $(z_1^*, z_2^*)(L,L) = (1,1)$, the buyer collects expected profit of $[1-(1-l)^2]r - 2c^L$ from the market. Note that $[1-(1-l)^2]r - 2c^L = (lr - c^L) + [l(1-l)r - c^L] = \psi^L + \psi^{LL}$, where $\psi^{LL} \stackrel{\text{def}}{=} \psi^{LL}(1,r) = l(1-l)r - c^L$ (see (9)). Objective (45c) equals

$$(\alpha^L)^2(\psi^L + \psi^{LL}) - [(\alpha^H)^2 + 2(\alpha^H\alpha^L)]\omega. \tag{53}$$

Finally, we compare the local values of objective (45c), under the three scenarios of penalty-production assignments, to identify the global optimal penalty $p(L,L)$. When (52) and (53) are both negative, it is optimal to set $p(L,L) \le c^H/h$, which induces no production assignment $(z_1^*, z_2^*)(L,L) = (0,0)$. This is equivalent to $q(L,L) = 0$ and $p(L,L) = 0$. When (52) is greater than both zero and (53), the optimal penalty is $p(L,L) = c^L/l$, which induces production assignment $(z_1^*, z_2^*)(L,L) = (1,0)$. When (53) is greater than both zero and (52), the optimal penalty is $p(L,L) = c^L/[l(1-l)]$, which induces production assignment $(z_1^*, z_2^*)(L,L) = (1,1)$. Finally, we can verify that the optimal $p(L,L)$ derived from this procedure is less than r, and hence are feasible.

The solution procedure for (49b)**.** Because $r > c^H/h$ (see Assumption 1), the profit of inducing sole-sourcing (from the high-type supplier), $\psi^H \overset{\text{def}}{=} hr - c^H$, is positive. Hence, it is suboptimal to induce the (H,L) type to make no production assignment. We focus on the other two cases: $(z_1^*, z_2^*)(H, L) = (1, 0)$ and $(z_1^*, z_2^*)(H, L) = (1, 1)$.

From Lemma 1, to induce assignments $(z_1^*, z_2^*)(H, L) = (1, 0)$, the buyer selects the penalty $p(H, L)$ from interval $[c^H/h, c^L/[l(1-h)])$. From Lemma 6, the information cost $\Gamma^{\frac{HH}{HL}}\big(1, p(H, L)\big)$ is minimized (to zero), if $p(H, L) \le c^H/[h(1-h)]$. Note from Lemma 5 that the model parameters must satisfy $c^H/h < c^H/[h(1-h)] < c^L/[l(1-h)]$. Any $p(L, L) \in \big[c^H/h, c^H/[h(1-h)]\big]$ is feasible. With the production assignments $(z_1^*, z_2^*)(H, L) = (1, 0)$, the buyer collects the expected profit of $\psi^H \overset{\text{def}}{=} hr - c^H$ from the market, and objective (45b) equals $2(\alpha^H\alpha^L)\psi^H$.

From Lemma 1, to induce $(z_1^*, z_2^*)(H, L) = (1, 1)$, the penalty should satisfy $p(H, L) \ge c^L/[l(1-h)]$. From Lemma 6, the information cost $\Gamma^{\frac{HH}{HL}}\big(1, p(H, L)\big)$ is increasing in $p(H, L)$ for $p(H, L) \ge c^L/[l(1-h)]$. Therefore, we set $p(H, L) = c^L/[l(1-h)]$ to minimize the information cost, obtaining $\Gamma^{\frac{HH}{HL}}(1, p(H, L)) = \phi$ (defined in (10b)). Under the production assignments $(z_1^*, z_2^*)(H, L) = (1, 1)$, the buyer collects the expected profit of $\psi^H + \psi^{HL}$, and objective (45b) equals $2(\alpha^H\alpha^L)\psi^H + \big[2(\alpha^H\alpha^L)\psi^{HL} - (\alpha^H)^2\phi\big]$.

We compare the two local values of objective (45b) to identify the global optimal $p(H, L)$ and impose the limited liability constraint. If $2(\alpha^H\alpha^L)\psi^{HL} - (\alpha^H)^2\phi < 0$, the optimal penalty is any $p(L, L) \in \big[c^H/h, c^H/[h(1-h)]\big]$. Under the limited liability constraint, $p(H, L) \le r$, the set of feasible $p(H, L)$ is $\big[c^H/h, \min\{r, c^H/[h(1-h)]\}\big]$. If $2(\alpha^H\alpha^L)\psi^{HL} - (\alpha^H)^2\phi \ge 0$, the optimal penalty is $p(H, L) = c^L/[l(1-h)]$. Note that $2(\alpha^H\alpha^L)\psi^{HL} - (\alpha^H)^2\phi \ge 0$ implies $r > c^L/[l(1-h)]$. Therefore, $p(H, L) = c^L/[l(1-h)]$ satisfies $p(H, L) \le r$.

The solution procedure for problem (49a)**.** Following the solution procedure for (49b) and (49c), we obtain the following results: when $(\alpha^H)^2\psi^{HH} \le 0$, the optimal penalty $p(H, H) \in \big[c^H/h, c^H/[h(1-h)]\big)$; otherwise, $p(H, H) \ge c^H/[h(1-h)]$.

We impose constraint $p(H, H) \le r$ to ensure the feasibility of $p(H, H)$. Inequality $(\alpha^H)^2\psi^{HH} \le 0$ implies $r \le c^H/[h(1-h)]$. Therefore, under $(\alpha^H)^2\psi^{HH} \le 0$, the optimal penalty must be $p(H, H) \in [c^H/h, r]$. Inequality $(\alpha^H)^2\psi^{HH} > 0$ implies $r > c^H/[h(1-h)]$. Under $p(H, H) \le r$, the optimal penalty must be $p(H, H) \in \big[c^H/[h(1-h)], r\big]$. ■

Lemma 9. *There exist $p(H, H)$, $p(H, L)$ and $p(L, L)$ that belong to the set of optimal penalties in Lemma 8, such that $p(H, H) \ge p(H, L) \ge p(L, L)$. For such penalties, IC constraints* (44), (45d), (41b) *and* (45e) *are satisfied.*

Proof. The proof comprises two steps.

In step 1, we prove the existence of optimal penalties satisfying $p(H,H) \geq p(H,L) \geq p(L,L)$.

For that we first prove that (a) for any $p(H,L)$, there exists $p(H,H) \geq p(H,L)$, and (b) for any $p(L,L)$ there exists $p(H,L) \geq p(L,L)$. Let's begin with the proof of (a). From Lemma 8, it is true that $p(H,L) \leq r$, and r belongs to the interval of the optimal $p(H,H)$. Therefore, $p(H,H) = r$ guarantees $p(H,H) \geq p(H,L)$.

Next, we prove (b). From Lemma 8, $p(L,L)$ can take three possible values. First, suppose that condition (LL.1) is satisfied and $p(L,L) = 0$. Observe that the possible values of $p(H,L)$ are $c^L/[l(1-h)]$ and any value in the non-empty interval $[c^H/h, \min\{r, c^H/[h(1-h)]\}]$. Therefore, $p(H,L) > 0$.

Second, suppose that condition (LL.2) is satisfied and $p(L,L) = c^L/l$. The value of $p(H,L)$ depends on which condition, (HL.1) or (HL.2), is true. If (HL.2) holds, then $p(H,L) = c^L/[l(1-h)] > c^L/l = p(L,L)$. If (HL.1) holds, then from Condition 1, $c^H/[h(1-h)] \geq c^L/l$. Therefore, there exists a $p(H,L) \geq p(L,L)$.

Lastly, suppose there condition (LL.3) is satisfied and $p(L,L) = \tilde{p}$. Following the analysis similar to the one for the case of (LL.2), and invoking Condition 1 when necessary, we prove that there exists $p(H,L) \geq p(L,L)$.

In step 2, we show the subset of solutions obtained under restriction $p(H,H) \geq p(H,L) \geq p(L,L)$ satisfy all IC constraints. We shall divide the proof into two parts: $(1-l)^2-(1-h) \geq 0$ and the converse condition. If $(1-l)^2-(1-h) \geq 0$, then $\Gamma^{HH}_{HL}(q,p)$ and $\Gamma^{HL}_{LL}(q,p)$ are both monotonic and increasing in p (see Lemma 6). Therefore, $\Gamma^{HL}_{LL}\big(1, p(H,L)\big) \geq \Gamma^{HL}_{LL}\big(1, p(L,L)\big)$, which is the IC constraint (41b). Similarly, we show the IC constraints (45d), (41b) and (45e) are satisfied.

On the contrary, if $(1-l)^2-(1-h) < 0$, then only $\Gamma^{HH}_{HL}(q,p)$ is monotonic and increasing in p. Constraints (44) and (45d) still follow from ranking $p(H,H) \geq p(H,L) \geq p(L,L)$. However, we need to prove (41b) and (45e) using a different approach.

Consider constraint (41b). From Lemma 8, under (HL.2), $p(H,L) = \frac{c^L}{l(1-h)}$. From Lemma 6, we can verify that $\Gamma^{HL}_{LL}\left(1, \frac{c^L}{l(1-h)}\right) \geq \Gamma^{HL}_{LL}\big(1, p(L,L)\big)$ for any value of $p(L,L)$ in Lemma 8. On the other hand, under (HL.1), we have $p(H,L) \in \left[p(L,L), \min\{r, \frac{c^H}{h(1-h)}\}\right]$. If (LL.1) is true and $p(L,L) = 0$, then constraint (41b) holds trivially. If (LL.2) is true and $p(L,L) = c^L/l$, then from Lemma 6, one can verify that for all $p \geq c^L/l$, $\Gamma^{HL}_{LL}(1,p) \geq \Gamma^{HL}_{LL}(1, c^L/l)$. Hence, (41b) follows from $p(H,L) \geq p(L,L) = c^L/l$. Finally, observe that $(1-l)^2-(1-h) < 0$ and (LL.3) cannot hold at the same time.

Consider constraint (45e). Given that (45d) holds, a sufficient condition for (45e) is $\Gamma^{HL}_{LL}(1, p(H,H)) \geq$

$\Gamma^{HL}_{LL}(1, p(L, L))$. We now prove this inequality. From Lemma 8, under (HL.2), $p(H, L) = \frac{c^L}{l(1-h)}$. From Lemma 6, because $\Gamma^{HL}_{LL}(1, p)$ is increasing in p for $p \geq \frac{c^L}{l(1-h)}$ and $p(H, H) \geq p(H, L) = \frac{c^L}{l(1-h)}$, we have $\Gamma^{HL}_{LL}(1, p(H, H)) \geq \Gamma^{HL}_{LL}(1, p(H, L))$. This and (41b) together imply inequality $\Gamma^{HL}_{LL}(1, p(H, H)) \geq \Gamma^{HL}_{LL}(1, p(L, L))$, which is the sufficient condition for constraint (45e). On the other hand, under (HL.1), we have $p(H, L) \in \left[p(L, L), \min\{r, \frac{c^H}{h(1-h)}\}\right]$. If (LL.1) is true, then $p(L, L) = 0$. Inequality $\Gamma^{HL}_{LL}(1, p(H, H)) \geq \Gamma^{HL}_{LL}(1, p(L, L))$ holds trivially. If (LL.2) is true, then $p(L, L) = c^L/l$. From Lemma 6, for all $p \geq c^L/l$, $\Gamma^{HL}_{LL}(1, p) \geq \Gamma^{HL}_{LL}(1, c^L/l)$ Because $p(H, H) \geq p(H, L) \geq p(L, L) = c^L/l$, we must have $\Gamma^{HL}_{LL}(1, p(H, H)) \geq \Gamma^{HL}_{LL}(1, p(L, L))$. Finally, observe that $(1-l)^2 - (1-h) < 0$ and (LL.3) cannot hold at the same time. ∎

Lemma 10. *The optimal solution to program (43) with an additional constraint $\Gamma^{HH}_{HL}(H, L) \geq \Gamma^{HH}_{HL}(L, L)$ dominates the solution of the problem with an additional constraint $\Gamma^{HH}_{HL}(H, L) < \Gamma^{HH}_{HL}(L, L)$.*

Proof. **In preparation for the proof**, we first simplify program (43) using the additional constraint (54):

$$\Gamma^{HH}_{HL}\big((q, p)(H, L)\big) \leq \Gamma^{HH}_{HL}\big((q, p)(L, L)\big), \tag{54}$$

obtaining:

$$\max_{\substack{(q,p)(t_1,t_2),\\ t_1,t_2\in\{H,L\}}}$$

$$(\alpha^H)^2\Big[rE\min\Big\{D, \min\big\{q(H,H), \rho^H z_1^*(H,H) + \rho^H z_2^*(H,H)\big\}\Big\} - c^H z_1^*(H,H) - c^H z_2^*(H,H)\Big] \tag{55a}$$

$$+ 2(\alpha^H\alpha^L)\Big[rE\min\Big\{D, \min\big\{q(H,L), \rho^H z_1^*(H,L) + \rho^H z_2^*(H,L)\big\}\Big\} - c^H z_1^*(H,L) - c^L z_2^*(H,L)\Big] \tag{55b}$$

$$\left.\begin{aligned} &+(\alpha^L)^2\Big[rE\min\Big\{D, \min\big\{q(L,L), \rho^L z_1^*(L,L) + \rho^L z_2^*(L,L)\big\}\Big\} - c^L z_1^*(L,L) - c^L z_2^*(L,L)\Big] \\ &\quad -(\alpha^H)^2\Gamma^{HH}_{HL}\big((q,p)(L,L)\big) - [(\alpha^H)^2 + 2(\alpha^H\alpha^L)]\Gamma^{HL}_{LL}\big((q,p)(L,L)\big)\end{aligned}\right\} \tag{55c}$$

Subject to

$$\Gamma^{HH}_{HL}\big((q,p)(H,L)\big) \leq \Gamma^{HH}_{HL}\big((q,p)(L,L)\big) \tag{55d}$$

$$\Gamma^{HH}_{HL}\big((q,p)(H,H)\big) \geq \Gamma^{HH}_{HL}\big((q,p)(L,L)\big) \tag{55e}$$

$$\Gamma^{HL}_{LL}\big((q,p)(H,L)\big) \geq \Gamma^{HL}_{LL}\big((q,p)(L,L)\big) \tag{55f}$$

$$\Gamma^{HH}_{HL}\big((q,p)(H,H)\big) + \Gamma^{HL}_{LL}\big((q,p)(H,H)\big) \geq \Gamma^{HH}_{HL}\big((q,p)(L,L)\big) + \Gamma^{HL}_{LL}\big((q,p)(L,L)\big) \tag{55g}$$

$$q(t_1,t_2) \geq 0,\ r \geq p(t_1,t_2) \geq 0,\ \text{for } (t_1,t_2) \in \{(H,H),(H,L),(L,L)\}.$$

Note that the constraint (55d) is identical to (54). We denote a solution (contract menu) to be $\sigma \overset{\text{def}}{=} \big((q,p)(H,H), (q,p)(H,L), (q,p)(L,L)\big)$. We use $\tau^{HH}\big((q,p)(H,H)\big)$, $\tau^{HL}\big((q,p)(H,L)\big)$ and $\tau^{LL}\big((q,p)(L,L)\big)$ to denote the three parts of the objective at solution σ, (55a), (55b) and (55c), and use $\Pi(\sigma)$ to denote the total value of the objective:

$$\Pi(\sigma) = \tau^{HH}\big((q,p)(H,H)\big) + \tau^{HL}\big((q,p)(H,L)\big) + \tau^{LL}\big((q,p)(L,L)\big). \tag{56}$$

We define the feasible region of program (55) to be F.

We now sketch the plan for the proof. To prove this lemma, we shall show that there exists an optimal solution of program (55) that satisfies the binding constraint (55d).

Lemma 11 shows that at any $\sigma \in F_N$ inequality (55d) is binding, where

$$\begin{aligned} F_N \stackrel{\text{def}}{=} F \cap \Big(&\{\sigma : 0 \le p(H,L) \le c^H/h\} \cup \{\sigma : q(H,L) = 0\} \\ &\cup \{\sigma : 0 \le p(L,L) \le c^H/[h(1-h)]\} \cup \{\sigma : q(L,L) = 0\}\Big). \end{aligned} \tag{57}$$

If there exists $\sigma \in F_N$ that is optimal to program (55), then the lemma is proved. Otherwise, in search for optimality all points in F_N are irrelevant, and we study set $F \setminus F_N$, defined as:

$$F^* \stackrel{\text{def}}{=} F \cap \left\{\sigma : p(H,L) > \tfrac{c^H}{h}, p(L,L) > \tfrac{c^H}{h(1-h)}, q(L,L) > 0, q(H,L) > 0\right\}. \tag{58}$$

Hereafter, the proof will be done by contradiction. We make the following assumption:
Assumption (a1): *Assume that all optimal solutions to program* (55) *must be in set*

$$F_a^* \stackrel{\text{def}}{=} F^* \cap \left\{\sigma : \Gamma^{HH}_{HL}\big((q,p)(H,L)\big) < \Gamma^{HH}_{HL}\big((q,p)(L,L)\big)\right\}. \tag{59}$$

We shall show that for any $\sigma_0 \in F_a^*$, one of the following two statements must be true:

1. There exists a solution $\sigma_1 \in F_a^*$, such that $\Pi(\sigma_1) > \Pi(\sigma_0)$.
2. There exists a solution σ_1 in set

$$F_b^* \stackrel{\text{def}}{=} F^* \cap \left\{\sigma : \Gamma^{HH}_{HL}\big((q,p)(H,L)\big) = \Gamma^{HH}_{HL}\big((q,p)(L,L)\big)\right\}, \tag{60}$$

   such that $\Pi(\sigma_1) \ge \Pi(\sigma_0)$.

Thus, we shall arrive at a contradiction to Assumption (a1).

Let's proceed with the proof. Region F_a^* is further partitioned into six regions:

$$F_{a1}^* \stackrel{\text{def}}{=} F_a^* \cap \{\sigma : q(L,L) > 1\} \tag{61a}$$

$$F_{a2}^* \stackrel{\text{def}}{=} F_a^* \cap \{\sigma : q(H,L) < 1\} \tag{61b}$$

$$F_{a3}^* \stackrel{\text{def}}{=} F_a^* \cap \left\{\sigma : q(L,L) \le 1 \le q(H,L), p(H,L) \le \tfrac{c^L}{l(1-h)}, p(L,L) \le \tfrac{c^L}{l(1-h)}\right\} \tag{61c}$$

$$F_{a4}^* \stackrel{\text{def}}{=} F_a^* \cap \left\{\sigma : q(L,L) \le 1 \le q(H,L), p(H,L) \le \tfrac{c^L}{l(1-h)}, p(L,L) > \frac{c^L}{l(1-h)}\right\} \tag{61d}$$

$$F_{a5}^* \stackrel{\text{def}}{=} F_a^* \cap \left\{\sigma : q(L,L) \le 1 \le q(H,L), p(H,L) > \tfrac{c^L}{l(1-h)}, p(L,L) \le \tfrac{c^L}{l(1-h)}\right\} \tag{61e}$$

$$F_{a6}^* \stackrel{\text{def}}{=} F_a^* \cap \left\{ \sigma : q(L,L) \le 1 \le q(H,L), p(H,L) > \tfrac{c^L}{l(1-h)}, p(L,L) > \tfrac{c^L}{l(1-h)} \right\}. \tag{61f}$$

Lemmas 12 establishes a contradiction to Assumption (a1) for all $\sigma \in F_{a1}^*$. Lemma 14 establishes a contradiction to Assumption (a1) for all $\sigma \in F_{a2}^*$. Lemmas 16 establishes a contradiction to Assumption (a1) for all $\sigma \in F_{a3}^*$. Lemma 17 establishes a contradiction to Assumption (a1) for all $\sigma \in F_{a4}^*$. Lemma 18 shows that region F_{a5}^* is empty. Lemma 19 establishes a contradiction to Assumption (a1) for all $\sigma \in F_{a6}^*$. ■

Lemma 11. *Any $\sigma \in F_N$ satisfies the binding constraint* (55d).

Proof. From its definition, set F_N is the union of the following two subsets:

$$F_N^{(1)} \stackrel{\text{def}}{=} F \cap \left(\left\{ \sigma : 0 \le p(H,L) \le c^H/h \right\} \cup \left\{ \sigma : q(H,L) = 0 \right\} \right), \text{ and} \tag{62a}$$

$$F_N^{(2)} \stackrel{\text{def}}{=} F \cap \left(\left\{ \sigma : 0 \le p(L,L) \le c^H/[h(1-h)] \right\} \cup \{ \sigma : q(L,L) = 0 \} \right). \tag{62b}$$

Thus, the proof has two parts.

First, we show that any $\sigma \in F_N^{(1)}$ satisfies the binding constraint (54). Given that $0 \le p(H,L) \le c^H/h$ or $q(H,L) = 0$, from Lemma 6, solution σ satisfies $\Gamma^{\frac{HL}{LL}}\big((q,p)(H,L)\big) = 0$. Applying $\Gamma^{\frac{HL}{LL}}\big((q,p)(H,L)\big) = 0$ and the fact that $\Gamma^{\frac{HL}{LL}}\big((q,p)(L,L)\big) \ge 0$ (see Lemma 6) to constraint (55e), we conclude that σ must satisfy $\Gamma^{\frac{HL}{LL}}\big((q,p)(L,L)\big) = 0$. For this equality to be true at solution σ, from Lemma 6, we must have $q(L,L) = 0$ or $p(L,L) \le c^H/h$. In either case, $\Gamma^{\frac{HH}{HL}}\big((q,p)(L,L)\big) = 0$ (see Lemma 6). Furthermore, because $c^H/h < c^H/[h(1-h)]$ (see Lemma 5), it follows $p(H,L) \le c^H/[h(1-h)]$. under $0 \le p(H,L) \le c^H/[h(1-h)]$ or $q(H,L) = 0$, from Lemma 6, it follows that solution σ also satisfies $\Gamma^{\frac{HH}{HL}}\big((q,p)(H,L)\big) = 0$. Therefore, it follows $\Gamma^{\frac{HH}{HL}}\big((q,p)(H,L)\big) = \Gamma^{\frac{HH}{HL}}\big((q,p)(L,L)\big)$. This is the binding (54) at σ.

Second, we show that at any $\sigma \in F_N^{(2)}$, the constraint (54) must be binding. From Lemma 6, because $p(L,L) \le c^H/[h(1-h)]$ or $q(L,L) = 0$, it follows that $\Gamma^{\frac{HH}{HL}}\big((q,p)(L,L)\big) = 0$ at solution σ. The result follows from the fact $\Gamma^{\frac{HH}{HL}}\big((q,p)(H,L)\big) \ge 0$ for all $q(H,L) \ge 0$ and $p(H,L) \ge 0$. ■

Lemma 12 (**Region $\boldsymbol{F_{a1}^*}$**). *Define solution $\sigma_0 \stackrel{def}{=} \big((q_0,p_0)(H,H),(q_0,p_0)(H,L),(q_0,p_0)(L,L)\big)$. Suppose $\sigma_0 \in F_{a1}^*$. Define*

$$\sigma_1 \stackrel{def}{=} \big((q_0,p_0)(H,H),(q_0,p_0)(H,L),(q_1,p_0)(L,L)\big). \tag{63}$$

Then, there exists a $q_1(L,L) < q_0(L,L)$, such that the following statements are true: (a) $\sigma_1 \in F_{a1}^*$, *and* (b) $\Pi(\sigma_1) > \Pi(\sigma_0)$ *(strict).*

Proof. **First**, we establish that there exists a $q_1(L, L)$ such that solution $\sigma_1 \in F^*_{a1}$. Because solution σ_1 deviates from σ_0 only by the value of $q(L, L)$, we verify whether or not σ_1 satisfies the constraints that are relevant to $q(L, L)$: the non-binding (55d), (55e), (55f), (55g), and $q(L, L) > 1$. Because at solution σ_0 constraint (55d) is strict and $q_0(L, L) > 1$, from the continuity of $\Gamma^{\frac{HH}{HL}}(q, p)$ and the fact $\Gamma^{\frac{HH}{HL}}(q, p)$ is increasing in q, it follows that there exists a $q_1(L, L) < q_0(L, L)$, such that $\Gamma^{\frac{HH}{HL}}((q_0, p_0)(H, L)) < \Gamma^{\frac{HH}{HL}}((q_1, p_0)(L, L))$ (i.e., constraint (55d) at σ_1), and $q_1(L, L) > 1$. Furthermore, because $\Gamma^{\frac{HH}{HL}}(q, p)$ and $\Gamma^{\frac{HL}{LL}}(q, p)$ are increasing in q, and $q_1(L, L) < q_0(L, L)$, (55e), (55f) and (55g) continue to be true for σ_1. Therefore, solution $\sigma_1 \in F^*_{a1}$.

Second, we show that for $q_1(L, L) < q_0(L, L)$ such that $\sigma_1 \in F^*_{a1}$, we have objective $\Pi(\sigma_1) > \Pi(\sigma_0)$ strict. Because solutions σ_0 to σ_1 differ only in the value of $q(L, L)$, in the objective only the value of (55c) changes between the two solutions. Therefore, it suffices to show $\tau^{LL}\big((q_1, p_0)(L, L)\big) > \tau^{LL}\big((q_0, p_0)(L, L)\big)$. Lemma 13 shows that $\tau^{LL}\big((q, p)(L, L)\big)$ is strictly decreasing in $q(L, L)$ for $q(L, L) > 1$. The result follows from $1 < q_1(L, L) < q_0(L, L)$. ■

Lemma 13. *For $p(L, L) > c^H/h$, the value of* (55c) *(i.e., $\tau^{LL}\big((q, p)(L, L)\big)$) is strictly decreasing in $q(L, L)$ for $q(L, L) > 1$.*

Proof. Because the production assignments for the (L,L) type depend on the value of $p(L, L)$, we consider three cases.

Case (1), $p(L, L) \geq c^L/[l(1 - l)]$. From Lemma 1, the (L,L) type's production assignments are $(z^*_1, z^*_2)(L, L) = \big(q(L, L), q(L, L)\big)$. Substituting such $(z^*_1, z^*_2)(L, L)$ into (55c), and applying $q(L, L) > 1$, we derive

$$\begin{aligned} \tau^{LL}\big((q, p)(L, L)\big) &= (\alpha^L)^2\big[(2l - l^2)r - 2c^L q(L, L)\big] \\ &\quad - (\alpha^H)^2\Gamma^{\frac{HH}{HL}}\big((q, p)(L, L)\big) - [(\alpha^H)^2 + 2(\alpha^H\alpha^L)]\Gamma^{\frac{HL}{LL}}\big((q, p)(L, L)\big). \end{aligned} \tag{64}$$

From Lemma 6, $\Gamma^{\frac{HH}{HL}}\big((q, p)(L, L)\big)$ and $\Gamma^{\frac{HL}{LL}}\big((q, p)(L, L)\big)$ are both increasing in $q(L, L)$. The result follows.

Case (2), $c^L/l \leq p(L, L) < c^L/[l(1 - l)]$. The proof is identical to that for case (1), except that, under such $p(L, L)$, the production assignments are $(z^*_1, z^*_2)(L, L) = \big(q(L, L), 0\big)$, and the value of (55c) is

$$\begin{aligned} \tau^{LL}\big((q, p)(L, L)\big) &= (\alpha^L)^2\big[lr - c^L q(L, L)\big] \\ &\quad - (\alpha^H)^2\Gamma^{\frac{HH}{HL}}\big((q, p)(L, L)\big) - [(\alpha^H)^2 + 2(\alpha^H\alpha^L)]\Gamma^{\frac{HL}{LL}}\big((q, p)(L, L)\big). \end{aligned} \tag{65}$$

Case (3), $c^H/h < p(L, L) < c^L/l$. Under such $p(L, L)$, the production assignments are

$(z_1^*, z_2^*)(L, L) = \big(0, 0\big)$, and the value of (55c) is

$$\tau^{LL}\big((q,p)(L,L)\big) = -(\alpha^H)^2 \Gamma^{\frac{HH}{HL}}\big((q,p)(L,L)\big) - [(\alpha^H)^2 + 2(\alpha^H\alpha^L)]\Gamma^{\frac{HL}{LL}}\big((q,p)(L,L)\big). \tag{66}$$

From Lemma 6, for $p(L, L)$ in this interval, $\Gamma^{\frac{HH}{HL}}\big((q,p)(L,L)\big)$ is strictly increase in $q(L, L)$, and $\Gamma^{\frac{HL}{LL}}\big((q,p)(L,L)\big)$ is increasing in $q(L, L)$ (strictly or weakly). The result follows. ■

Lemma 14 (**Region F^*_{a2}**). *Define solution* $\sigma_0 \overset{def}{=} \big((q_0, p_0)(H, H), (q_0, p_0)(H, L), (q_0, p_0)(L, L)\big)$. *Suppose* $\sigma_0 \in F^*_{a2}$. *Define*

$$\sigma_1 \overset{def}{=} \big((q_0, p_0)(H, H), (q_1, p_0)(H, L), (q_0, p_0)(L, L)\big). \tag{67}$$

Then, there exists a $q_1(H, L) > q_0(H, L)$*, such that the following statements are true:* (a) $\sigma_1 \in F^*_{a2}$, *and* (b) $\Pi(\sigma_1) > \Pi(\sigma_0)$.

Proof. **First**, we establish that there exists a $q_1(H, L)$ such that solution $\sigma_1 \in F^*_{a2}$. Because solution σ_1 deviates from σ_0 only by the value of $q(H, L)$, we only have to verify whether or not σ_1 satisfies the constraints that are relevant to $q(H, L)$: the non-binding (55d), (55f), and $q(H, L) < 1$. Because at solution σ_0 constraint (55d) is strict and $0 < q_0(H, L) < 1$, from the continuity of $\Gamma^{\frac{HH}{HL}}(q, p)$ and the fact $\Gamma^{\frac{HH}{HL}}(q, p)$ is increasing in q, it follows that there exists a $q_1(H, L) > q_0(H, L)$, such that $\Gamma^{\frac{HH}{HL}}((q_1, p_0)(H, L)) < \Gamma^{\frac{HH}{HL}}((q_0, p_0)(L, L))$ (i.e., constraint (55d) at σ_1), and $q_1(H, L) < 1$. Furthermore, because $\Gamma^{\frac{HL}{LL}}(q, p)$ is increasing in q, and $q_1(H, L) > q_0(H, L)$, (55f) continue to be true for σ_1. Therefore, solution $\sigma_1 \in F^*_{a2}$.

Second, we show that for $q_1(H, L) > q_0(H, L)$ such that $\sigma_1 \in F^*_{a2}$, we have objective $\Pi(\sigma_1) > \Pi(\sigma_0)$ strict. Because only $q(H, L)$ is changed as we move from solution σ_0 to σ_1, in the objective only the value of (55b) changes between the two solutions. Therefore, it suffices to show $\tau^{HL}\big((q_1, p_0)(H, L)\big) > \tau^{HL}\big((q_0, p_0)(H, L)\big)$. Lemma 15 shows that $\tau^{HL}\big((q, p)(H, L)\big)$ is strictly increasing in $q(H, L)$ for $q(H, L) < 1$. The result follows from $1 > q_1(H, L) > q_0(H, L)$. ■

Lemma 15. *For* $p(H, L) > c^H/h$*, the value of* (55b) *(i.e.,* $\tau^{HL}\big((q, p)(H, L)\big)$*) is strictly increasing in* $q(H, L)$ *for* $0 \le q(H, L) < 1$.

Proof. Because the production assignments for the (H,L) type depend on the value of $p(H, L)$, we consider two cases.

Case (1), $p(H, L) \ge c^L/[l(1 - h)]$. From Lemma 1, the (H,L) type's production assignments are $(z_1^*, z_2^*)(H, L) = \big(q(H, L), q(H, L)\big)$. Substituting such $(z_1^*, z_2^*)(H, L)$ into (55b), and applying $q(H, L) < 1$, we derive

$$\tau^{HL}\big((q, p)(H, L)\big) = 2(\alpha^H\alpha^L)\big[(hr - c^H) + (l(1 - h)r - c^L)\big]q(H, L) \tag{68}$$

For $p(H,L) \geq c^L/[l(1-h)]$ to be feasible, we must have $r \geq p(H,L) \geq c^L/[l(1-h)]$, that is, $l(1-h)r - c^L \geq 0$. Because $r \geq c^L/[l(1-h)] > c^H/h$, we have $hr - c^H > 0$. Therefore, $\tau^{HL}\big((q,p)(H,L)\big)$ is strictly increasing in $q(H,L)$.

Case (2), $\boldsymbol{c^H/h < p(H,L) < c^L/[l(1-h)]}$. Following the same analysis for case (1), we obtain

$$\tau^{HL}\big((q,p)(H,L)\big) = 2(\alpha^H\alpha^L)(hr - c^H)q(H,L). \tag{69}$$

For $p(H,L) > c^H/h$ to be feasible, we must have $r \geq p(H,L) > c^H/h$. This implies $hr - c^H > 0$. The result follows. ■

Lemma 16 (**Region $\boldsymbol{F^*_{a3}}$**). *Define solution* $\sigma_0 \stackrel{def}{=} \big((q_0,p_0)(H,H),(q_0,p_0)(H,L),(q_0,p_0)(L,L)\big)$. *Suppose* $\sigma_0 \in F^*_{a3}$. *Define* $\sigma_1 \stackrel{def}{=} \big((q_0,p_0)(H,H),(q_0,p_1)(H,L),(q_0,p_0)(L,L)\big)$, *in which* $p_1(H,L)$ *satisfies:*

$$\Gamma^{\frac{HH}{HL}}\big((q_0,p_1)(H,L)\big) = \Gamma^{\frac{HH}{HL}}\big((q_0,p_0)(L,L)\big). \tag{70}$$

Then, the following statements are true: (a) $\sigma_1 \in F^*_b$, *and* (b) $\Pi(\sigma_1) \geq \Pi(\sigma_0)$.

Proof. Because $\Gamma^{\frac{HH}{HL}}(q,p)$ is strictly monotonic in p and $\Gamma^{\frac{HH}{HL}}(q,p) \in \mathbb{R}^+$, there exist a unique solution to (70).

We shall first show that $\sigma_1 \in F^*_b$, and then show that $\Pi(\sigma_1) \geq \Pi(\sigma_0)$.

In the first step, we shall show that $\sigma_1 \in F^*_b$, that is, σ_1 satisfies all constraints that define F^*_b. Because solution σ_1 deviates from σ_0 only by the value of $p(H,L)$, we only have to verify the constraints that are relevant to $p(H,L)$: the binding (55d), (55f), $r \geq p(H,L)$ and $c^H/h < p(H,L) \leq c^L/[l(1-h)]$.

From Lemma 20, it follows $p_0(H,L) < p_1(H,L) \leq p_0(L,L)$. We shall use this ordering repeatedly in this proof.

Solution σ_1 satisfies (55d) trivially, because $(q_0,p_1)(H,L)$ satisfies (70), which is the binding form of (55d) at solution σ_1.

We now show that σ_1 satisfies (55f), which has the following form:

$$\Gamma^{\frac{HL}{LL}}\big((q_0,p_1)(H,L)\big) \geq \Gamma^{\frac{HL}{LL}}\big((q_0,p_0)(L,L)\big). \tag{71}$$

From Lemma 6, $\Gamma^{\frac{HL}{LL}}(q,p)$ may, or may not, be monotonic increasing in p, depending on the value of $(1-l)^2 - (1-h)$. We consider two cases of the model parameter values: $(1-l)^2 - (1-h) < 0$ and $(1-l)^2 - (1-h) \geq 0$.

If $(1-l)^2 - (1-h) < 0$, from Lemma 6, $\Gamma^{\frac{HL}{LL}}(q,p)$ is increasing in p for all $p \in P_1 \stackrel{\text{def}}{=} \big(c^H/h, c^L/[l(1-l)]\big)$, and is decreasing for $p \in P_2 \stackrel{\text{def}}{=} \big[c^L/[l(1-l)], c^L/[l(1-h)]\big]$.

Case (1), $p_1(H,L) \in P_2$. Because $c^L/[l(1-h)] \geq p_0(L,L) \geq p_1(H,L)$, it follows $p_0(L,L) \in P_2$. Inequality (71) follows from the facts that $q_0(H,L) \geq q_0(L,L)$, $p_1(H,L) \leq p_0(L,L)$, and $\Gamma^{\frac{HL}{LL}}(q,p)$ is increasing in q, but decreasing in p for $p \in P_2$ (by Lemma 6).

Case (2), $p_1(H,L) \in P_1$. Because $p_0(H,L) < p_1(H,L)$, we have $p_0(H,L) \in P_1$ as well. Because $\Gamma^{\frac{HL}{LL}}(q,p)$ is increasing in p for $p \in P_1$ (see Lemma 6) and $p_0(H,L) < p_1(H,L)$, it follows:

$$\Gamma^{\frac{HL}{LL}}\big((q_0,p_1)(H,L)\big) \geq \Gamma^{\frac{HL}{LL}}\big((q_0,p_0)(H,L)\big). \tag{72}$$

Solution σ_0 satisfies constraint (55f). Therefore,

$$\Gamma^{\frac{HL}{LL}}\big((q_0,p_0)(H,L)\big) \geq \Gamma^{\frac{HL}{LL}}\big((q_0,p_0)(L,L)\big). \tag{73}$$

Inequality (71) follows from (72) and (73).

If $(1-l)^2-(1-h) \geq 0$, from Lemma 6, $\Gamma^{\frac{HL}{LL}}(q,p)$ is increasing in p for all $p > c^H/h$. The proof of constraint (55f) at σ_1 is identical to that for case (2) under $(1-l)^2-(1-h) < 0$.

We now show that σ_1 satisfies $p_1(H,L) \leq r$ and $c^H/h < p_1(H,L) \leq c^L/[l(1-h)]$. Inequalities $p_1(H,L) \leq r$ and $p_1(H,L) \leq c^L/[l(1-h)]$ follow from the ordering $p_1(H,L) \leq p_0(L,L)$ and the fact that solution σ_0 satisfies $p_0(L,L) \leq r$ and $p_0(L,L) \leq c^L/[l(1-h)]$. Inequality $p_1(H,L) > c^H/h$ follows from $p_1(H,L) > p_0(H,L)$ and $p_0(H,L) > c^H/h$.

In summary of the first step, we have $\sigma_1 \in F_b^*$.

In the second step, we show that the objective value at σ_1, $\Pi(\sigma_1) \geq \Pi(\sigma_0)$. Because solutions σ_1 and σ_0 have the same $(q_0,p_0)(H,H)$ and $(q_0,p_0)(L,L)$, the two solutions obtain the same values for (55a) and (55c), respectively. To prove $\Pi(\sigma_1) \geq \Pi(\sigma_0)$, it is sufficient to show that σ_1 attains the same or better value for (55b) than σ_0, that is,

$$\tau^{HL}\big((q_0,p_1)(H,L)\big) \geq \tau^{HL}\big((q_0,p_0)(H,L)\big). \tag{74}$$

From the first step of the proof, solution σ_1 is feasible, and thus $c^H/h < p_1(H,L) \leq c^L/[l(1-h)]$. We shall consider two cases: (1) $c^H/h < p_1(H,L) < c^L/[l(1-h)]$, and (2) $p_1(H,L) = c^L/[l(1-h)]$.

Case (1), $c^H/h < p_1(H,L) < c^L/[l(1-h)]$. Note that $c^H/h < p_0(H,L) < p_1(H,L) < c^L/[l(1-h)]$. From Lemma 1, solutions σ_1 and σ_0 generate the same production assignments: $(z_1^*, z_2^*)(H,L) = (q_0(H,L), 0)$. Therefore, (55b) is the same at solutions σ_1 and σ_0:

$$\tau^{HL}\big((q_0,p_1)(H,L)\big) = \tau^{HL}\big((q_0,p_0)(H,L)\big). \tag{75}$$

Case (2), $p_1(H,L) = c^L/[l(1-h)]$. First, we show this equality can be true only if $p_0(L,L) = c^L/[l(1-h)]$. This follows from the ordering $p_1(H,L) \leq p_0(L,L)$ and the fact that σ_0 satisfies

$p_0(L,L) \le c^L/[l(1-h)]$. Second, we show $q_0(H,L) = q_0(L,L) = 1$. From its definition, solution σ_1 satisfies equality (70). Because $\Gamma^{\frac{HH}{HL}}(q,p)$ is strictly increasing in p (see Lemma 6), and $p_1(H,L) = p_0(L,L)$, it follows $q_0(H,L) = q_0(L,L)$. From the fact $\sigma_0 \in F^*_{a3}$ satisfies $q_0(L,L) \le 1 \le q_0(H,L)$, it follows $q_0(H,L) = q_0(L,L) = 1$ must be true. Next, we shall compare $\tau^{HL}\big((q_0,p_1)(H,L)\big)$ at σ_1 and $\tau^{HL}\big((q_0,p_0)(H,L)\big)$ at σ_0. From Lemma 1, under $p_1(H,L) = c^L/[l(1-h)]$ and $q_0(H,L) = 1$, the production assignments at solution σ_1 are $(z_1^*, z_2^*)(H,L) = (1,1)$. Therefore,

$$\tau^{HL}\big((q_0,p_1)(H,L)\big) = 2(\alpha^H\alpha^L)\big[(hr - c^H) + (l(1-h)r - c^L)\big]. \tag{76}$$

Under $p_0(H,L) < p_1(H,L) = c^L/[l(1-h)]$, the production assignments at solution σ_0 are $(z_1^*, z_2^*)(H,L) = (1,0)$. Therefore,

$$\tau^{HL}\big((q_0,p_0)(H,L)\big) = 2(\alpha^H\alpha^L)(hr - c^H). \tag{77}$$

The difference between (76) and (77) is

$$\tau^{HL}\big((q_0,p_1)(H,L)\big) - \tau^{HL}\big((q_0,p_0)(H,L)\big) = 2(\alpha^H\alpha^L)\big[l(1-h)r - c^L\big]. \tag{78}$$

(78) must be positive, because $r \ge c^L/[l(1-h)]$, which follows from $p_0(L,L) = c^L/[l(1-h)]$ and the fact that the feasible σ_0 must satisfy $p_0(L,L) \le r$.

Therefore, the objective $\Pi(\sigma_1) \ge \Pi(\sigma_0)$. ■

Lemma 17 (**Region F^*_{a4}**). *Define solution* $\sigma_0 \overset{def}{=} \big((q_0,p_0)(H,H),(q_0,p_0)(H,L),(q_0,p_0)(L,L)\big)$. *Suppose* $\sigma_0 \in F^*_{a4}$. *Define* $\sigma_1 \overset{def}{=} \big((q_0,p_0)(H,H),(q_0,p_0)(H,L),(q_0,p_1)(L,L)\big)$. *Then, there exists a* $p_1(L,L) < p_0(L,L)$, *such that* (a) $\sigma_1 \in F^*_{a4}$, *and* (b) $\Pi(\sigma_1) > \Pi(\sigma_0)$.

Proof. **First**, we show there exists a $p_1(L,L) < p_0(L,L)$ such that $\sigma_1 \in F^*_{a4}$. We have to check whether or not σ_1 satisfies the constraints for F^*_{a4} that are relevant to $p(L,L)$: the non-binding (55d), (55e), (55f), (55g), and $r \ge p(L,L) > c^L/[l(1-h)]$. Because (55d) is non-binding at σ_0, and $p_0(L,L) > c^L/[l(1-h)]$, from the continuity of $\Gamma^{\frac{HH}{HL}}(q,p)$ and the fact $\Gamma^{\frac{HH}{HL}}(q,p)$ is increasing in p, it follows that there exists a $p_1(L,L) < p_0(L,L)$ such that $\Gamma^{\frac{HH}{HL}}\big((q_0,p_0)(H,L)\big) < \Gamma^{\frac{HH}{HL}}\big((q_0,p_1)(L,L)\big)$, and $p_1(L,L) > c^L/[l(1-h)]$. Therefore, σ_1 satisfies the non-binding (55d). Because $\Gamma^{\frac{HH}{HL}}(q,p)$ and $\Gamma^{\frac{HL}{LL}}(q,p)$ are increasing in p (see Lemma 6) and $p_1(L,L) < p_0(L,L)$, (55e), (55f) and (55g) continue to be true at σ_1. Therefore, σ_1 satisfies all constraints that define F^*_{a4}.

Second, we show that the objective value at σ_1, $\Pi(\sigma_1) > \Pi(\sigma_0)$ strict. The only difference between σ_0 and σ_1 is the value of $p(L,L)$. Therefore, only (55c) changes as we move from σ_0 to

σ_1. To prove $\Pi(\sigma_1) > \Pi(\sigma_0)$, it is sufficient to show that σ_1 attains a strictly better value for (55c) than σ_0, that is,

$$\tau^{LL}\big((q_0, p_1)(L, L)\big) > \tau^{LL}\big((q_0, p_0)(L, L)\big). \tag{79}$$

Because $q(L, L) = q_0(L, L)$ and $p(L, L) > c^L/[l(1-h)]$ at both solutions, the production assignments are identical at σ_1 and σ_0: $(z_1^*, z_2^*)(L, L) = (q_0(L, L), q_0(L, L))$ (see Lemma 1). Therefore, σ_1 and σ_0 generate the same market profit (expected revenue minus production costs). But, because $\Gamma^{\frac{HH}{HL}}(q, p)$ and $\Gamma^{\frac{HL}{LL}}(q, p)$ are both strictly increasing in penalty p (see Lemma 6) and $p_1(L, L) < p_0(L, L)$, at σ_1 information costs are strictly smaller. Therefore, (79) is true, and thus $\Pi(\sigma_1) > \Pi(\sigma_0)$. ■

Lemma 18 (**Region $\boldsymbol{F^*_{a5}}$**). *Set F^*_{a5} is empty.*

Proof. We prove by contradiction. Assume there exists a solution $\sigma \in F^*_{a5}$. Then, σ must satisfy (55d). From the facts that $\Gamma^{\frac{HH}{HL}}(q, p)$ is strictly increasing in both q and p (see Lemma 6), and that $\sigma \in F^*_{a5}$ satisfies the orderings $q(L, L) \le q(H, L)$ and $p(L, L) < p(H, L)$, it follows $\Gamma^{\frac{HH}{HL}}\big((q, p)(H, L)\big) > \Gamma^{\frac{HH}{HL}}\big((q, p)(L, L)\big)$. This violates (55d). Therefore, $\sigma \notin F^*_{a5}$. This contradicts the assumption. ■

Lemma 19 (**Region $\boldsymbol{F^*_{a6}}$**). *Define solution* $\sigma_0 \overset{def}{=} \big((q_0, p_0)(H, H), (q_0, p_0)(H, L), (q_0, p_0)(L, L)\big)$. *Suppose* $\sigma_0 \in F^*_{a6}$. *Define* $\sigma_1 \overset{def}{=} \big((q_0, p_0)(H, H), (q_0, p_1)(H, L), (q_0, p_0)(L, L)\big)$, *in which* $p_1(H, L)$ *satisfies*

$$\Gamma^{\frac{HH}{HL}}\big((q_0, p_1)(H, L)\big) = \Gamma^{\frac{HH}{HL}}\big((q_0, p_0)(L, L)\big). \tag{80}$$

Then, the following statements are true: (a) $\sigma_1 \in F^*_b$, *and* (b) $\Pi(\sigma_1) = \Pi(\sigma_0)$.

Proof. Because $\Gamma^{\frac{HH}{HL}}(q, p)$ is strictly monotonic in p and $\Gamma^{\frac{HH}{HL}}(q, p) \in \mathbb{R}^+$, there exist a unique solution to (80).

In the first step, we show that $\sigma_1 \in F^*_{a6}$, that is, σ_1 satisfies all constraints that defines F^*_{a6}. Because solution σ_1 differs from σ_0 only by the value of $p(H, L)$, we only have to verify the constraints that are relevant to $p(H, L)$: the binding (55d), (55f) and $r \ge p(H, L) > c^L/[l(1-h)]$.

From Lemma 20, it follows $p_0(H, L) < p_1(H, L) \le p_0(L, L)$. We shall use this ordering repeatedly in this proof.

Solution σ_1 satisfies the binding (55d) trivially, because $(q_0, p_1)(H, L)$ satisfies (80), which is the binding form of (55d) at solution σ_1.

We show that σ_1 satisfies (55f), which has the following form:

$$\Gamma^{\frac{HL}{LL}}\big((q_0, p_1)(H, L)\big) \ge \Gamma^{\frac{HL}{LL}}\big((q_0, p_0)(L, L)\big). \tag{81}$$

Because $p_0(H,L) < p_1(H,L)$ and $\sigma_0 \in F^*_{a6}$ satisfies $p_0(H,L) > c^L/[l(1-h)]$, we have $p_1(H,L) > c^L/[l(1-h)]$ as well. Because $\Gamma^{\frac{HL}{LL}}(q,p)$ is increasing in p for $p > c^L/[l(1-h)]$ (see Lemma 6) and $p_0(H,L) < p_1(H,L)$, it follows:

$$\Gamma^{\frac{HL}{LL}}\big((q_0,p_1)(H,L)\big) \geq \Gamma^{\frac{HL}{LL}}\big((q_0,p_0)(H,L)\big). \tag{82}$$

Solution σ_0 satisfies constraint (55f). Therefore,

$$\Gamma^{\frac{HL}{LL}}\big((q_0,p_0)(H,L)\big) \geq \Gamma^{\frac{HL}{LL}}\big((q_0,p_0)(L,L)\big). \tag{83}$$

Inequality (81) follows from (82) and (83). Therefore, σ_1 satisfies (55f).

We now show that σ_1 satisfies $r \geq p_1(H,L) > c^L/[l(1-h)]$. Inequality $p_1(H,L) > c^L/[l(1-h)]$ follows from the fact that $\sigma_0 \in F^*_{a6}$ satisfies $p_0(H,L) > c^L/[l(1-h)]$ and the ordering $p_1(H,L) > p_0(H,L)$. $r \geq p_1(H,L)$ follows from the fact that $\sigma_0 \in F^*_{a6}$ satisfies $r \geq p_0(L,L)$ and the ordering $p_1(H,L) \leq p_0(L,L)$.

In summary of the first step, we have $\sigma_1 \in F^*_b$.

In the second step, we show that the objective value at σ_1, $\Pi(\sigma_1) = \Pi(\sigma_0)$. Because solutions σ_1 and σ_0 have the same $(q_0,p_0)(H,H)$ and $(q_0,p_0)(L,L)$, the two solutions obtain the same values for (55a) and (55c), respectively. To prove $\Pi(\sigma_1) = \Pi(\sigma_0)$, it is sufficient to show that σ_1 attains the same value for (55b) as σ_0, that is,

$$\tau^{HL}\big((q_0,p_1)(H,L)\big) = \tau^{HL}\big((q_0,p_0)(H,L)\big). \tag{84}$$

Because $q(H,L) = q_0(H,L)$ and $p(H,L) > c^L/[l(1-h)]$ at both solutions, the production assignments are identical at σ_1 and σ_0: $(z_1^*, z_2^*)(H,L) = (q_0(H,L), q_0(H,L))$ (see Lemma 1). Therefore, σ_1 and σ_0 generate the same market profit (expected revenue minus production costs). It follows that (84) is true, and thus $\Pi(\sigma_1) = \Pi(\sigma_0)$. ∎

Lemma 20. *Define solution* $\sigma_0 \overset{def}{=} \big((q_0,p_0)(H,H), (q_0,p_0)(H,L), (q_0,p_0)(L,L)\big)$. *Suppose* $\sigma_0 \in F^*_{a3} \cup F^*_{a4} \cup F^*_{a5} \cup F^*_{a6}$. *Define* $\sigma_1 \overset{def}{=} \big\{(q_0,p_0)(H,H), (q_0,p_1)(H,L), (q_0,p_0)(L,L)\big\}$, *in which* $p_1(H,L)$ *satisfies*

$$\Gamma^{\frac{HH}{HL}}\big((q_0,p_1)(H,L)\big) = \Gamma^{\frac{HH}{HL}}\big((q_0,p_0)(L,L)\big). \tag{85}$$

Then, we must have $p_0(H,L) < p_1(H,L) \leq p_0(L,L)$.

Proof. Because $\Gamma^{\frac{HH}{HL}}(q,p)$ is strictly monotonic in p and $\Gamma^{\frac{HH}{HL}}(q,p) \in \mathbb{R}^+$, there exist a unique solution to (85).

First, $p_1(H,L) \le p_0(L,L)$ must be true. Because $\Gamma^{HH}_{HL}(q,p)$ is monotonic increasing in q and p (see Lemma 6), and $\sigma_0 \in F^*_{a3} \cup F^*_{a4} \cup F^*_{a5} \cup F^*_{a6}$ implies $q_0(H,L) \ge q_0(L,L)$, for $p_1(H,L)$ to satisfy equality (85) we must have $p_1(H,L) \le p_0(L,L)$.

Second, $p_0(H,L) < p_1(H,L)$ must be true. Because σ_0 satisfies $\Gamma^{HH}_{HL}\big((q_0,p_0)(H,L)\big) < \Gamma^{HH}_{HL}\big((q_0,p_0)(L,L)\big)$, and $p_1(H,L)$ satisfies (85), it follows $\Gamma^{HH}_{HL}\big((q_0,p_0)(H,L)\big) < \Gamma^{HH}_{HL}\big((q_0,p_1)(H,L)\big)$. Ordering $p_0(H,L) < p_1(H,L)$ follows from the monotonicity property that $\Gamma^{HH}_{HL}(q,p)$ is increasing in p. ■

Proof of Proposition 3. (17) equals (11) minus (15). First, (11a) equals (15a). Second, (17a) equals (11b) less (15b). Third, (17b) equals (11c) less (15c). ■

Proof of Proposition 4. First, we show (17a) is positive. (17a) equals (11b) less (15b). (11b) can be written as

$$\max\Big\{0, 2\left(\alpha^H\alpha^L\right)\psi^{HL} - (\alpha^H)^2\phi\Big\}, \tag{86}$$

and (15b) can be written as

$$\max\Big\{0, 2\left(\alpha^H\alpha^L\right)\psi^{HL} - 2(\alpha^H)^2\phi\Big\}. \tag{87}$$

Because $2\left(\alpha^H\alpha^L\right)\psi^{HL} - (\alpha^H)^2\phi > 2\left(\alpha^H\alpha^L\right)\psi^{HL} - 2(\alpha^H)^2\phi$, it follows that (11b) is greater than (15b). Therefore, (17a) is positive.

Second, we show (17b) is negative. (17b) equals (11c) less (15c). One can verify that (11c) can be equivalently written as the maximum among the three values:

$$\max\Big\{0, (\alpha^L)^2\psi^L - [1-(\alpha^L)^2]\phi, (\alpha^L)^2(\psi^L+\psi^{LL}) - [1-(\alpha^L)^2]\omega\Big\}. \tag{88}$$

Similarly, (15c) can be written as

$$\max\Big\{0, \left(\alpha^L\right)^2\psi^L - \left(\alpha^H\alpha^L\right)\phi, \left(\alpha^L\right)^2(\psi^L+\psi^{LL}) - 2\left(\alpha^H\alpha^L\right)\phi\Big\}. \tag{89}$$

To prove (11c) is less than (15c), we shall show that the three items in (88), respectively, are smaller than the corresponding items in (89). It is trivial that the first item in (88) and (89) are equals. We compare the second item in (88) and (89). From the fact that $\alpha^H + \alpha^L = 1$, it follows $1-(\alpha^L)^2 = (\alpha^H)^2 + 2(\alpha^H\alpha^L) > \alpha^H\alpha^L$. Therefore,

$$(\alpha^L)^2\psi^L - [1-(\alpha^L)^2]\phi < \left(\alpha^L\right)^2\psi^L - \left(\alpha^H\alpha^L\right)\phi \tag{90}$$

We now compare the third item in (88) and (89). Because $1-(\alpha^L)^2 > 2(\alpha^H\alpha^L)$ and $\omega > \phi$ (see Lemma 4), it follows

$$(\alpha^L)^2(\psi^L+\psi^{LL}) - [1-(\alpha^L)^2]\omega < \left(\alpha^L\right)^2(\psi^L+\psi^{LL}) - 2\left(\alpha^H\alpha^L\right)\phi. \tag{91}$$

Therefore, (17b) is negative. ■

Proof of Proposition 5. We define the set of r of the lemma to be $A \stackrel{\text{def}}{=} \{r : r \geq \tilde{r}^{HL}, r \geq \min\{r^L, r^{LL}\}\}$. We dissect the set A into the following two subsets:

$$A_1 \stackrel{\text{def}}{=} A \cap \{r : r \geq r^{LL}\} \tag{92a}$$

$$A_2 \stackrel{\text{def}}{=} A \cap \{r : r < r^{LL}\} \tag{92b}$$

First, we prove the lemma for all $r \in A_1$. In the definition of A_1, the inequality $r \geq r^{LL}$ implies $r \geq \min\{r^L, r^{LL}\}$. Therefore, $A_1 = \{r : r \geq \tilde{r}^{HL}, r \geq r^{LL}\}$. From (17a), for $r \geq \tilde{r}^{HL}$ the PSP value from the (H,L) and (L,H) types is $(\alpha^H)^2\phi$. From (17b), for $r \geq r^{LL}$ the PSP value from the (L,L) type is:

$$\begin{aligned}&\left\{(\alpha^L)^2(\psi^L + \psi^{LL}) - [1-(\alpha^L)^2]\omega\right\}\\&-\left[\left(\alpha^L\right)^2\psi^L - \left(\alpha^H\alpha^L\right)\phi\right]^+ - \left[\left(\alpha^L\right)^2\psi^{LL} - \left(\alpha^L\alpha^H\right)\phi\right]^+.\end{aligned} \tag{93}$$

Because $\tilde{r}^{HL} > \tilde{r}^{LL} > \tilde{r}^L$ (see Lemma 7), condition $r \geq \tilde{r}^{HL}$ implies $r > \tilde{r}^{LL} > \tilde{r}^L$. It follows from these inequalities and the definitions of $\tilde{r}^{LL}$ and $\tilde{r}^L$ that $\left(\alpha^L\right)^2\psi^L - \left(\alpha^H\alpha^L\right)\phi > 0$ and $\left(\alpha^L\right)^2\psi^{LL} - \left(\alpha^L\alpha^H\right)\phi > 0$. We apply these inequalities to the PSP value from the (L,L) type (93), obtaining

$$-[1-(\alpha^L)^2]\omega + \left(\alpha^H\alpha^L\right)\phi + \left(\alpha^L\alpha^H\right)\phi. \tag{94}$$

We sum the PSP values from the (H,L) and (L,H) types and from the (L,L) type, obtaining the total PSP value: $-[1-(\alpha^L)^2](\omega - \phi)$. It follows from Lemma 4 that the total PSP value is negative.

Second, we proof the lemma for all $r \in A_2$. From its definition, A_2 is non-empty only if $r^{LL} > r^L$. Therefore, $A_2 = \{r : r \geq \tilde{r}^{HL}, r^L \leq r < r^{LL}\}$. From (17b), for $r^L \leq r < r^{LL}$ the PSP value from the (L,L) type is:

$$\left\{(\alpha^L)^2\psi^L - [1-(\alpha^L)^2]\phi\right\} - \left[\left(\alpha^L\right)^2\psi^L - \left(\alpha^H\alpha^L\right)\phi\right]^+ - \left[\left(\alpha^L\right)^2\psi^{LL} - \left(\alpha^L\alpha^H\right)\phi\right]^+. \tag{95}$$

Because $\left(\alpha^L\right)^2\psi^{LL} - \left(\alpha^L\alpha^H\right)\phi > 0$ and $\left(\alpha^L\right)^2\psi^{LL} - \left(\alpha^L\alpha^H\right)\phi > 0$, the PSP value from (L,L) (96) equals

$$-[1-(\alpha^L)^2]\phi + \left(\alpha^H\alpha^L\right)\phi - \left[\left(\alpha^L\right)^2\psi^{LL} - \left(\alpha^L\alpha^H\right)\phi\right]. \tag{96}$$

Adding this formula and the PSP value from the (H,L) and (L,H) types, $(\alpha^H)^2\phi$, we obtain the total PSP values to be $-\left(\alpha^L\right)^2\psi^{LL}$. Because $r > \tilde{r}^{LL}$, we have $\left(\alpha^L\right)^2\psi^{LL} > 0$. The result follows. ■

Proof of Proposition 6. We shall first prove the monotonicity of the PSP values by the PSP-type in h and l, which is illustrated in Figure 4. The expressions for these PSP values are provided in Tables 1 and 2. Using the monotonicity of the PSP values by type, we shall then prove the monotonicity of the total PSP value.

First, we prove that the PSP value from the (H,L) and (L,H) types increases in h (and decreases in l) for $r \geq \tilde{r}^{HL}$, and decreases in h (and increases in l) for $r^{HL} \leq r < \tilde{r}^{HL}$. Using their expressions, one can verify that ϕ increases in h and decreases in l, and ψ^{HL} decreases in h and increases in l. We apply these properties to the expressions for the PSP value in Table 1. The result follows.

Second, we prove that the PSP value from the (L,L) type decreases in h (and increases in l) for $r \geq r^{LL}$, and increases in h (and decreases in l) for $r < r^{LL}$. From Table 2, the result would follow if ψ^L and ψ^{LL} are constant in h and increases in l; and ϕ, ω and $\omega - \phi$ increase in h and decrease in l. Using the expressions for ψ^L and ψ^{LL}, one can verify that their monotonicity stated above is true. The monotonicity of ω and $\omega - \phi$ remains to be proved.

We now prove that ω increases in h and decreases in l. From (10a) and (10c), the expression for ω can be one of three cases, depending on the value of $\tilde{p}$: $\tilde{p} = \frac{c^L}{l(1-l)}$, r (only if $(1-l)^2 - (1-h) < 0$ and $\frac{c^L}{l(1-l)} \leq r < \frac{c^L}{l(1-h)}$) or $\frac{c^L}{l(1-h)}$. Applying $\tilde{p}$ to (21), we obtain the expression for ω:

$$\omega = \begin{cases} (h-l)\frac{c^L}{l(1-l)} - (c^H - c^L) & \tilde{p} = \frac{c^L}{l(1-l)} \\ [(1-l)^2 - (1-h)]r - c^H + 2c^L & \tilde{p} = r \\ (h-l)(1-l)\frac{c^L}{l(1-h)} - (c^H - c^L) & \tilde{p} = \frac{c^L}{l(1-h)} \end{cases}. \tag{97}$$

One can verify without much difficulty that ω increases in h and decreases in l in all three cases.

From there, we derive the expression for $\omega - \phi$, where $\phi = \frac{h}{l}c^L - c^H$ (see (10b)):

$$\omega - \phi = \begin{cases} c^L\left(1 - \frac{1-h}{1-l}\right) & \tilde{p} = \frac{c^L}{l(1-l)} \\ [(1-l)^2 - (1-h)]r + 2c^L - \frac{h}{l}c^L & \tilde{p} = r \\ \frac{c^L}{l}\frac{(h-l)^2}{1-h} & \tilde{p} = \frac{c^L}{l(1-h)} \end{cases}. \tag{98}$$

In the cases $\tilde{p} = \frac{c^L}{l(1-l)}$ and $\tilde{p} = \frac{c^L}{l(1-h)}$, one can be verified the monotonicity without difficulty. We focus on the case $\tilde{p} = r$, which is used under $(1-l)^2 - (1-h) < 0$ and $\frac{c^L}{l(1-l)} \leq r < \frac{c^L}{l(1-h)}$. The first order derivative of $\omega - \phi$ with respective to h, obtaining $\frac{\partial}{\partial h}(\omega - \phi) = r - \frac{c^L}{l}$. Because $r \geq \frac{c^L}{l(1-l)} > \frac{c^L}{l}$, we have $\frac{\partial}{\partial h}(\omega - \phi) > 0$. The first order with respect to l is $\frac{\partial}{\partial l}(\omega - \phi) = -2(1-l)\left[r - \frac{hc^L}{2(1-l)l^2}\right]$. From $r \geq \frac{c^L}{l(1-l)}$, it follows $\frac{\partial}{\partial l}(\omega - \phi) \leq -\frac{c^L}{l}\left(2 - \frac{h}{l}\right)$. From $(1-l)^2 - (1-h) < 0$, it follows that $2 - \frac{h}{l} > 0$. Therefore, $\frac{\partial}{\partial l}(\omega - \phi) < 0$.

Lastly, we prove the monotonicity of the total PSP value. The results for $\tilde{r}^{HL} \leq r < r^{LL}$ and $r^{HL} \leq r < \tilde{r}^{HL}$ follow immediately from the monotonicity of the PSP values by type. For $r \geq r^{LL}$, the total PSP value equals $-[1 - (\alpha^H)^2](\omega - \phi)$. The result follows from the monotonicity of $\omega - \phi$. ■